

## **Project: The Mathematics of Pipe Organ Sound Production**

This investigation looks into the mathematics behind how sound is produced by the pipes of a traditional pipe organ. It also considers ways in which sound can be altered and the resulting affect on overall timbre and pitch perceived. All recordings used were made from an Austin and Rogers organ and analyzed by Praat. Please note that the error in cents (c) of the data collected can be accounted for by the fact that the pipes were not all in tune at the time of data collection, and by human error when measuring in Praat.

The pipes of the pipe organ can be categorized into two categories: flue pipes and reed pipes. Please see DIAGRAM 1 for reference. In *flue pipes*, air is blown into the base of the pipe and forced out through the slit at point A. This causes the pressure inside the pipe (area labeled B) to lower. As a result, air is sucked back into the pipe and a standing wave is created. The flue acts as an open-open pipe, the length being the distance from (A) to the top of the pipe. *Reed pipes* produce sound using a reed, similar to many reed-based woodwind instruments. Air is blown into the pipe base, causing a metal reed to flap against a small piece of metal. The resulting sound produced is reinforced by a conical resonator, resting above the reed apparatus. The resonator must have a Q-factor such that it responds to a relatively wide range of frequencies (within a given note value) in order to accommodate different tuning styles. Reed pipes typically mimic any sound normally produced by a brass or woodwind instrument (barring flutes) while flue pipes are meant to produce flute and string instrument sounds. This study will focus primarily on the flue pipe (henceforth “pipe”).

The frequency of the sound produced by a pipe is primarily affected by the pipe’s length. The lowest key on a 61-note organ keyboard is C2, which corresponds to a pipe of length 8’. This can be confirmed using the following mathematics, given that 8ft approximately equals 2.44m. Of course, organ builders must take end-correction into account when making such calculations while designing pipes

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Open-open pipe:  $C/2L = F$

$$\lambda = 2L = 2(2.44\text{m}) = 4.88\text{m}$$

$$340\text{m/s} / 4.88\text{m} = 69.67\text{Hz} \approx 65.41\text{Hz (actual C2)}$$

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Intuitively, increasing pipe length will lower the frequency while decreasing pipe length will raise the frequency. However there are other ways of manipulating the frequency without changing physical pipe length.

*Harmonic pipes* are pipes that sound an octave above where they should. That is, their length is double what it should be. For instance, an 8’ harmonic pipe will sound a C3 instead of a C2. This is due to a small hole that is cut into the pipe at the first harmonic (an octave above), functioning in a similar way to a register key.

On the other hand, *stopped pipes* are closed on the top causing them to sound an octave lower (they are half the length of a normal pipe). For instance, a stopped 8' pipe will sound at C1 instead of C2. This is because stopping a pipe causes it to now become an open-closed pipe and thus:

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Closed-open pipe:  $F = C/4L$   
 $F = 340 / (4 \times 2.44)$   
 $= 34.83\text{Hz} \approx \mathbf{C1}$

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The pipes described above are often made with pipe organs for both practical and artistic reasons. For instance, if a set of pipes is to be made aesthetically prominent, they may be made as harmonic pipes (to appear larger) whereas if a room has a low ceiling, stopped pipes may be used to save space.

With this said, not all pipes are designed to sound the same pitch correlating with the key played on the keyboard. As one might guess, if you play the key "C2" and activate a simple 8' and 4' pipe you will hear the pitches C2 and C3 sound at once. However, pipe lengths are not always in octaves. Some pipes are designed to reinforce certain other harmonics of the fundamental, and thus create a difference in timbre. See DIAGRAM 2. In this recording, I first play a fundamental, C4 by itself. I then activate pipes sounding not only 2 octaves above, but also 2 octaves and a 5<sup>th</sup> above. Table 1 summarizes the harmonics and frequencies that are primarily reinforced by activating these pipes. Clearly the timbre is also made to sound quite a bit harsher and "tinnier" when more emphasis is put on these high harmonics.

Table 1:

Frequency (Hz)	Note	dB
1042	C6-7c	34.2
1570	G6+2c	35.3
2085	C7-7c	44.1
3128	G7-4c	20.9

Another common way to change the timbre of the sound produced is to add a *celeste* pipe. A celeste pipe is tuned just slightly sharp of normal flue pipe. Exact tuning is at the discretion of the builder. It is so close in frequency to the normally tuned pipe that the pitch difference is not even audible when the pipes are sounded separately. However, when the normally tuned pipe is played simultaneously with its celeste pipe, beating occurs. This is illustrated in DIAGRAM 3. Notice the close pairing of peaks. A good organ tuner will tune the celeste pipe with the normally-

tuned pipe in such a way that the beating creates a soft, pulsating sound that is of a warm timbre and not hard on the ears.

Changing pipe width also changes the timbre of the sound the pipe produces. As a general rule, the thinner the pipe, the “harsher” the timbre. That is, more high harmonics are stronger in thinner pipes. Flute-sounding pipes tend to be wider while string-sounding pipes tend to be thinner, producing a more nasal sound. Referring to DIAGRAM 4, it is clear that as 3 pipes are played (in order from widest to narrowest) at pitch C4, higher partials become more prominent. In the case of the thinnest pipe, some of the mid-range partials even become weaker as well. Table 2 lists the primary partials that were strengthened between the first and second pipe (the most dramatic change). Access was not granted to the pipe chamber, so the actual widths could not be measured.

Table 2

Frequency (Hz)
+526 (C5+9c)
+2349 (D7)
+2613 (E7-16c)
+2864 (F#7-57c)
+3128 (G7-4c)

Finally, it must be mentioned that certain pipes are enclosed in a box with shutters, known as a *swell box*. The shutters are controlled from the organ console. One may think that closing the shutters will simply make the sound quieter. However, there are changes in timbre as well. That is to say, some harmonics are less muted than others by the shades. This is shown in DIAGRAM 5 in which the swell box is opened, closed, and then opened again. Table 3 traces two specific harmonics of C4 as the box opens and closes. These two harmonics were specifically chosen in order to clearly illustrate the effect. Clearly G6 was muted significantly more than C5 relative to the starting values. In general, more distant (higher) harmonics are muted the most. Hence, a duller timbre can be expected in addition to quieter sound in general.

Table 3

	Swell box open	Swell box shut
509.5Hz (C5-46c)	43.8dB	25.2dB
1563Hz (G6-6c)	17.9dB	3.2dB

DIAGRAM 1: <http://home.comcast.net/~pqboom/tour/organphysics/menu3.html>

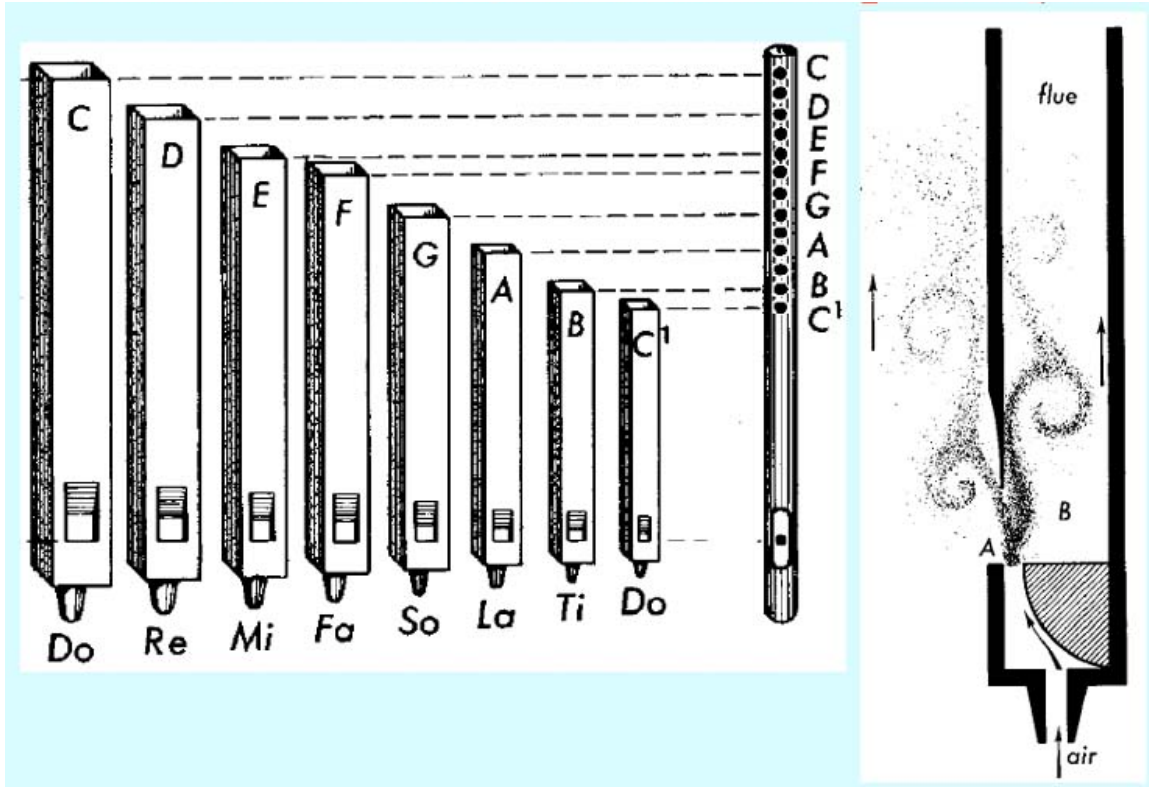


DIAGRAM 2:

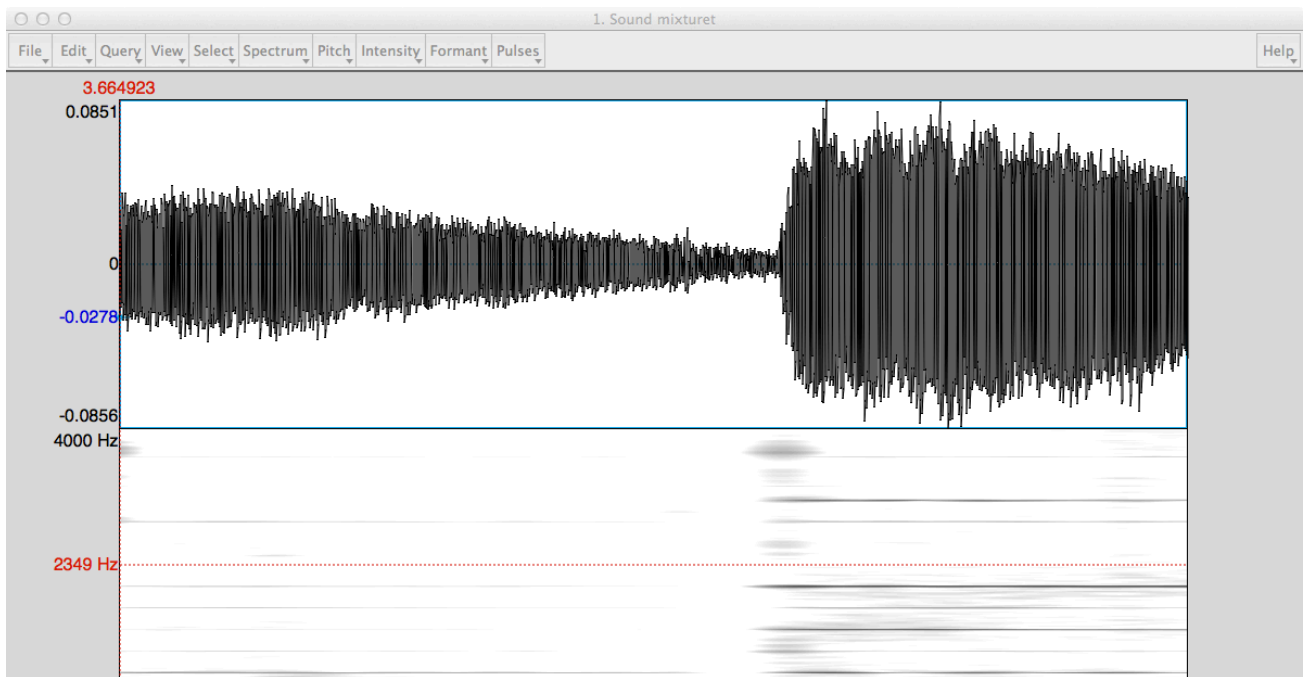


DIAGRAM 3:

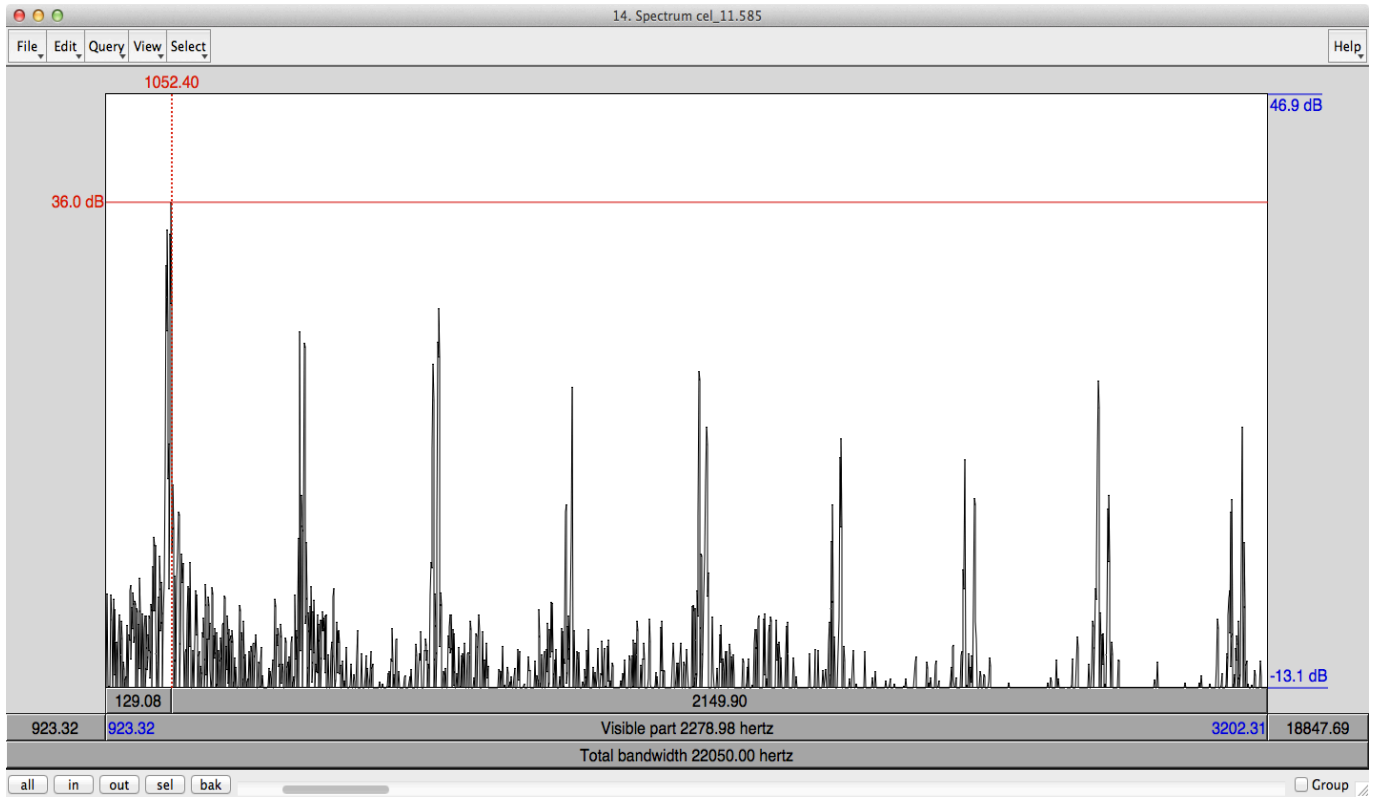


DIAGRAM 4:

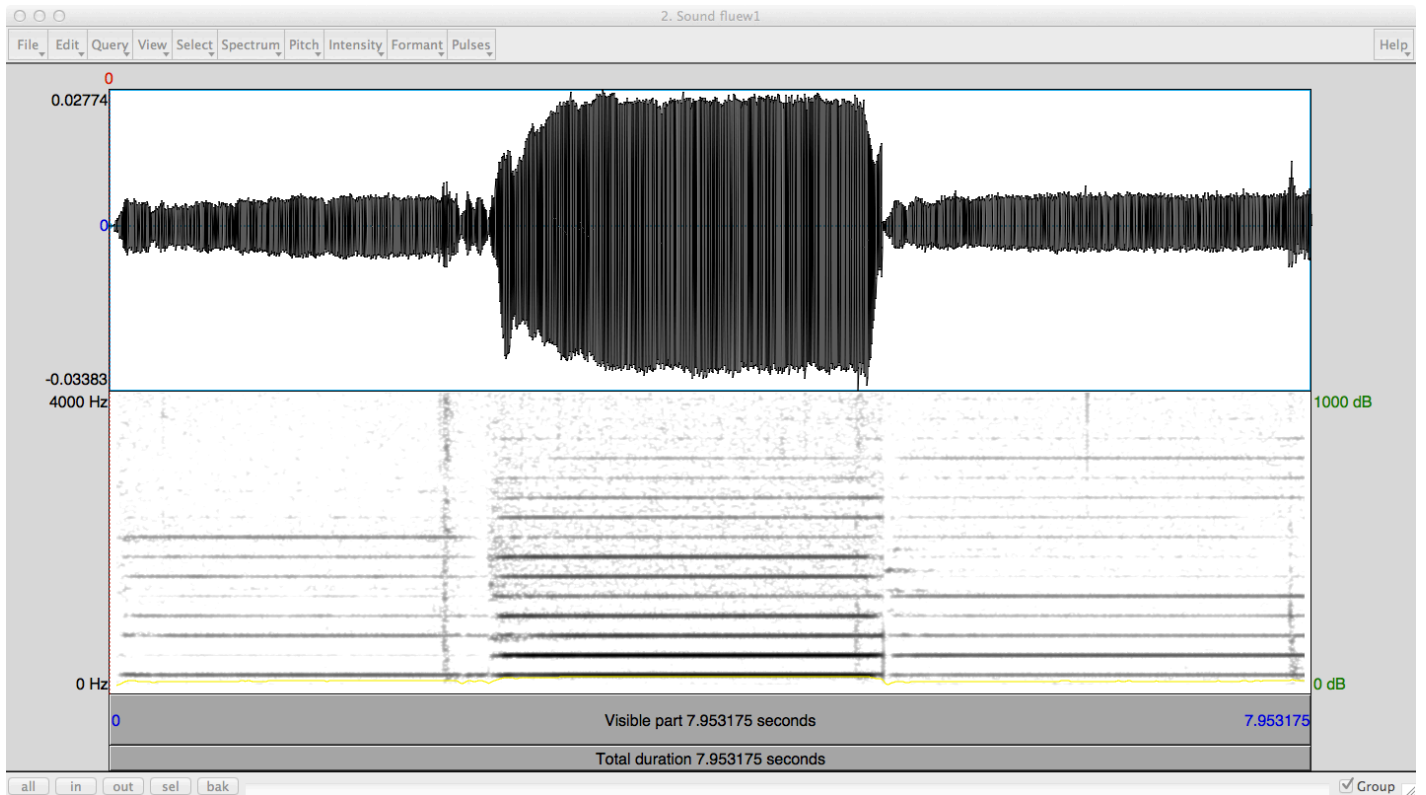
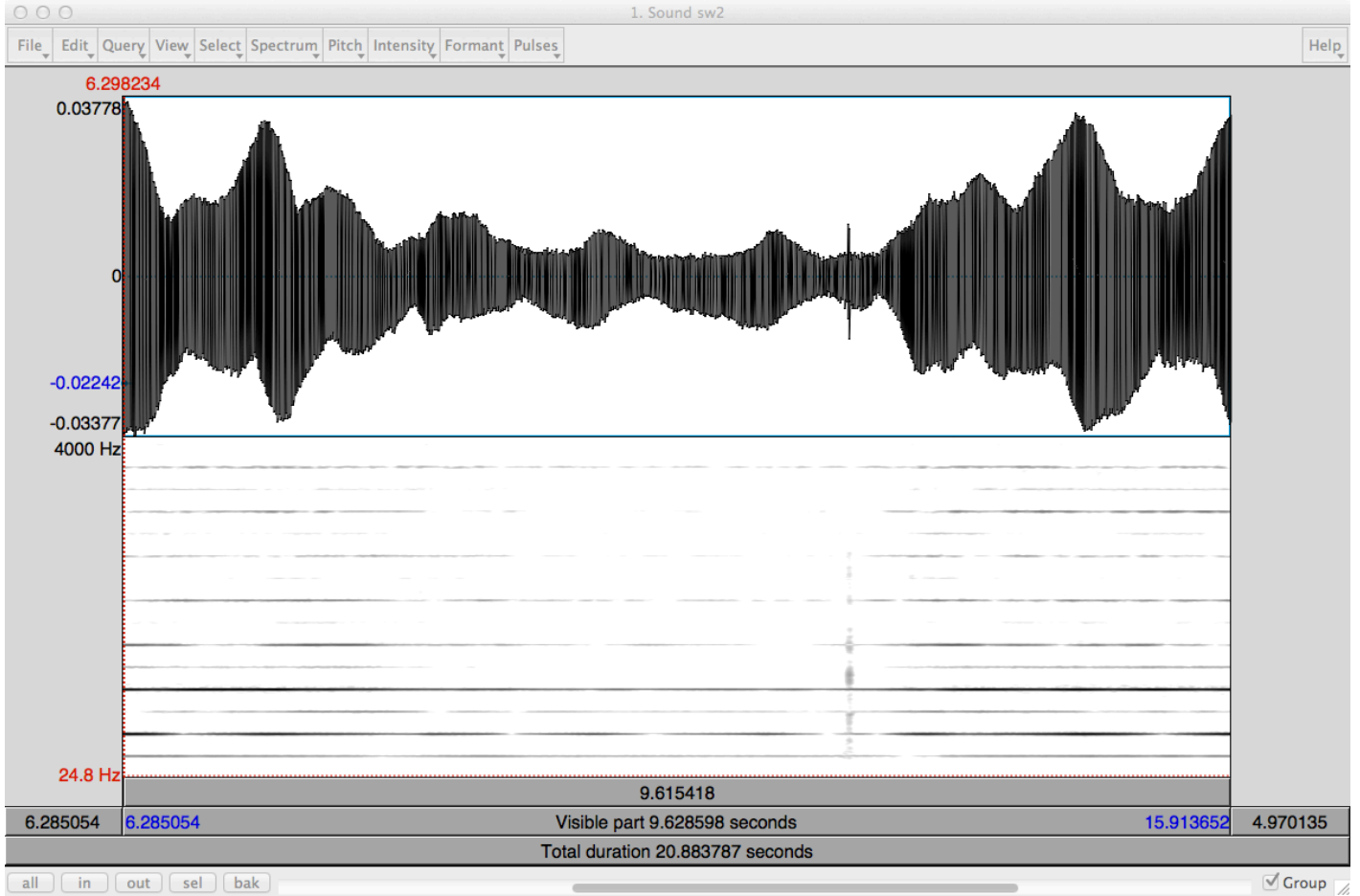


Diagram 5:



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