Math 50 Stat Inf: Homework 7

due Wed Feb 22

First few are exam review so focus on them before Monday.

5.3 : 11,

12 (both review, but very important)

5.5 : 1 (review).

A. Samples $\{y_i\}$, $i = 1 \ldots n$ are taken from a normal with parameters μ and σ. Let's do Bayesian inference on the variance. This is also relevant practise for the exam. [No matlab].

i) Write down the likelihood function $L(\mu, \sigma|y)$. This is exactly what you plotted in HW5. Replace any constant (parameter-independent) terms by an overall constant; we will not bother keeping track of such constants.

ii) Assuming a uniform (improper) prior on (μ, σ), write the posterior $g(\mu, \sigma|y)$.

iii) Find the *marginal* posterior pdf on σ alone: $g(\sigma|y) = \int_{-\infty}^{\infty} g(\mu, \sigma|y) d\mu$. This corresponds to inference on σ with μ unknown. [Hint: This is the trickiest part. You may leave an unknown constant σ-independent factor out front. You'll need the Gaussian integral $\int_{-\infty}^{\infty} e^{-(a^2x^2/2)-bx}dx = (\sqrt{2\pi}/a)e^{b^2/2a^2}$, which can be proven simply by completing the square]

iv) Take $(d/d\sigma) \ln g(\sigma|y)$ to find the *maximum a-posteriori* estimate (*i.e.* peak of the posterior pdf) for σ. It should be familiar—where have you seen it? Note how the skewness of the $L(\mu, \sigma)$ peak you plotted in HW5 made the peak of the marginal on σ move from the ML joint estimate of (μ, σ).

B. Bayesian prediction for bus waiting time. [No matlab].

i) Solve part D) of Worksheet 2/13/06 to get the predictive pdf $h(y|y_1)$ of a future sample drawn from the bus wait-time pdf. As you can see the Bayesian method is to follow the laws of probability by averaging (marginalizing) over all the possible biases θ weighted according to their posterior pdf deduced from the data y_1. Have you seen the resulting pdf recently?

ii) Write down the predictive pdf for y following the 'frequentist' (non-Bayesian) approach of finding a single ML estimate then using that to predict.

iii) Practical consequences. You start taking a new bus route with unknown arrival rate θ (use a uninformative prior). Today (day 1) you wait $y_1 = 10$ mins. Let Y be tomorrow's wait time. Compute the Bayesian's and frequentist's predictions for $p(Y \geq 60\text{mins})$. [Hint: you should be able to integrate both on paper—think about $\tan^{-1}$.]

iv) Discuss why they differ. Which would you trust?

6.2 : 1 (careful whether one- or two-sided),

2.

6.3 : 1,

6.

6.4 : 1,
3,
7.