Final Project: Composition

Michael Blum
Math 5: Music and Sound
Prof. Barnett
Fall 2011

For my project, I chose to write a musical composition using ideas and methods learned in class. The composition I wrote employed the analysis of sounds using spectrums and spectrograms, the concept of partials, Helmholtz's theory of dissonance, and the mathematical reflection of a melody in the x-axis (musically regarded as transposition).

The composition begins with an eerie organ sound, created with a software instrument of Apple's Logic Pro (Figure 1). The software instrument is controlled with drawbars, which represent the harmonic content of the organ sound produced. It also allows the user to control the quantity and quality of vibrato added to the instrument. This accounts for the oscillations that appear in the organ sound. This instrument plays an A2 throughout the entire piece (although it has a partial at 51.35 Hz, close to A1—I will therefore call A1, 55 Hz, the fundamental frequency), acting as a pedal note. As you can see in the data below, the organ sound contains partials that are not multiples of the fundamental frequency, 55 Hz. All other elements of the composition were written after the organ's pedal note was established throughout the piece.

Figure 1 – Software Instrument Controls (Organ)

Below, I have included an image of the spectrogram of the organ sound (Figure 2); the spectrum of frequencies within the organ sound (Figure 3); a chart that details many of the most prominent partials that appear in the organ's spectrum, along with their corresponding intensities, note names, and octave numbers (Figure 4); and a bar graph depicting the number of occurrences of notes which appear in said chart (Figure 5).

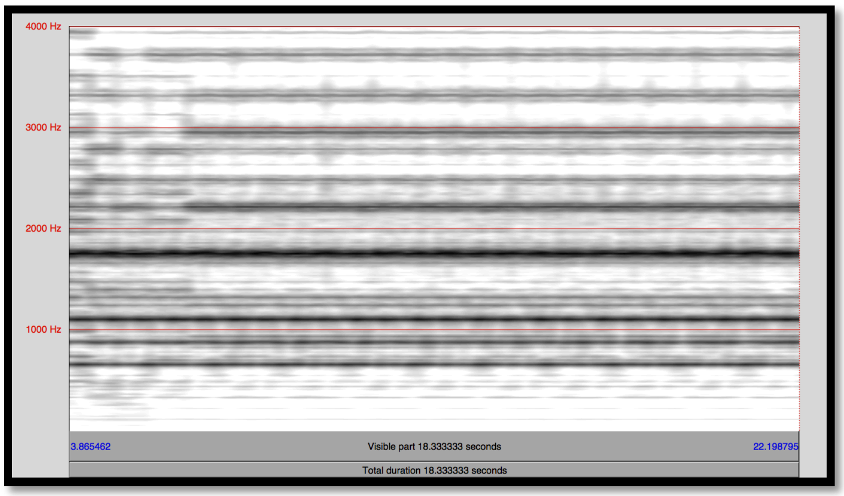

Figure 2 – Spectrogram of Organ Sound

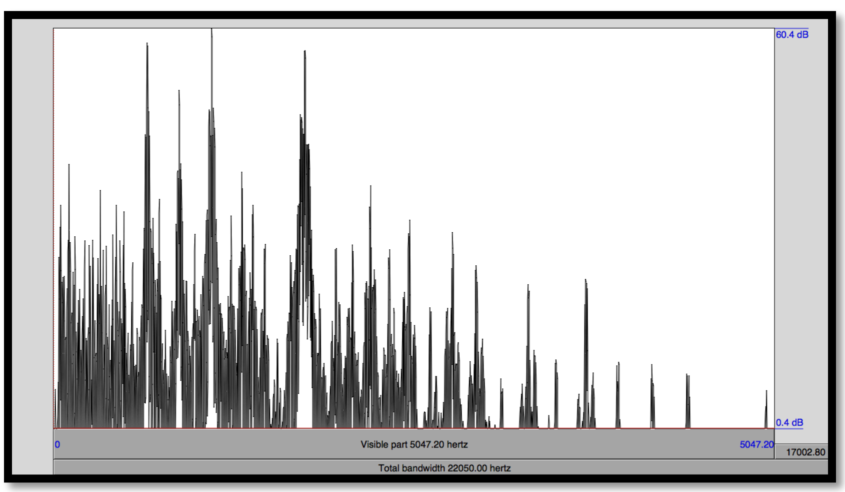

Figure 3 – Spectrum of Organ Partials

Ratio to f_0 (55 Hz)	Frequency (Hz)	Intensity (dB)	Semitones from A440	Octaves Away from A440	Semitones from Nearest A	Note	
1	51.35	26	-37.19	-3	-1.19	G#	1
1.3	70.2	12	-31.78	-2	-7.78	C#	2
1.6	84.87	9.2	-28.49	-2	-4.49	F	2
2.1	110.53	28.7	-23.92	-1	-11.92	A	2
2.8	149.81	17.3	-18.65	-1	-6.65	D	2
3.4	186.47	14.2	-14.86	-1	-2.86	F#	3
4	218.42	16.7	-12.12	-1	-0.12	A	3
4.6	248.27	14.4	-9.91	0	-9.91	B	3
5.1	277.6	18.3	-7.97	0	-7.97	C#	4
5.5	298.83	16	-6.70	0	-6.70	D	4
6	328.92	20	-5.04	0	-5.04	E	4
6.8	369.25	18.3	-3.03	0	-3.03	F#	4
7.2	393.34	18	-1.94	0	-1.94	G	4
7.6	414.84	15	-1.02	0	-1.02	G#	4
8	439.42	25	-0.02	0	-0.02	A	4
8.5	465.61	13.6	0.98	0	0.98	A#	4
9	492.84	27.9	1.96	0	1.96	B	4
10.1	555.16	14.8	4.02	0	4.02	C#	5
10.7	587.08	7.9	4.99	0	4.99	D	5
12	659.88	51.1	7.02	0	7.02	E	5
13.5	740.53	38.2	9.01	0	9.01	F#	5
14.3	786.09	22.2	10.05	0	10.05	G	5
15.2	831.63	15.3	11.02	0	11.02	G#	5
16.1	880.33	46	12.01	1	0.01	A	5
18	988.15	30.3	14.01	1	2.01	B	5
20.2	1108.9	53.8	16.00	1	4.00	C#	6
21.4	1173.61	23.1	16.98	1	4.98	D	6
22.7	1244.31	25.1	18.00	1	6.00	D#	6
24	1319.2	36.8	19.01	1	7.01	E	6
25.5	1397.23	25.5	20.00	1	8.00	F	6
27	1481	30.7	21.01	1	9.01	F#	6
28.6	1569.51	15.2	22.02	1	10.02	G	6
30.3	1664.63	19.2	23.04	1	11.04	G#	6
32.1	1763.22	51.1	24.03	2	0.03	A	6
35.9	1974.09	21.1	25.99	2	1.99	B	6
38.1	2091.53	26.2	26.99	2	2.99	C	7
40.4	2218.4	33	28.01	2	4.01	C#	7
42.8	2350.42	29.5	29.01	2	5.01	D	7
45.3	2489.45	21.2	30.00	2	6.00	D#	7
48	2637.75	22.9	31.00	2	7.00	E	7
53.9	2960.07	23	33.00	2	9.00	F#	7
72	3959.58	18.5	38.04	3	2.04	B	7

Figure 4 – Chart of Partials in Organ Sound

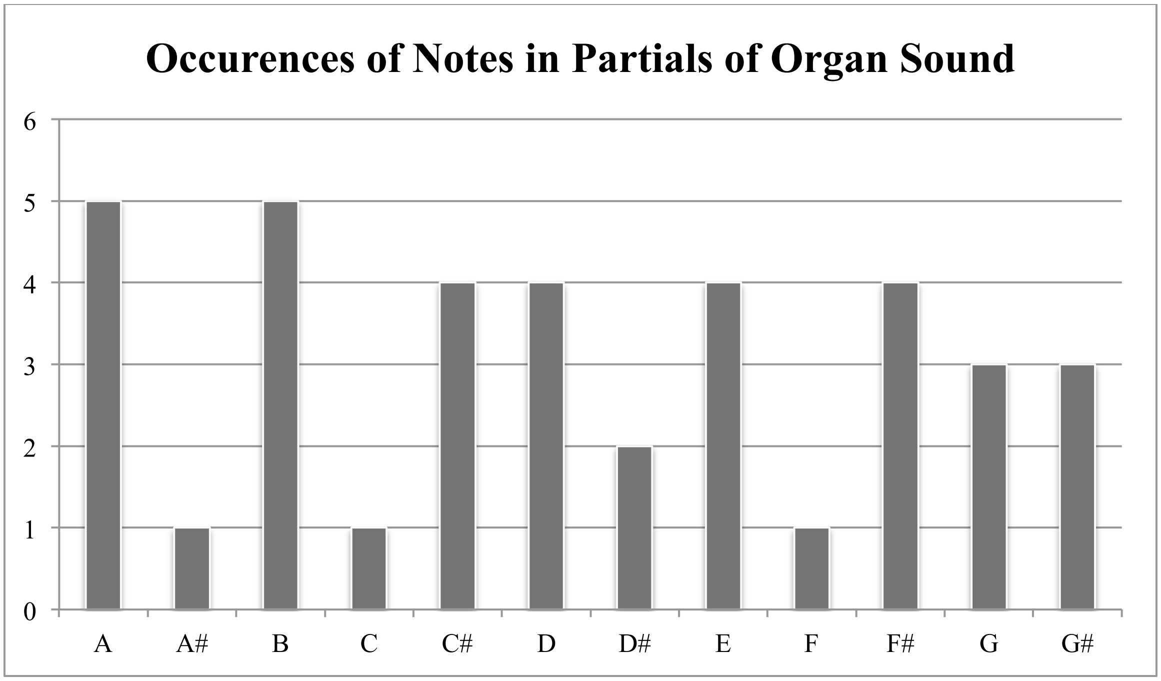

Figure 5

I used the above data to write the melodies in the piece. With the melodies, I intended to highlight many of the strange partials that appeared in the organ sound. For example, in the first melody (see Figure 6) of the piece (which I have termed "Motif A"), I stress the tritone of A, D#. This highlights two partials of the organ sound: the 22.7th and the 45.3rd multiples of f_0. The first melody also highlights the major second of A, B, which is one of the more frequently occurring notes in the partials of the organ sound (B appears at $4.6f_0$, $9f_0$, $18f_0$, $36f_0$, $72f_0$). I also emphasized A and C#, both of which are prominent in the partials of the organ sound. In a variation on Motif A (Figure 7), I begin to stress the note F# as well, whose partials occur four times in the spectrum of the organ sound.

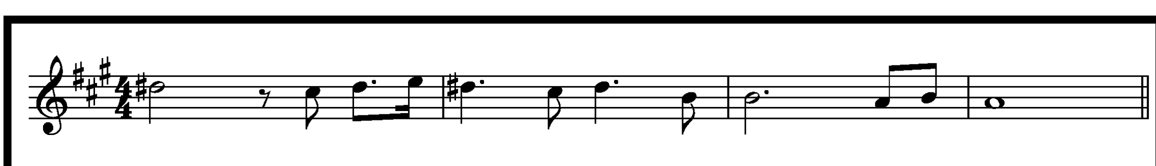

Figure 6 – Motif A

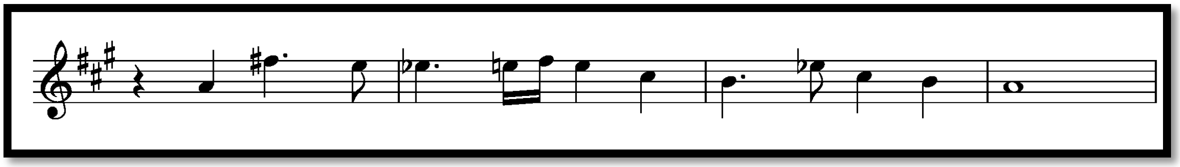

Figure 7 – Variation on Motif A

In the second distinct melody (see Figure 8) of the piece (Motif "B"), I utilized some of the partials apparent in the organ pedal in a duet between a harp and a vibraphone. This melody juxtaposes many notes separated by one semitone, for example, Eb (or D#) and E, C and C#, and F and F#. Because the two instruments that play this melody have long sustain times, these juxtaposed notes create beating and dissonance in the melody. In Figure 8, I have highlighted one portion of the melody that is especially dissonant.

Figure 8 – Motif B (vibraphone on first staff, harp on second staff), with dissonant portion highlighted

This dissonance of Motif B can be explained mathematically, using Helmholtz's theory of dissonance. When I examined the spectrogram (see Figure 9) for that portion of the composition, I found seven partials that were each less than or roughly equal to 10% away from the frequency of the preceding partial. I calculated this by recording the partials whose intensities exceeded 25dB and then finding the relationships in percentage between every adjacent partial. My calculations can be found below, in Figure 10.

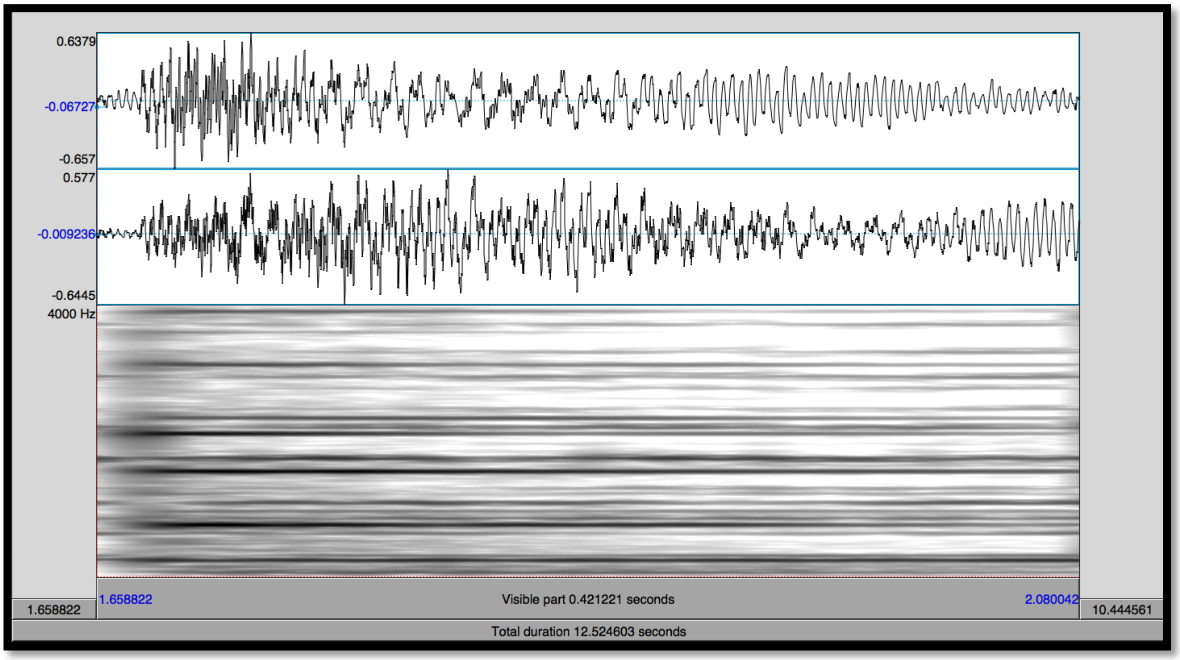

Figure 9 – Spectrogram for the third beat of the first measure of Motif B

Strongest Partials (above 25dB)		
Frequency (Hz)	Intensity (dB)	% Relative to Previous Partial
41.86	34.3	n/a
60.76	31.7	31.1%
82.08	55.5	26.0%
111.87	40.8	26.6%
125.02	42.1	10.5%
166.78	37.1	25.0%
192.48	25.9	13.4%
207.96	29.2	7.4%
261.41	56.3	20.4%
277.18	39.8	5.7%
290.91	25	4.7%
329.75	36	11.8%
783.16	47.4	57.9%
796.59	31.6	1.7%
880.41	31.8	9.5%
982.45	25.6	10.4%
1109.79	32	11.5%
1568.21	39.6	29.2%
Dissonances (≤~10%) are highlighted in yellow		

Figure 10 – Calculations of Dissonance in the third beat of the first measure of Motif B

Later in the piece, I transform Motif A to fit the progressing harmony by transposing it down a perfect fifth, which would be referred to mathematically as a reflection in the *x*-axis (see Figure 11). This transformation is not an exact reflection; there are minor asymmetries.

Figure 11 – Comparison of Motif A and its melodic transformation

When I chose to write a musical composition for this project, I aimed at creating a piece of artistic expression while integrating mathematical concepts that relate to music. In the writing process, I often found that my musical 'ear' and my knowledge from this class complemented each other harmoniously, especially when deriving the melodies from the common notes that appeared in the spectrum of the organ sound, creating dissonance and transposing melodies.