Math 68
Exam #1
Fall term, 1999

This exam is to be turned in before noon in Ken Bogart's mailbox on Friday Oct. 29, 1999. You are on your honor not to discuss how to do any problems (or what their answers are) with any other people. You may ask your instructors for clarification; in the event that neither is around you may ask an advanced graduate student or another faculty member for clarification, and in this second case you should explain who you asked, how you asked them, and what they replied. You may use your problem notebook, any notes you have from class, and any notes you have taken while working on problems. This is an open book exam, but if you use a book, you are expected to reference it and make it clear what you got from it. Some of these problems have more than one solution. Extra credit will be given for each additional way in which you solve a problem.

1. In how many ways can you pass out k identical apples to n children? What if each child must get at least one? At least j?

2. In how many ways can you pass out k identical apples to n children so that no child gets more than j apples? Solve this problem in at least two ways.

3. Let $P(n,k)$ be the number of ways to make k nonempty lists of the elements of an n-element set S such that each element of S is in one and only one of these lists. Tell us as much as you can about $P(n,k)$.

4. Using m colors, in how many ways can we color the four (three-dimensional) diagonals of a cube free to move in space?

5. The group $\mathcal{C}$ of symmetries of the cube acts on the set D of (three-dimensional) diagonals of the cube. Thus we have a permutation representation $\varphi : \mathcal{C} \to S_D$, where S_D is the symmetric group on the set D. Is this representation faithful? Does φ help you say anything about the group $\mathcal{C}$? (If so, what does it help you say and why, and if not, why not?)