

Chaos, Fractals, and Art

This project was aimed at pursuing research in the relatively new field of chaos, fractals, and art. The goals were to explore the idea of using fractals to generate art, to study fractal dimensions of art and natural images that possess self-similarity, and to use an Iterated Function System (IFS) to create fractal art.

Though the term *fractal* was not coined until the 1970's, "art has always been fractal. The science of chaos is helping to newly define an aesthetic that has always lain beneath the changing artistic ideas of different periods, cultures, and schools." Long before the formal study of chaos arose, such fractal images existed and were captured in various art forms, such as "The Great Wave off Kanagawa" by Katsushika Hokusai shown in Fig 1. This Japanese painter managed to capture "all these aspects of the fractal world we're about to enter" as early as the eighteenth century in an image that shows clear self-similarity.ⁱⁱ



FIG. 1: "The Great Wave off Kanagawa" by Katsushika Hokusai, ca. 1830-1832ⁱⁱⁱ

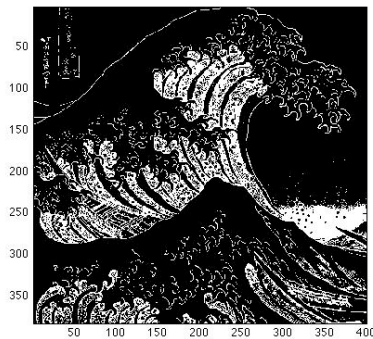


FIG. 2: cropped section of wave in Fig. 1 binarized to input in MATLAB

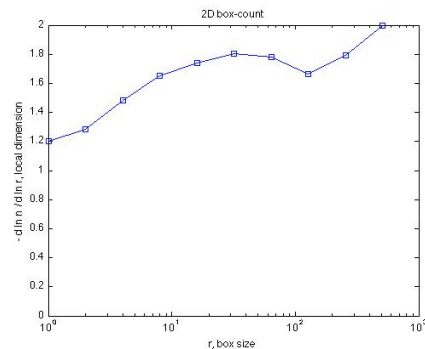


FIG. 3: 2D boxcount slope of fractal outline in Fig 2.

After selecting the rectangular section of Fig. 1 that exhibits the fractal properties of self-similarity, MATLAB was used to create a binarized version of the image (Fig. 2) where any pixel above a certain threshold of color intensity was passed through as a one (true, shown in white) and the pixels below the threshold were passed through as a zero (false, shown in black). Passing this image through F. Moisy's boxcount package, boxcount.m, (in Appendix A)

outputted Fig. 3, the two-dimensional boxcounting slope of the fractal image shown in Fig 2. Using this code, the fractal dimension of the wave in Hokusai's painting was calculated to be 1.7264 ± 0.093 , clearly in the chaotic realm since the closest space-filling dimension is two.

Continuing on the path of finding fractals in old works of art, one might come across abstract artist Jackson Pollock, famous for his unique style. Though the art and math world has seen much controversy over the claims that Pollock's work exhibits chaotic properties, scientist Richard P. Taylor found one way to prove the existence of a fractal dimension in Pollock's "Blue Poles: Number II" shown in Fig. 4.

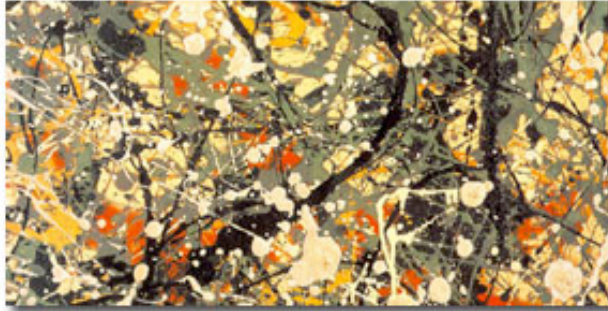


FIG. 4: "Blue Poles: Number II" by Jackson Pollock, 1952.

Taylor "started the investigation by scanning a Pollock painting into the computer; he then covered it with a computer-generated mesh of identical squares. By analyzing, which squares were occupied by the painted pattern and which were empty, he was able to calculate the statistical qualities of the pattern. And by reducing the square size he was able to look at the pattern at what amounts to a finer magnification.^{iv}" MATLAB was used to confirm Taylor's findings. Fig. 5 was created to simulate the test Taylor created by finding a threshold that separated light strokes from dark; passing this through boxcount.m revealed a fractal dimension of roughly 1.8.

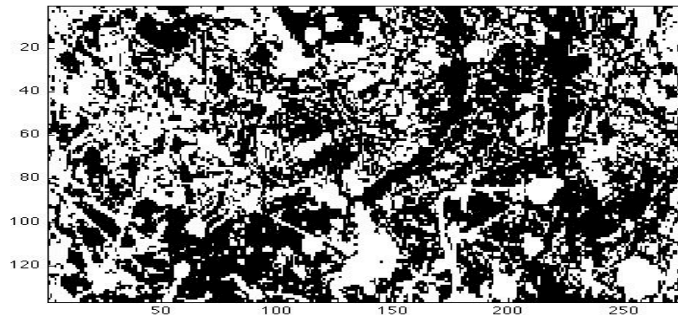


FIG. 5: binarized version of Fig. 4

The overlapping field of art and chaos continued to evolve past the fractal properties seen in various works in art history; the field has been supported by continuous exploration from both scientists and artists alike. For example, "Gottfried Mayer-Kress of the Santa Fe Institute is one of the world's experts on nonlinear systems" who collaborated with graphic designer Jenifer Bacon. Together they studied an equation that is "used to model the behavior of subatomic particles," and developed a "portrait of [this] nonlinear equation." The image depicts the immense range of behavior of a system that exhibits sensitive dependence on initial conditions

(electronic copy of image not found. See Briggs' text for picture).^v However, while Briggs' book exemplifies a product of collaboration between the scientific and artistic worlds, the author provides no background on the actual math involved.

Artist Jenifer Bacon supplied an article entitled "Collaboration" from the magazine *IRIS Universe* (in Appendix B) that describes a bit more of both the scientific and artistic processes involved in the Mayer-Kress and Bacon collaboration. Included in the article is an abstract landscape that Bacon calls "Canyons and Mesas," Fig. 6. Interestingly though, the image was "originally conceived in the context of the 'beam-beam interactions' of interacting storage rings in particle physics."^{vi}

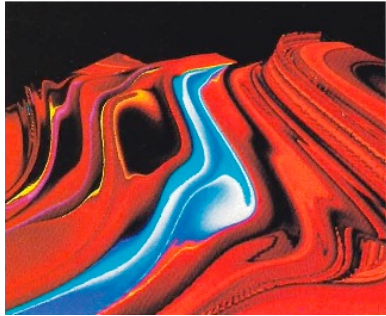


FIG. 6: Bacon's "Canyons and Mesas" Ca.1990
Product of collaboration with Dr. Mayer-Kress.

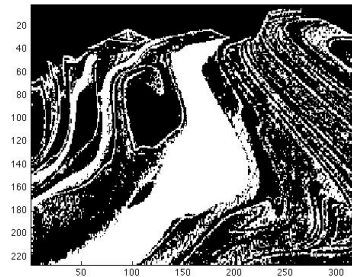


FIG. 7: Binarized version of Fig. 6 for use in MATLAB boxcount.m

Again using a binarized version of the image as the input (Fig. 7), MATLAB was used to compute numerical values that characterize the chaotic properties of "Canyons and Mesas." Fig. 8 shows a graph of the number of boxes N as a function of the box size r for the fractal input (blue) as compared to the space-filling box count shown in red. The slope of the blue graph is displayed in Fig. 9, where the plateau region of the boxcount slope shows the fractal dimension, 1.7926 ± 0.032 .

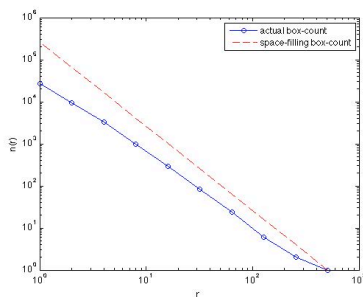


FIG. 8: Number of boxes N as a function of box size r
 $N(r)$ of image in blue. $N(r)$ space-filling dimension in red.

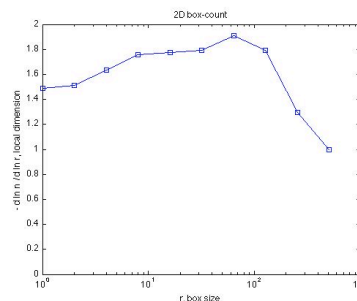


FIG. 9: Slope of $N(r)$. Fractal dimension taken from plateau region.

Furthermore, fractal geometry has appeared in a wide range of works by various artists using different mediums. John Lewis used fractal graphics to create a chaos landscape in his animated short film *Aliens*.^{vii} Photographer Lawrence Hudetz captures "the fractal shapes of chaos," while colleague Joseph Cantrell "describes photographing nature as a process of sensitizing himself to the subtle movements of nature's creative chaos."^{viii} Though the photographers do not create these chaotic images, they capture amazing self-similar patterns found in nature.

An example of one such fractal created by natural means is the glue fractal shown in Fig. 10, which was “formed by pulling two glue-covered acrylic sheets apart. The air travels in the direction the horns are pointing, creating dendritic structures into the glue in an effort to equalize the pressures in the room and between the sheets.”^{ix}

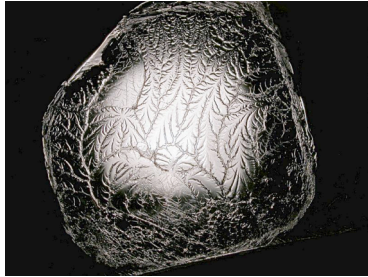


FIG. 10: Naturally created fractal of art patterns formed when pulling apart two glue-covered sheets.

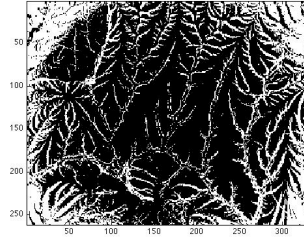


FIG. 11: Binarized center section of Fig. 10.

Analyzing the binarized central piece shown in Fig. 11 revealed a fractal dimension of 1.77. Even without the use of MATLAB, the glue image shows clear patterns of self-similarity in the branching patterns of the air’s path.

Turning from natural to computer-generated images, Clifford Pickover, author of *Computers Pattern Chaos and Beauty*, delves a bit more into the mathematical and computer graphical side of the chaotic art world. Pickover claims, “Not only can computers and graphics be used in counting and measuring, but they also are of enormous help in producing visual art.” He illustrates system behavior of functions such as $z \rightarrow z^7 - 1$ and $z \rightarrow z^2 + \mu$ in graphic images.^x Briggs quotes Pickover on computers and art: “The computer is a tool that lets artists, mathematicians and scientists see unexpected and strange new worlds that they couldn’t have appreciated before. It also lets nonartists participate in what we might call art. Art critics might not call it art, but the works I do are, to me, in the realm of art.”^{xi}

For example, Martin Pflingstl used the software ChaosPro 3.3 to produce a fractal art image from a section of the Mandelbrot Set between $(-0.77016296, -0.11594849i)$ and $(-0.77211609, -0.11448364i)$, called “Galaxy of Galaxies”, shown in Fig. 12.

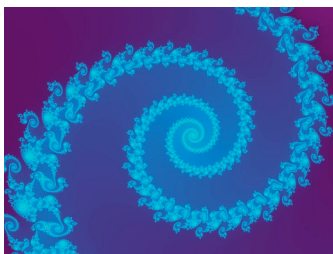


FIG. 12: Mandelbrot Set between $(-0.77016296, -0.11594849i)$ and $(-0.77211609, -0.11448364i)$

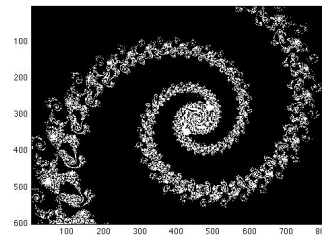


FIG. 13: Binarized version of Fig. 12

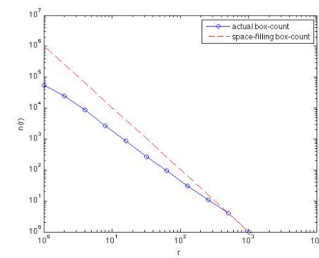


FIG. 14: $N(r)$ of Fig. 13

The discrepancy between the two lines in Fig. 13 confirms the visible fractal properties of “Galaxy of Galaxies.” The fractal dimension was calculated to be 1.6218 ± 0.072 .

Michael Barnsley’s book, *Fractals Everywhere*, presents the equations and variable parameters used in Iterated Function Systems (IFS) to create fractal images in simple computer programs. Equation 1 shows the equation for an IFS of an affine map

$$w_j(x, y) = \begin{bmatrix} a_j & b_j \\ c_j & d_j \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_j \\ f_j \end{bmatrix} = A_j * (x, y) + t_j \quad (1)$$

where $j = 1, 2, 3, 4$, and the probability p of $w_1 + w_2 + w_3 + w_4 = 1$. The values and probabilities used in creating a fractal fern (Fig. 15) are shown in Table I, a fractal tree (Fig. 16) in Table II, and a fractal castle (Fig. 17) in Table III. Tables are taken from Barnsley's book.^{xii} The MATLAB code used can be found in Appendix C.

Table I: IFS code for a Fern.

w	a	b	c	d	e	f	p
1	0	0	0	0.16	0	0	0.01
2	0.85	0.04	-0.04	0.85	0	1.6	0.85
3	0.2	-0.26	0.23	0.22	0	1.6	0.07
4	-0.15	0.28	0.26	0.24	0	.44	0.07

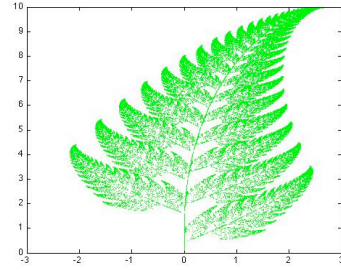


FIG. 15: Fractal Fern

Table II: IFS code for a Fractal Tree

w	a	b	c	d	e	f	p
1	0	0	0	0.5	0	0	0.05
2	0.42	-0.42	0.42	0.42	0	0.2	0.4
3	0.42	0.42	-0.42	0.42	0	0.2	0.4
4	0.1	0	0	0.1	0	0.2	0.15

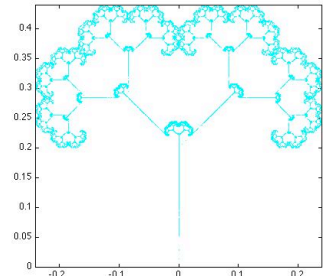


FIG. 16: Fractal Tree

Table III: IFS code for a Fractal Castle.

w	a	b	c	d	e	f	p
1	0.5	0	0	0.5	0	0	0.25
2	0.5	0	0	0.5	2	0	0.25
3	0.5	0	0	0.5	0	1	0.25
4	0.4	0	0	0.4	2	1	0.25

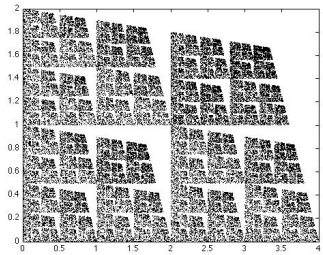


FIG. 17: Fractal Castle

In conclusion, the magnitude of the field of chaos and art is expansive. Though a newly researched theory, ideas of chaos clearly prevailed even before characteristic terms were invented to describe such properties found in various art forms. From paintings, old and new, to image found in nature and those created on the computer artwork has experienced a fair share of influence from chaos theory and its mathematical background. In return, scientists and mathematicians have added to art history by recognizing the artistic dimensions of chaotic, fractal images.

Appendix A

Boxcount.m

```
function [n,r] = boxcount(c,varargin)
%BOXCOUNT Box-Counting of a D-dimensional array (with D=1,2,3).
% [N, R] = BOXCOUNT(C), where C is a D-dimensional array (with D=1,2,3),
% counts the number N of D-dimensional boxes of size R needed to cover
% the nonzero elements of C. The box sizes are powers of two, i.e.,
% R = 1, 2, 4 ... 2^P, where P is the smallest integer such that
% MAX(SIZE(C)) <= 2^P. If the sizes of C over each dimension are smaller
% than 2^P, C is padded with zeros to size 2^P over each dimension (e.g.,
% a 320-by-200 image is padded to 512-by-512). The output vectors N and R
% are of size P+1. For a RGB color image (m-by-n-by-3 array), a summation
% over the 3 RGB planes is done first.
%
% The Box-counting method is useful to determine fractal properties of a
% 1D segment, a 2D image or a 3D array. If C is a fractal set, with
% fractal dimension DF < D, then N scales as R^(-DF). DF is known as the
% Minkowski-Bouligand dimension, or Kolmogorov capacity, or Kolmogorov
% dimension, or simply box-counting dimension.
%
% BOXCOUNT(C,'plot') also shows the log-log plot of N as a function of R
% (if no output argument, this option is selected by default).
%
% BOXCOUNT(C,'slope') also shows the semi-log plot of the local slope
% DF = - dlnN/dlnR as a function of R. If DF is constant in a certain
% range of R, then DF is the fractal dimension of the set C. The
% derivative is computed as a 2nd order finite difference (see GRADIENT).
%
% The execution time depends on the sizes of C. It is fastest for powers
% of two over each dimension.
%
% Examples:
%
% % Plots the box-count of a vector containing randomly-distributed
% % 0 and 1. This set is not fractal: one has N = R^-2 at large R,
% % and N = cste at small R.
% c = (rand(1,2048)<0.2);
% boxcount(c);
%
% % Plots the box-count and the fractal dimension of a 2D fractal set
% % of size 512^2 (obtained by RANDCANTOR), with fractal dimension
% % DF = 2 + log(P) / log(2) = 1.68 (with P=0.8).
% c = randcantor(0.8, 512, 2);
% boxcount(c);
% figure, boxcount(c, 'slope');
%
% F. Moisy
% Revision: 2.10, Date: 2008/07/09

% History:
% 2006/11/22: v2.00, joined into a single file boxcountn (n=1,2,3).
% 2008/07/09: v2.10, minor improvements

% control input argument
error(nargchk(1,2,nargin));

% check for true color image (m-by-n-by-3 array)
if ndims(c)==3
    if size(c,3)==3 && size(c,1)>=8 && size(c,2)>=8
        c = sum(c,3);
    end
end
```

```

    end
end

warning off
c = logical(squeeze(c));
warning on

dim = ndims(c); % dim is 2 for a vector or a matrix, 3 for a cube
if dim>3
    error('Maximum dimension is 3.');
```

```

end

% transpose the vector to a 1-by-n vector
if length(c)==numel(c)
    dim=1;
    if size(c,1)~=1
        c = c';
    end
end

width = max(size(c)); % largest size of the box
p = log(width)/log(2); % nbre of generations

% remap the array if the sizes are not all equal,
% or if they are not power of two
% (this slows down the computation!)
if p~=round(p) || any(size(c)~=width)
    p = ceil(p);
    width = 2^p;
    switch dim
        case 1
            mz = zeros(1,width);
            mz(1:length(c)) = c;
            c = mz;
        case 2
            mz = zeros(width, width);
            mz(1:size(c,1), 1:size(c,2)) = c;
            c = mz;
        case 3
            mz = zeros(width, width, width);
            mz(1:size(c,1), 1:size(c,2), 1:size(c,3)) = c;
            c = mz;
    end
end

n=zeros(1,p+1); % pre-allocate the number of box of size r

switch dim

    case 1    %----- 1D boxcount -----%

        n(p+1) = sum(c);
        for g=(p-1):-1:0
            siz = 2^(p-g);
            siz2 = round(siz/2);
            for i=1:siz:(width-siz+1)
                c(i) = ( c(i) || c(i+siz2));
            end
            n(g+1) = sum(c(1:siz:(width-siz+1)));
        end

    case 2    %----- 2D boxcount -----%
```

```

n(p+1) = sum(c(:));
for g=(p-1):-1:0
    siz = 2^(p-g);
    siz2 = round(siz/2);
    for i=1:siz:(width-siz+1)
        for j=1:siz:(width-siz+1)
            c(i,j) = ( c(i,j) || c(i+siz2,j) || c(i,j+siz2) || c(i+siz2,j+siz2) );
        end
    end
    n(g+1) = sum(sum(c(1:siz:(width-siz+1),1:siz:(width-siz+1))));
end

case 3    %----- 3D boxcount -----%

n(p+1) = sum(c(:));
for g=(p-1):-1:0
    siz = 2^(p-g);
    siz2 = round(siz/2);
    for i=1:siz:(width-siz+1),
        for j=1:siz:(width-siz+1),
            for k=1:siz:(width-siz+1),
                c(i,j,k)=( c(i,j,k) || c(i+siz2,j,k) || c(i,j+siz2,k) ...
                    || c(i+siz2,j+siz2,k) || c(i,j,k+siz2) || c(i+siz2,j,k+siz2) ...
                    || c(i,j+siz2,k+siz2) || c(i+siz2,j+siz2,k+siz2));
            end
        end
    end
    n(g+1) = sum(sum(sum(c(1:siz:(width-siz+1),1:siz:(width-siz+1),1:siz:(width-siz+1))));
end

end
n = n(end:-1:1);
r = 2.^(0:p); % box size (1, 2, 4, 8...)

if any(strncmpi(varargin,'slope',1))
    s=-gradient(log(n))./gradient(log(r));
    semilogx(r, s, 's-');
    ylim([0 dim]);
    xlabel('r, box size'); ylabel('- d ln n / d ln r, local dimension');
    title([num2str(dim) 'D box-count']);
elseif nargin==0 || any(strncmpi(varargin,'plot',1))
    loglog(r,n,'s-');
    xlabel('r, box size'); ylabel('n(r), number of boxes');
    title([num2str(dim) 'D box-count']);
end
if nargin==0
    clear r n
end
end

```


Collaboration

SCIENCE, ART, & CHAOS

BY DR. GOTTFRIED JOSEF MAYER-KRESS, Ph.D.

Traditional tools, such as mathematical equations and formulas, are no longer sufficient in describing the science of complex and chaotic systems. Graphical and even multi-media tools have become increasingly important. In the early 1980s I used comparatively primitive tools (three-pen plotters) to create graphic representations of nonlinear, chaotic structures. Long before publication of *The Beauty of Fractals* I saw the potential in this material for artists. But I also knew that I — a scientist without artistic talent or training — was not the person who would uncover these other, non-scientific layers of chaotic structures.

I had a friend at that time who was an art student. One day I invited her over to the computer lab and showed her the fractal images. However, she was not especially impressed, though she did sit down at the computer and experiment with mixing colors.

Not long afterwards, I gave a talk in New Mexico at the Santa Fe Center for Contemporary Art where I showed slides of fractal images. I had a similar response: Decorative but definitely not art. What surprised me was that the artists got excited over old view-graphs I had with me of mechanical chaos experiments and chaotic brain wave data. I realized that to create something other than visual fast food requires an artistic eye.

In creating the image of the Chirikov standard map (figure 1) my intention was to demonstrate two different properties of chaotic systems. Starting with a

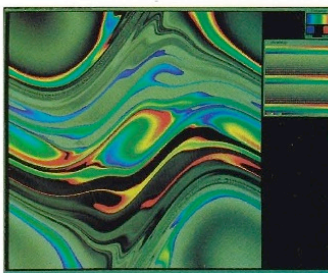


figure 1

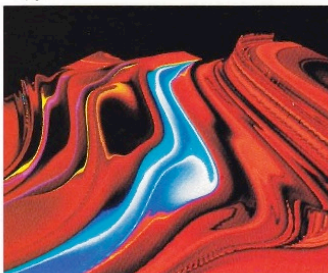
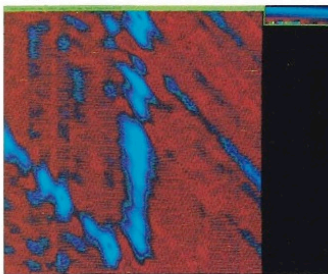


figure 2



few hundred horizontal lines of identical color we observe how these lines are deformed and finally completely mixed up, but only in certain “chaotic” bands (in Jenifer Bacon’s landscapes, such as “Canyons and Mesas,” figure 2, this corresponds to very rugged areas), whereas there are always regions for which stable order persists and forms islands or chains of islands. This is a typical metaphor for chaotic systems. It also has some real applications — and was originally conceived in the context of the “beam-beam interactions” of interacting storage rings in particle physics. There the particles move approximately at the speed of light and therefore “feel” each other’s presence only through an instantaneous kick, which then can be described by a discrete map such as the “standard map” described above. The standard map has also been used to describe phenomena of multi-photon absorption and quantum chaos, a very recent and exciting field of active research in which the photo-electric effect, for which Einstein received the Nobel prize, appears in a completely new light.

An interesting recent development in the visualization of complex processes is represented by the recurrence plots of figure 3. Here an observed process (in this case a measured heart rate, evolving in time — no, the heart is not a regular clock, it develops complex, chaotic patterns, at least as long as it’s healthy) is represented by its recurrence patterns. A pattern is defined by a vector consisting of a set of points in the

(continued on page 46)

Calligraphy by Inja Ink

In a fascinating convergence of art and science, an artist using an IRIS workstation has created landscape "paintings" based on visualizations developed by a scientist studying chaos theory

ART, SCIENCE, & CHAOS

BY JENIFER BACON

Last year while at Princeton University I began a collaboration with Gottfried Josef Mayer-Kress working with computer images that he had generated for scientific research in chaos. It was an opportunity for me as a graphic artist to not only work directly with a scientist, but also to learn to use the IRIS workstation and Princeton Visualization Tools. When I began the project my sole purpose was to work with the images in order to clarify them for scientific visualization. The images would still be computer models of chaotic systems to be used for science. I had seen chaos and fractal images in the past. I regarded them as intriguing scientific images but not art.

In one afternoon I learned to use the Princeton Visualization Tools, the IRIS workstation, and began work. The standard "makemap" color map was used for the initial images I received from Mayer-Kress (figure 1). At first I felt there were too many colors all mixed together, which was distracting. However, there was something intriguing about the structure of the images. They seemed to flow and move like liquid or vapor and I began to see them as raw material. It was as if I were a painter and had gone outdoors with canvas and oil colors to paint a landscape. The chaos images on the computer put me in mind of the land and sky — I could paint and interpret them as I wished. The flexibility and power of the IRIS workstation enhanced the work, allowing the process to become spontaneous and playful.

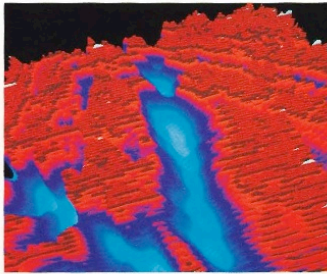


figure 3

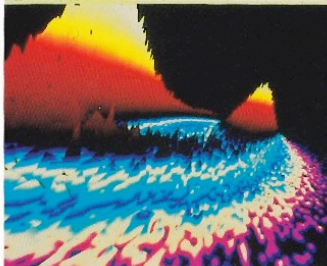
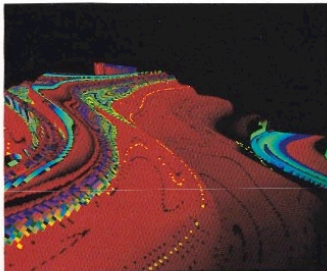


figure 4



My first step was to search the image and determine if there were any inherent patterns. In the interest of simplification I eliminated the mix and jumble of colors. I wasn't interested in merely creating pretty pictures; I was looking for a movement or a visual tension in the image that sparked my personal vision. Still, I questioned the value of searching for personal visions in images that are essentially models of particle accelerators, chemical reactions, and heart rates. Painting was far more immediate and allowed absolute freedom along with the challenge of blank canvas. Was there then any value to using these images of chaos to create a personal vision?

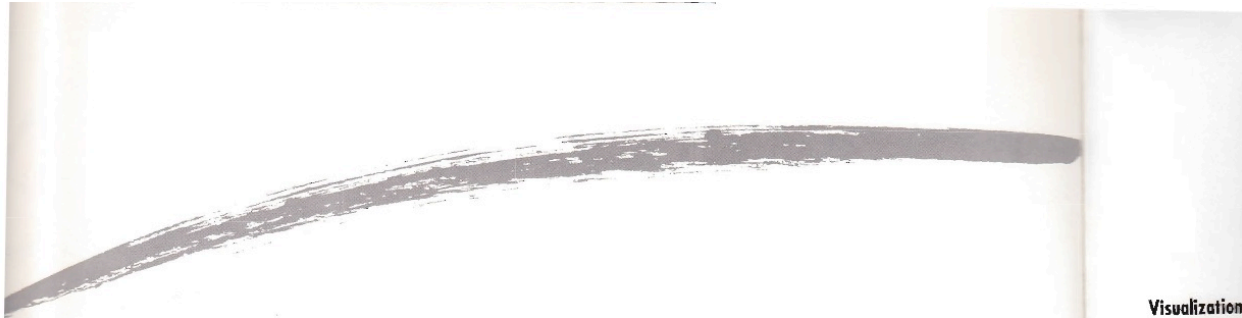
The image titled *Canyons and Mesas* (figure 2), originated from the nonlinear mathematical "standard map" of a torus, was transformed into a brilliant landscape full of waves, motion and color. This torus slice, which had formed a strange attractor in the science model, had become a strange surreal landscape in its art version.

In the image *Blue Canyons* (figure 3) the color map is a recurrence plot of the heart rate of a pig, but the image was manipulated to create a rugged red landscape creased with deep, mysterious canyons.

Islands (figure 4) began as a model of a chemical reaction and was transformed into ethereal islands floating down a liquid pathway of pink and blue.

In the creating of these abstract landscapes I had discovered new visual tools which made it possible for me to create unique images that could not have been

(continued on page 46)



SCIENCE *(continued from page 44)*



time series separated by a fixed time interval. The time of the process evolves along the horizontal axis, on the vertical axis we represent the time interval between the pattern at a certain time given by the x-coordinate and a new pattern occurring later. Thus both axes represent times at which patterns are observed. The color indicates how similar these patterns are. If there is some deterministic structure in the evolution of the patterns this then becomes immediately visible in the recurrence plot.

For example, according to a hypothesis expressed by A. Goldberger of the Harvard Medical School, as a heart comes close to a pathological state — such as a heart attack — the complexity, or “chaos,” of the heart rhythms should decrease which may then be observed in the more regular structure of the recurrence plot. (For an article on this topic see *Discover* magazine, May 1990.)

In the third example (figure 4) computer graphics were used to describe the stability of a model of an oscillating chemical reaction measured against external, randomly variable agitations. Especially in large chemical reactors it is extremely important to understand how sensitive the system is to small irregularities in the control parameters in order to prevent dangerous run-away reactions or explosions.

Starting from a stable condition for the reaction, we can apply a Chapman-Kolmogorov method to compute the probabilities for the behavior of the system at a given time, averaged over the ensemble of external agitations. The peaks in the figure correspond to states of the chemical system for which there exist constant reaction rates. Adding small external agitations leads to the formation of the “atoll,” a situation in which the reaction rate changes in an

oscillatory pattern, still with a high probability (z-coordinate) to be found near the original steady fixed points.

Thus we visually observe the evolution of landscapes which describe the statistical properties of the reactions and the conditions under which instabilities might occur. This class of non-linear equations is very general. Such equations can also model the stability of ecologies and also have been used to describe crisis in the strategic arms race.

ART *(continued from page 45)*

achieved in any other way.

The results of this art/science collaboration were entirely unexpected and went far beyond what Dr. Mayer-Kress and I had anticipated. My intent was to use Mayer-Kress's images as a point of departure, a basis on which to create something more than brightly colored computer graphics — as a vehicle for self expression.

While computer visualization technology is encouraging the convergence of science and art, we are also reminded that age old, fundamental differences exist between the two disciplines. Visualization in the sciences is typically used as a means of representing data so that it may be better understood or communicated to others. Visualization as applied in the arts is not so much a representation of data as it is a representation of the artist's imagination and feelings.

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Jenifer Bacon is Art Director at Tygenthof & Partners, a Southern California advertising agency.

Visualization

The Visuali and Science held August ing, Sweden is to demor promote gr diversity of A further o meeting pl researchers ment agen world. A n ternational ers will be

For mor Ingrid Nyri sity, 5-581 46 (13)28 1

Computer A

Computer third confi mation on sity of Ge Institute c The May Geneva, paper pre festival.

Origin applica sented in systems, animatio ics-based systems, special a mation f animatic blur and based a mation, effects s speech r recording

For Daniel Graphi Institut

Appendix C

Fern.m

```
%Code credit to
%http://www.mathworks.com/matlabcentral/fileexchange/16618-barnsleys-fern
%parameters from Fractals Everywhere by Michael Barnsley
%comments by Ali Lavine
%Notation for IFS of affine map:
%  $w_j(x) = w_j[x;y] = [a_j \ b_j; \ c_j \ d_j] * [x;y] + [e_j; \ f_j] = A_j * x + t_j$ 
% where each j is a subscript, j = 1,2,3,4, and the total probability of
% the  $w_j$ 's equals 1.
n=70000; % number of iterations
%IFS code for a Fern
%a = [a1 a2 a3 a4]
a=[0 0.85 0.2 -0.15];
b=[0 0.04 -0.26 0.28];
c=[0 -0.04 0.23 0.26];
d=[0.16 0.85 0.22 0.24];
e=[0 0 0 0];
f=[0 1.6 1.6 0.44];
%initialize variables
x(1)=0;
y(1)=0;
j=0;
%loop for each iteration
for i=1:n
prob=rand;
%prob < p1
if prob<0.01
j=1;
% p1 < prob < p1 + p2
elseif prob>0.01 && prob<0.86
j=2;
% p1 + p2 < prob < p1 + p2 + p3
elseif prob>0.86 && prob<0.93
j=3;
else
j=4;
end
x(i+1)=a(j)*x(i)+b(j)*y(i)+e(j);
y(i+1)=c(j)*x(i)+d(j)*y(i)+f(j);
end
plot(x,y,'g','MarkerSize',1)
```

-
- ⁱ Briggs, John. *Fractals the patterns of chaos: a new aesthetic of art, science, and nature*. New York: Simon & Schuster, 1992. 28.
- ⁱⁱ "The Mirror", chapter 0 of *Turbulent Mirror, an illustrated guide to chaos theory and the science of wholeness*, by John Briggs and F. David Peat, 1989 *Editor's note: the mirrorlike structure of this volume makes chapter 0 the fifth chapter in the book.*, pp 83-113. 112.
*Citation from compilation by Professor Wallace entitled Mathematics 7, Attractive Disorder: Chaos in Literature, Mathematics and Beyond. Winter, '98.
- ⁱⁱⁱ "The Great Wave at Kanagawa (from a Series of Thirty-six Views of Mount Fuji) | Katsushika Hokusai | All | Asian Art | Collection Database | Works of Art | The Metropolitan Museum of Art, New York." *The Metropolitan Museum of Art, New York: metmuseum.org*. Web. 04 Dec. 2009.
<http://www.metmuseum.org/works_of_art/collection_database/asian_art/the_great_wave_at_kanagawa_from_a_series_of_thirty_six_katsushika_hokusai/objectview.aspx?collID=6&OID=60013238>.
- ^{iv} Txopitea, Ainize. "Www. yochaos.com." *AINIZE TXOPITEA*. Web. 02 Dec. 2009.
<http://www.ainizetxopitea.com/link_websites/yochaos/tesis.html>.
- ^v Briggs. *Fractals*. 44.
- ^{vi} Mayer-Kress, Ph.D., Gottfried J., and Jenifer Bacon. "Collaboration: SCIENCE, ART, & CHAOS." *IRIS Universe*: 44-46. Print. Number Sixteen
- ^{vii} Briggs. *Fractals*. 92.
- ^{viii} Briggs. *Fractals*. 102-103.
- ^{ix} "File:Glue1 800x600.jpg -." *Wikipedia, the free encyclopedia*. Web. 04 Dec. 2009.
<http://en.wikipedia.org/wiki/File:Glue1_800x600.jpg>.
- ^x Pickover, Clifford A. *Computers, Pattern, Chaos and Beauty Graphics from an Unseen World*. Boston: St Martins Pr, 1991. 8-11.
- ^{xi} Briggs. *Fractals*. 156.
- ^{xii} Barnsley, Michael F. *Fractals everywhere*. Boston: Academic, 1988. Print. 87, 188.