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Erik Satie's *Gymnopédie No.1* and the Acoustics of Piano Chords

In his review of a concert by the French pianist Jean-Yves Thibaudet at the Kennedy Center, *Washington Post* critic Tom Huizenga offered much more than his assessment of the performance. He also took time to comment on the legacies of some of the composers whose works Thibaudet included in the program. "Returning to the Concert Hall from intermission," Huizenga writes, "Thibaudet turned the tables on Erik Satie. In some circles it's fashionable to dismiss Satie as merely an iconoclast…But Thibaudet's detailed, committed performances were enough to disprove any Satie naysayers."[1] Huizenga especially finds this to be the case with Satie's *Gymnopédie No.1.* "Even the 'Gymnopédie No.1'—Satie's biggest hit—sounded fresh, played with a gently rocking left hand and pearls of wistful notes placed in the right."[2]

With the exception of Thibaudet's mastery, the biggest thing I took away from Huizenga's review is just how affecting Satie's music, especially *Gymnopédie No.1*, can be when done properly. It was with this idea in mind that I decided to make my project a mathematical analysis of the piece. Surely, I thought, something do beautiful had to utilize some important acoustic principals.

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[1] Tom Huizenga, "Thibaudet in His Native Musical Language; At the Kennedy Center, A Piano Tour de Force," *The Washington Post*, February 19, 2007. Accessed December 7, 2010, http://proquest.umi.com/pqdweb?index=23&did=1218612611&SrchMode=1&sid=7&Fmt=3&VInst=PROD&VType=PQD&RQT=309&VName=PQD&TS=1291722798&clientId=4347.

[2] Ibid.

But as I went through my research, what I began to find out was that the piece was not as simple as I presumed. Because an analysis of the entire piece, or even a large chunk of it, would have simply be too great of a task, I instead choose to narrow my focus and only look at the piece's opening eight measures. For me, this small section felt representative of the piece as a whole, in that it relied on fairly simple musical ideas to make a tremendous impact.

Gymnopèdie No.1 begins with an alternating series of two-measure major seventh chords. By definition, a major seventh chord is a four note series, where a "root" note is played along with those note that are a major third, a perfect fifth, and major seventh above it. In the case of *Gymnopédie No.1*, the first and third chords are built on the root G, while the second and fourth are built on the root D.[3]

However, as I soon found out these chords don't fit the previous definition perfectly. Looking at the score and then converting it into pitches, the G root noted in the score corresponds to the pitch to the pitch G2 while the D root corresponds to D2. Yet, when I found pitches for the other notes, they did not line up as expected. All of the notes that were part of the G chord line up with the pitch G3 instead G2, while all the notes in D chord lined up with D3 instead of D2. As someone who isn't particularly well versed in music theory, and thus assumes that the rules regarding music theory had to be followed exactly, I found this surprising. But as I soon found out, the fact that G3 and D3 aren't played is insignificant. Because pitches naturally excite all their harmonics, G3 and D3 will still be present and thus be able to form the major chords.

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[3] Erik Satie, "*Gymnopédie No.1*," in *Piano Music: Erik Satie* (New York: Editions Salabert, 1974): 11.

While harmonics resolved the first problem I encountered, they also helped create another. Now knowing which pitches were to be played according to the score, I then compiled a list of all the frequencies that should have been present, assuming that for that for any frequency f_n its second harmonic is equal to $2f_n$, its third frequency is equal to $3f_n$, etc. It was during this process, that I noticed something was amiss. Looking at the data, there appeared to be several instances where pairs of harmonics would be so close in frequencies as to cause beating or disharmony , as laid out it by the Helmholtz Theory of Dissonance. For example, with the chord based on D beating would likely occur between the 1st harmonic of A3 (220 Hz) and the 3rd harmonic of D2 (220.2 Hz). Meanwhile dissonance, but not beating, would likely occur between the 1st harmonic of C#4 (277.2 Hz) and the fourth harmonic of D2 (293.6 Hz), to name just one place.

When I actually analyzed the spectrums for the chords using Praat, some of these trouble spots disappeared, but a majority did not. Regarding the chord at G, beating should have been noticeable between the first harmonic of D4 (293.8 Hz) and the third harmonic of G2 (295.9 Hz), while dissonance should have been apparent because of the first harmonic of F#4 (371 Hz) and the fourth harmonic of G2 (392.6 Hz), to name one pairing.

Taken together, these results shattered my idea that the opening chords sounded pleasant because of the harmonics were spaced out so as to create consonance. In reality, these harmonics provided several opportunities for the chords to sound nasty.

So then, why didn't they? At first glance, the answer would seem to lie within the chords' power spectrums, which not only display the partials present, but also decibel levels. What becomes apparent is that in instances where two partials are close enough together to potentially cause either beating or dissonance, they do not wind up having the same intensity. Therefore, when potentially dissonant partials such as the 1st harmonic of F#4 (376.7 Hz) and the 5th harmonic of D2 (368.2 Hz) are both heard, but they have very different intensities, 29 dB and 11.2 dB respectively. There are several more instances of this phenomenon in both chords.

Although it's unclear whether this happens because of the piano's design, the piano does help ease the dissonance between partials in certain. Take for instance, how the piano deals with the problem of stretched partials. Although the strings are usually treated as being perfectly flexible, in reality they all have a certain amount of stiffness. As a result instead of harmonics being multiples, so that $f_n=nf_1$, they in fact become "stretched" so that f_n is in fact greater than nf_1. In theory, this should cause beating within pitches. But as Arthur H. Benade explains in his book *Fundamentals of Musical Acoustics*, no one notices this problem because, "the rapid decay of impulsive strong sounds…. gives somewhat less discordant results in piano and harpsichords than it does in pipe organs, where one is dealing with sustained tones have harmonic components."[4] Inharmonicity doesn't affect the sound, because the clashing partials decay too fast to be noticeable. While I could not look at the individual rate of decays of partials, it's not far-fetched to imagine that a similar

[4] Arthur H. Benade, *Fundamentals of Musical Acoustics* (New York: Dover, 1990), 324

process helps prevent dissonance within Satie's chords. It's unclear though what aspect of the piano's design would cause this to happen

All this doesn't mean however, that listeners play passive roles in resolving dissonant partials. As Benade explains, in a specialized situation, like when a string is repeatedly excited over an instrument that produces harmonic partials, "Our ears apply the minimum-beat criterion in a way that requires the fundamental components of the two tones to have simple, whole-number frequency ratios exactly like those that are found between pairs of ordinary (harmonic) musical tones."[5] In other words, our ear's design also helps makes dissonant partials sound nicer.

Yet despite all these aids, a certain amount of inharmonicity remains. But instead of warping the "true" sound of the piano, some researchers have proposed that this inharmonicity may be essential to it. As work by the noted physicist Harvey Fletcher proposed, real piano's stretched partials may be responsible for the supposed "warmth" of its sound.[6] This idea is an interesting one to close on because just like my initial belief in the acoustic simplicity of *Gymnopédie No.1* it deals with a it tires to explain a subjective response to the sound a piano makes. That a positive aspect of the piano sound could in fact be the result of a lack of harmony created by the instrument shows how presumptuous my initial conjecture was. Because although both the piano and myself as a listener have ways to combat the dissonance found in piece, it may just have always been inharmonicity that caused me to enjoy Satie's masterpiece.

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[5] Ibid, 324

[6] Alexander Galembo and Lola L. Cuddy, "Large Grand and Small upright Pianos: What Makes them Sound Different?"(paper presented at the 134th Meeting of the Acoustical Society of America, San Diego, California, USA, December 2, 1997).