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Math 5: The Math of Music & Sound
Final Project Write-Up

<center>Plastic Bottles, Q-Factors & Volume Ratios</center>

Does the ratio between the volume of the neck of a bottle to the volume of its body correlate with the Q-factor of its resonant frequency?

Our project sought to answer this question using two distinct approaches. Our original hypothesis posited that a higher neck to body volume ratio would directly correlate to a lower Q-factor relative to other volume ratios and Q-factors measured. Having theorized this, we chose six bottles of varying neck to body ratios and measured their experimental (observed) Helmholtz resonance frequencies and the accompanying Q-factors. We expected to see a linear relationship between increasing ratio and decreasing Q-factor.

| 2 L | 1 L | 750 mL | 450 mL | 237 mL | 4 oz |

Determine Neck:Body Volume Ratio

By measuring the neck length, neck area, neck volume, and body volume for each bottle, we could determine the ratios.

Body Volume	Neck Length	Neck Area	Neck Volume	N:B Ratio
.002 m^3	0.035	0.0003801	0.00001368	0.00684
.001 m^3	0.028	0.0003801	0.000010263	0.010263
.00075 m^3	0.0175	0.00212	0.00003847	0.051293333
.00045 m^3	0.026	0.000774	0.00001944	0.0432

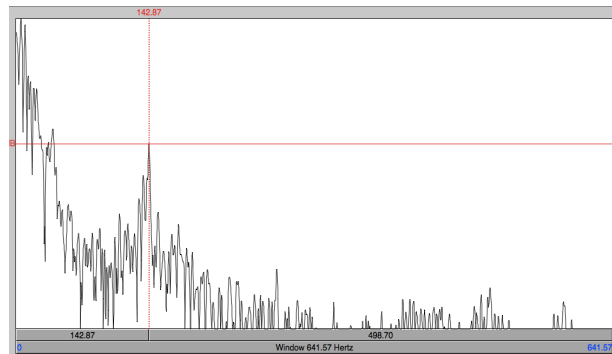| .000237 m^3	0.0235	0.00003767	0.000009154	0.038624473
.000118 m^3	0.01	0.000095	0.00000095	0.008050847

Determine Natural Frequencies

With a microphone in each bottle, and Audacity's white noise, we measured the response signal
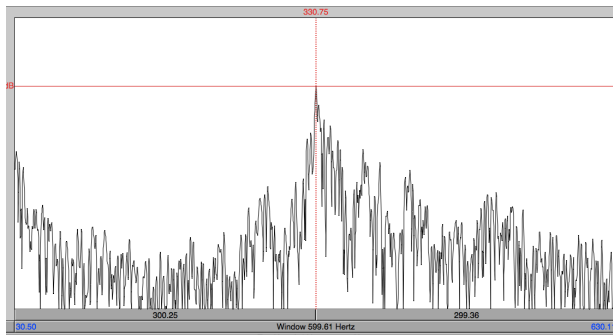
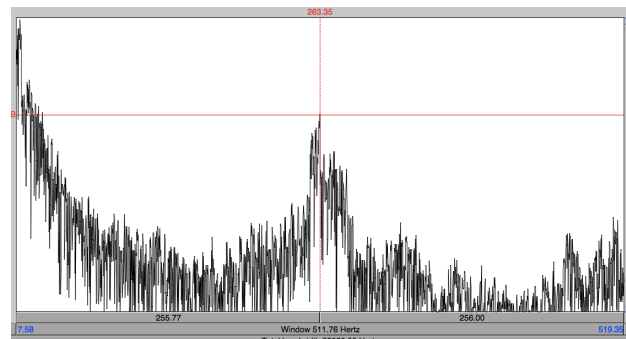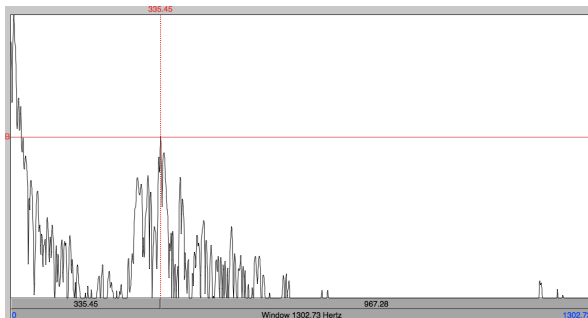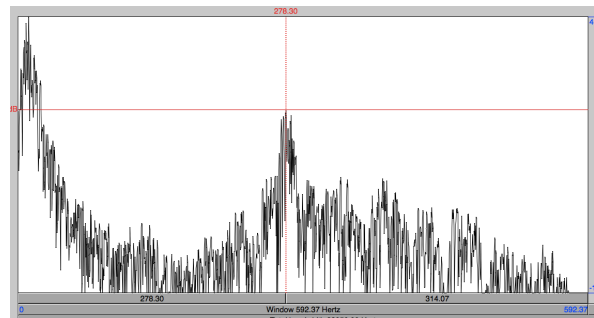of each of these bottles.

2 L – 778.9 Hz
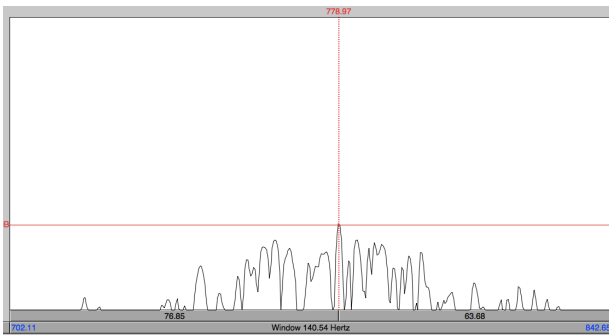
1 L – 142.87 Hz

750 mL- 330.75 Hz

450 mL – 263.35 Hz

237 mL – 335.45 Hz

4 oz. – 278.3 Hz

Determine Frequency Ranges

We used these same graphs to estimate the frequency at a 3-dB change in intensity in either direction, in order to determine the frequency range. We double-checked this estimate by producing a tone at the end of this frequency range and measuring the amplitude of it in comparison to the amplitude of the resonant frequency. The amplitude should be approximately $1/\sqrt{2}$ (=.707) in comparison to the resonant amplitude.
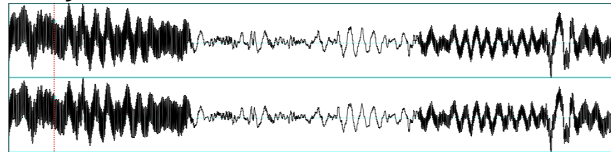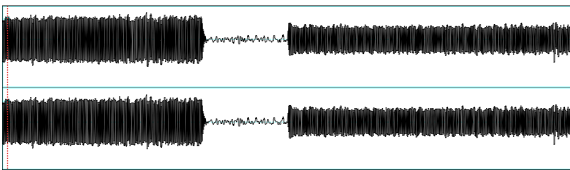
2 L - $f_0 \pm 15$ Hz   1 L- $f_0 \pm 5.41$ Hz
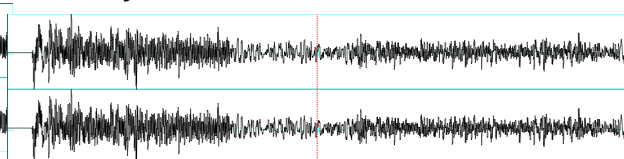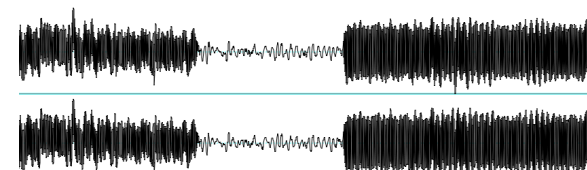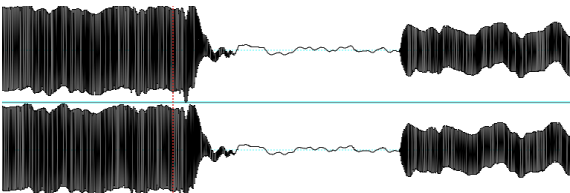

750 mL- $f_0 \pm 2.25$ Hz   450 mL- $f_0 \pm 7.9$ Hz
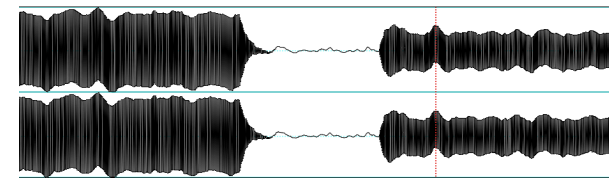

237 mL- $f_0 \pm 13.04$ Hz   4 oz.- $f_0 \pm 8.85$ Hz

Determine the Q-Factor

Using the data from above and the formula, $Q = f_0/\Delta f$, we determined the Q-factor for each bottle.
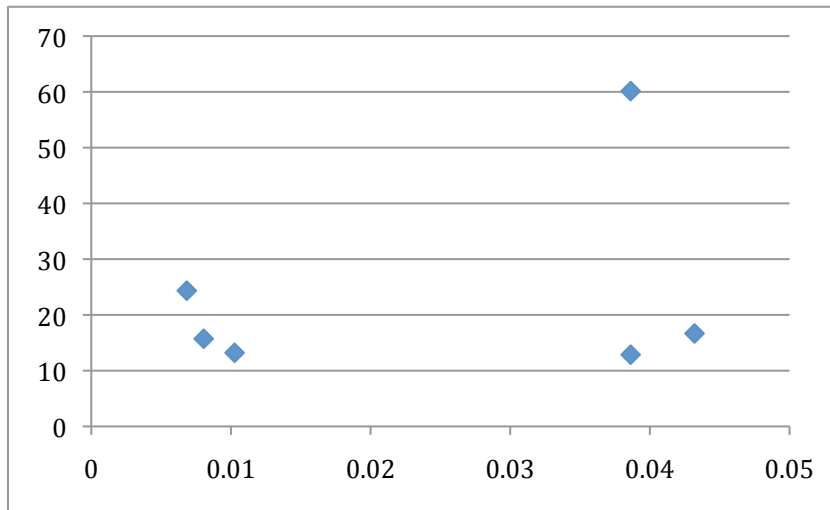
2L = 24.340625

1L = 13.20425139

750 mL = 60.13636364

450 mL = 16.66772152

237 mL = 12.86234663

4 oz = 15.72316384

Compare the Data

We plotted the neck to body volume ratios versus the Q factor in excel to see if we could see a

pattern. The Q factors are on the y-axis; the neck to body volume ratios are on the x-axis.

Our efforts using different bottle shapes and sizes yielded what appeared to be uncorrelated

results, summarized by the graph above. We thought there could be a few explanations for the

disconnect between our hypothesis and the results, the first of which being that perhaps the Q-

factor was related solely to the area of the neck to body volume ratio (i.e. the size of the opening

vs. the size of the bottle). However, there was no apparent correlation. Our second inclination

was that perhaps we had not controlled error properly. The main issue that we didn't control

was the size and shape of the bottles. The effective length of the neck considering the tapered

width was somewhat subjective. This is because for several of the bottles it was difficult to

determine where exactly we should consider the beginning of the neck. Another consideration is

the shape of the bottles. After some research, we discovered that the irregular shapes of the

bottles, particularly the flask, made them less-than-ideal Helmholtz resonators. Or rather,

though they still made effective resonators, the observed frequency differed significantly due to

the differing geometries of the bottles.[1]

Helmholtz Frequencies

Due to the inordinately high Q-factors we measured for the 750 mL bottle and the two-liter soda

bottle, we decided to calculate the theoretical Helmholtz frequency of each bottle, and compare

the calculated frequencies with the observed frequencies. It is possible, for example, that we

were actually calculating the resonance of a higher mode, which would explain the high

frequency we recorded for the two-liter bottle.

$$\frac{340}{2\pi} \times \sqrt{\left(\frac{\text{Area of Neck}}{(\text{Volume of Body})(\text{Length of Neck})}\right)}$$

2 L Bottle = $\frac{340}{2\pi} \times \sqrt{\left(\frac{0.0003801}{(.002 \text{ m}^3)(0.035)}\right)}$ = 126.1 Hz vs. 778.9 Hz

750 mL Bottle = $\frac{340}{2\pi} \times \sqrt{\left(\frac{0.00212}{(.00075 \text{ m}^3)(.0175)}\right)}$ = 687.7 Hz   vs. 330.75 Hz

For the 2 L bottle, based on this formula, we seem to have picked up on a higher mode in our

attempt to calculate the natural frequency of the bottle.

For the 750 mL bottle, we posit that the Helmholtz frequency is so high because the neck length

is so low, especially in comparison to the neck area. The odd shape of the neck thus affects the

resonating frequency.

Adjusting for Error

The only way to determine whether or not our hypothesis was valid was to control for the size

and shape of the body. Thus we decided to conduct a second round of experiments that

controlled the body volume and shape while altering the solely the neck length. Using the two-

liter bottle, we added a cardboard neck of the same radius of the original neck. We used adjusted

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[1] Chanaud, R. "Effects Of Geometry On The Resonance Frequency Of Helmholtz Resonators, Part Ii." *Journal of Sound and Vibration* 204.5 (1997): 829-34. Online, pg 833.

the neck to three different lengths, measuring the frequency and Q-factor of each with the same method described previously. We used the same methods to find the natural frequencies, the frequency ranges and the Q-factor.

Longest Neck Length- .123 m, f_0 = 762.8 Hz, Δf = 21.88 Hz
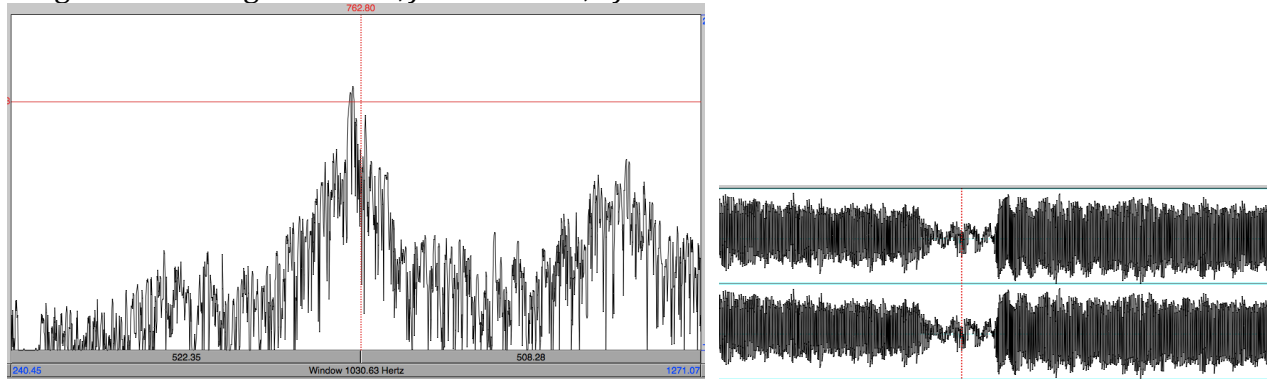
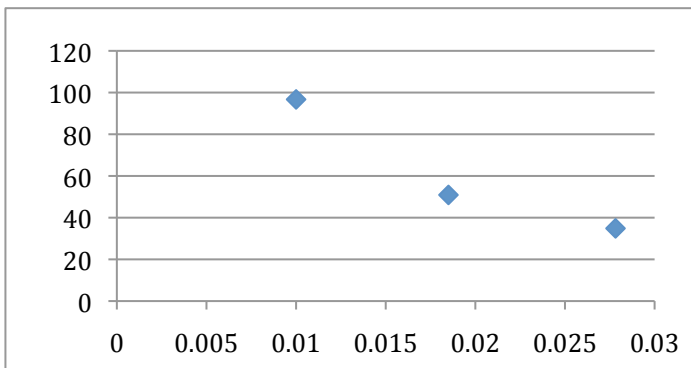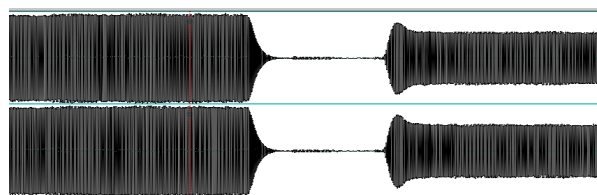Middle Neck Length- .082 m, f_0 = 760.53 Hz, Δf = 14.94 Hz
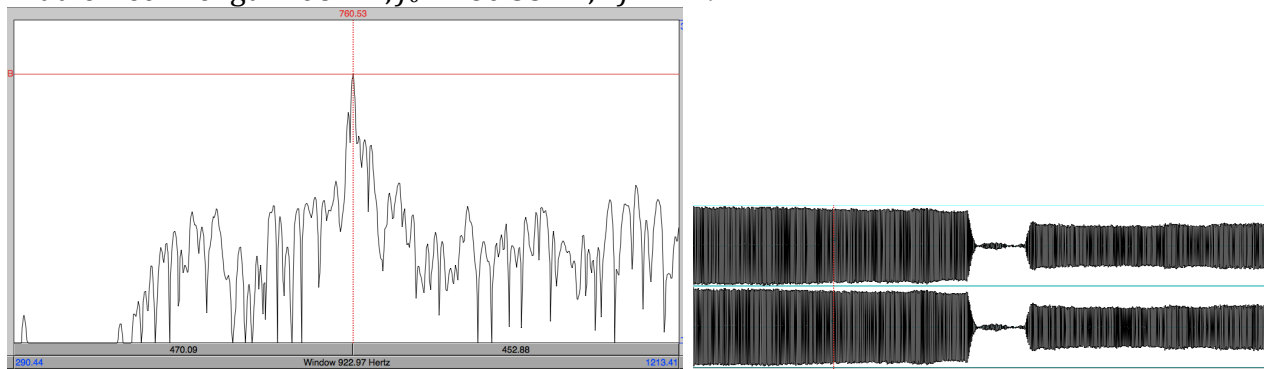
Shortest Neck Length- .044 m

Graph Missing, Bottle deconstructed
f_0 = 764.02 Hz
Δf = 7.9 Hz

The plot of the neck:body ratios (x-axis) versus the Q-factor (y-axis) shows a much stronger correlation than previously. This correlation corresponds with our hypothesis that the Q factor gets lower, as the neck:body volume

ratio decreases. Despite this seemingly conclusive information, the only part of the neck:body volume ratio that we truly altered was the neck length; therefore, the correlation cannot be attributed to neck volume without taking into account the neck shape.

The observed resonating frequencies for these bottles resembled one another very closely, considering that the change in neck length, a key element in the Helmholtz frequency formula. Again, we seem to have observed higher modes than the fundamental resonant frequency of the bottles, altering the Q-factor.

Conclusion

Our experiment clearly did not account for all possible error nor did it isolate satisfactorily each possible contributing factor in determining Q-factor. We can say conclusively that there is a loose inverse correlation between neck:body volume ratios and q-factors for Helmholtz resonators. To examine these other factors, it would have been useful to keep the same bad size but to alter the neck width as well as the neck length, separately, to determine to what degree each of these factors had on correlating with the Q-factor.

Bibliography:

Chanaud, R. "Effects Of Geometry On The Resonance Frequency Of Helmholtz Resonators, Part

Ii." *Journal of Sound and Vibration* 204.5 (1997): 829-34.