MATH 5, 11F
Prof. Barnett

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Building a Hammer Dulcimer: Construction and Mathematics

My final project involved beginning to construct a functional hammered dulcimer, and using knowledge from Math 5 to apply concepts therein. Physically building the piece was a challenge separate from the mathematical component and deserves some attention. Dulcimers are rare and quite expensive (part of the reason I took this opportunity to build one, rather than buy one), so information on constructing dulcimers is sparse and at times contradictory. Paired with my novice woodworking skills, the planning and building process was painfully slow-going (but nonetheless enjoyable).

The dimensions of my dulcimer are as follows:

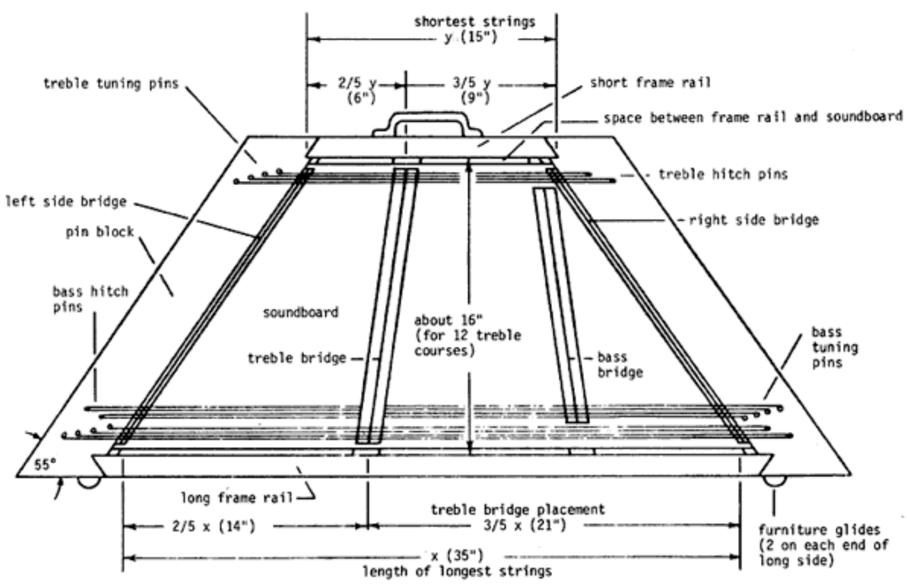


FIGURE 1

Hammer dulcimer diagram - top view

The longest string will be 35”, divided by a bridge that separates the string into 14” and 21”. The left section of the string should produce the note G#3, and the right section C#3. The frequencies of these notes are 207.65 Hz and 138.59 Hz, respectively. I wanted to calculate the tension needed on this first string to produce these notes, but first I needed to know the mass per unit length of the string. I calculated this by 1. Assuming that the string will be made of stainless steel, as most dulcimer strings are, 2. Assuming that the gauge (diameter) of the string is .024", which is a gauge used for low notes (these notes will be the lowest in the dulcimer), 3. Finding the density of stainless steel, and using this density to find the mass per unit length (mu or μ). The diameter of the string, in cm, is 0.061. I used this number to get the area of a cross-section of the string, which I found to be $0.003 cm^2$. This needed to be multiplied by 333 in order to get $1 cm^2$. The density of stainless steel is $7.85 g/cm^3$. I found the mass per unit length to be 0.0236 g/cm, or 0.00236 kg/m.

Using this μ, we can calculate the tension needed on each side of the string to produce these notes.

Cstring = sqrt(T/μ)

Cstring = f/2L

sqrt(T/μ) = f/2L.

<u>For G# (207.65 Hz):</u>

Sqrt(T/0.00236) = 207.65Hz/2(0.3556m) → Sqrt(T/0.00236) = 291.97

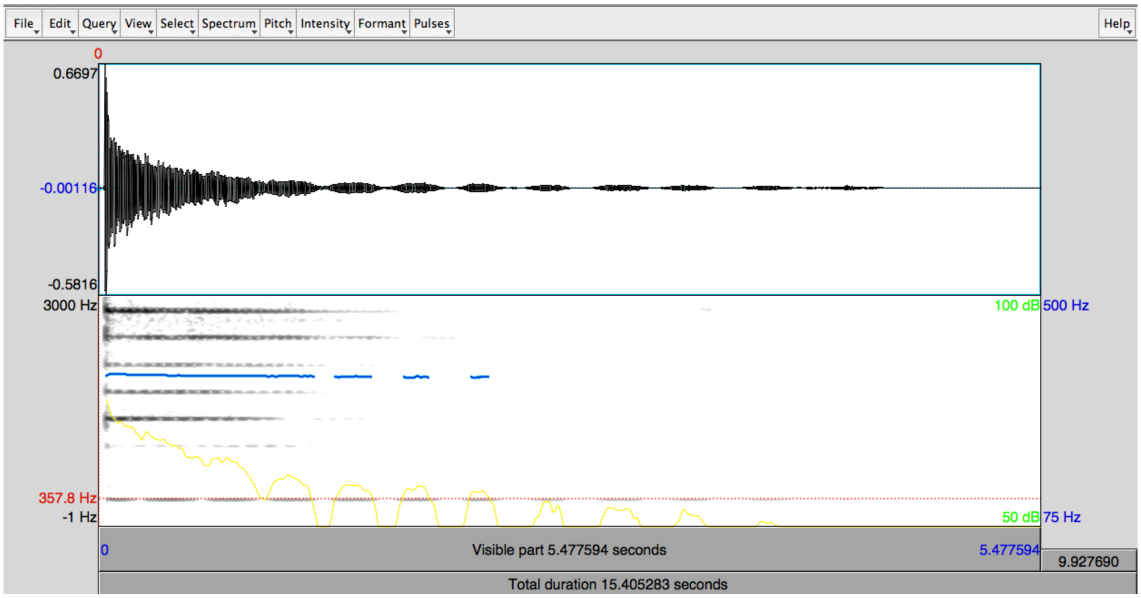
T = 201.2 Newtons for G#.

<u>For C# (138.59 Hz):</u>

Sqrt(T/0.00236) =138.59 Hz/2(0.5334m) → Sqrt(T/0.00236) = 129.9

T = 39.8 Newtons for C#.

Decay time



Above is a graph (in Praat) of a hit dulcimer string decaying over time (beating is present because the sound is taken from a video of someone tuning one string against another).

At 0.2 seconds (after the initial "attack" of the hit has died down), the maximum intensity is 49.7 dB. At 0.9 seconds, intensity is 42.4 dB.

t = 0.7 s

$-7.3 \text{ dB} = 10 \log_{10}(I_1/I_2) \rightarrow I_1/I_2 = 10^{-.73} \rightarrow \text{sqrt}(I_1/I_2) = A_1/A_2 = 0.4315$

$e^{-t/\tau} = .4315$, where t = 0.7 seconds.

$e^{-0.7/\tau} = 0.4315 \rightarrow$ take logs $\rightarrow -0.7/\text{tau} - \log(0.4315)/\log(e) \rightarrow -0.7\log(e)/\log(0.4315) = \tau$

$\tau = 0.832$ seconds.

An ideal dulcimer is designed to have a loud "attack" when the string is first struck (this can stand out even in a large band, depending on the type of strings and wood used for the soundboard). But this attack dies away quickly so not to overpower the next notes, which can be played in quick succession. The decay time to *inaudibility* is about 5 seconds in this case.

I spent a lot of time planning, designing and starting to build the dulcimer, and wish I had more time to apply more of what we learned in class to the dulcimer. I look forward to continuing this project in upcoming terms.