Image Deconvolution

Aron Liu

Problems from HW5

$$h = f * g + \varepsilon$$

$$\tilde{f}_m = \frac{\tilde{h}_m}{\tilde{g}_m} + \frac{\tilde{\varepsilon}_m}{\tilde{g}_m}$$

- When adding a small amount of Gaussian white noise, the noise overwhelms the image after dividing by the point spread function's (PSF's) FFT
- In most useful applications, the PSF isn't known. How do you reconstruct both the original image and the PSF?

Regularization

- What is an ill-posed problem?
  - A solution does not exist or is not unique, or
  - The solution is highly sensitive to the initial conditions & may not change continuously when the initial conditions change
- Inverse problems such as image deconvolution are commonly ill-posed
- What is regularization?
  - Adding additional constraints to an ill-posed problem to find a favorable solution

Tikhonov Regularization

- Also called the Tikhonov-Miller method, the Phillips-Twomey method, the Tikhonov-Phillips method, the constrained linear inversion method, linear regularization, or ridge regression
- Invented independently in many different contexts
- Commonly used for linear ill-posed problems

Tikhonov Regularization

- Basic problem:

$$Ax = b$$

$$\hat{b} = b + \varepsilon$$

$$\min_x \|Ax - b\|_2$$

- Tikhonov regularization:

$$\min_x \{\|Ax - b\|_2 + \lambda^2 \|x\|_2^2\}$$

- λ = acts on the size of the solution, has some optimal value
- This extra constraint prevents pixel values of x from becoming too large
- Acts as a “weight” on each pixel value

Tikhonov Regularization

- Alternate forms:

$$(A^T A + \lambda^2 I)x = A^T b$$

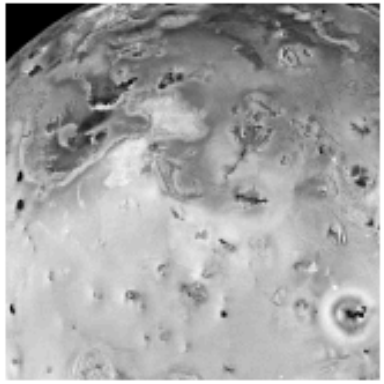
$$\min \left\| \begin{bmatrix} A \\ \lambda I \end{bmatrix} x - \begin{bmatrix} b \\ 0 \end{bmatrix} \right\|$$

- Intuition:

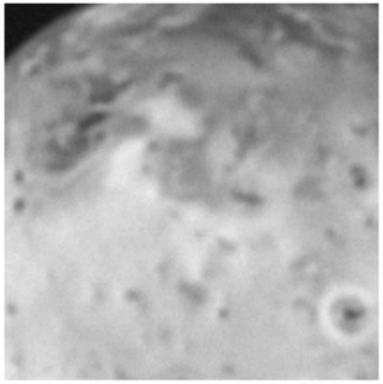
$$\text{As } \tilde{f}_m = \frac{\tilde{h}_m}{\tilde{g}_m} + \frac{\tilde{\varepsilon}_m}{\tilde{g}_m} \text{ increases, "weight" decreases}$$

Tikhonov Regularization

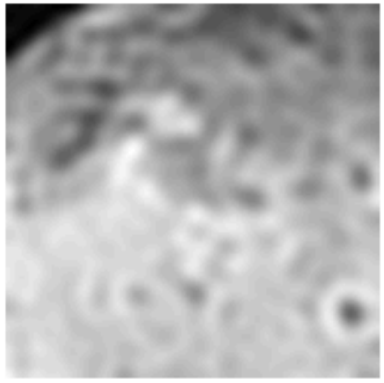
Exact



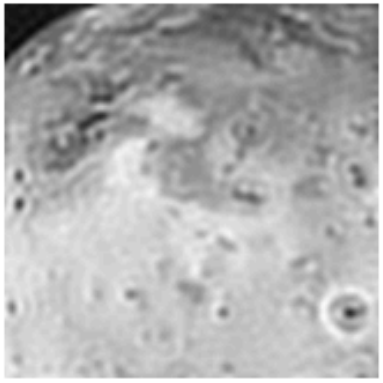
Blurred



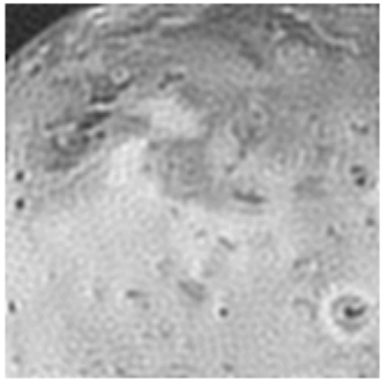
λ too large



$\lambda \approx$ ok



λ too small



What about when the PSF is unknown?

- This problem is called “blind image deconvolution” and many algorithms exist to solve it
- Iterative algorithms
  - operate on the result of the previous iteration and converge toward the final result
  - More versatile than linear methods BUT at the cost of computing time

Iterative Blind Deconvolution

- Start with the blurry image and an initial estimate of the deconvolved image or PSF
- Given some information about the deconvolved image and PSF, create a new estimate of the image and PSF

Iterative Blind Deconvolution

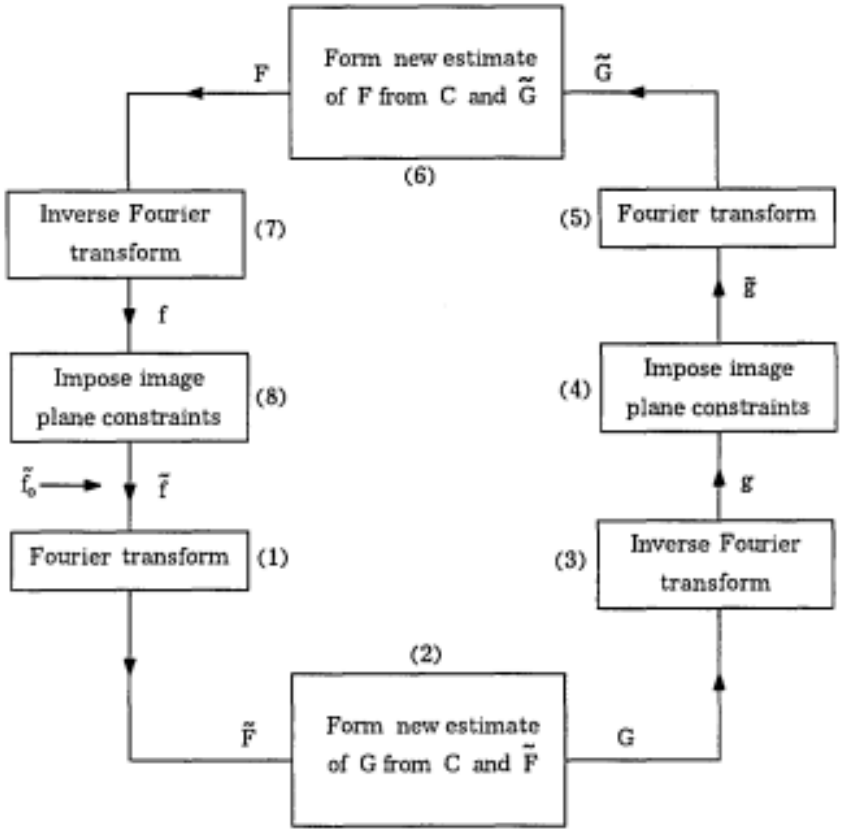
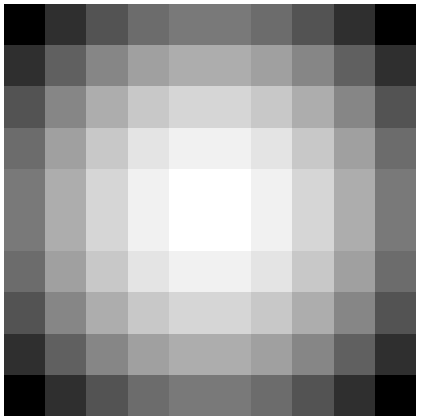


Fig. 1. General deconvolution algorithm.

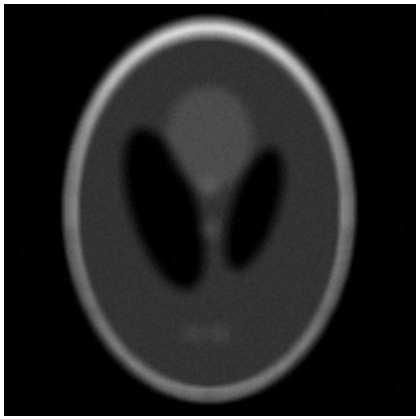
Blind Image Deconvolution

Original

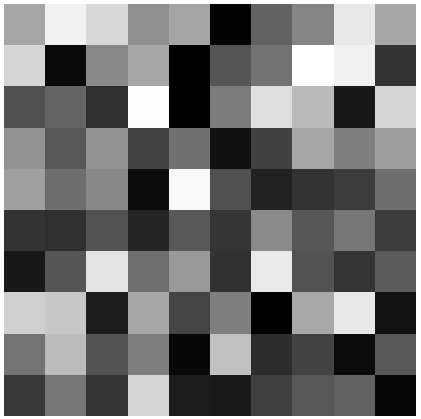
True PSF



Blurred Image



Initial PSF



Blind Image Deconvolution