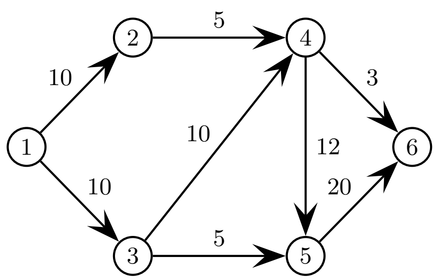
EXERCISES FOR HOMEWORK #7

Solutions to these problems are due on Wednesday, November 18.

1. Find a maximal flow from the source (vertex 1) to the sink (vertex 6) in the following network using the Ford-Fulkerson Algorithm. Prove that your flow is optimal by exhibiting a cut with the same capacity.



2. A graph is *bipartite* if its vertices can be partitioned into two sets such that each edge has one end in each set. A *vertex cover* of a graph is a set of vertices that includes at least one end of each edge, and a vertex cover is *minimum* if no other vertex cover has fewer vertices. A *matching* in a graph is a set of edges none of which share an end, and a matching is *maximum* if no other matching has more edges. The König-Egervàry Theorem states that in any bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover. Prove this from the Max-Flow Min-Cut Theorem.

3. Prove that in any simple graph, there are two vertices with the same degree.

4. A simple graph is called *regular* if all its vertices have the same degree. Let G be a connected regular graph with 22 edges. How many vertices can G have? (Note: there may be more than one possibility.)

5. In class we defined a *tree* as a connected graph without cycles. Prove that the following are equivalent for a graph T:

(1) T is a tree,

(2) every two vertices of T are connected by a unique path,

(3) T is minimally connected, i.e., $T - e$ is disconnected for every edge e of T,

(4) T is maximally acyclic, i.e., if u and v are disconnected in T, then adding the edge uv to T creates a cycle.

6. Let $\Delta(G)$ denote the maximum degree of G, i.e., $\max \deg v$ over all $v \in G$. Prove that every tree T has at least $\Delta(T)$ leaves.