

## *Voting Systems and Desirable Properties of Voting Systems*

This list is not complete—there are many other voting systems and additional desirable properties of systems. I believe it covers the most important ones. Comments are welcome!

### *Voting systems*

I am going to describe some voting systems for a set  $V$  of voters and a set  $C$  of candidates. The systems are for elections that elect a single candidate. I am going to assume that each voter ranks the set of candidates and that the rankings are transitive. I'm leaving out the possibilities of having the voters rank two candidates equally—i. e., allowing a tie. There is almost no difference in allowing ties. I'll call the set of voters' candidates a "profile," a term used by Kenneth Arrow, who may have initiated it.

A system of voting is a procedure that converts a profile into a group ranking. I am also omitting consideration of a tie in the group ranking. If there is a tie for first place, Roberts Rules of Order say that the decision should be made by some random device, such as drawing straws or tossing a coin.

*Plurality:* The winner is the candidate with the largest number of first-place votes.

*Plurality with runoff:* If no candidate has a majority of the set of first-place votes, the decision is made by pairwise comparison of the rankings of the two candidates who get the largest number of first-place votes.

*Single transferable vote (STV):* If no candidate has a majority of the set of first-place votes, then the candidate with the smallest number of first-place votes is eliminated and the process is repeated. Eliminating a candidate amounts to altering the set of voters' rankings by crossing off the name of that candidate, and then examining the resulting rankings.

*Borda Count:* For each voter, each candidate,  $X$ , in the voter's ranking is assigned a number of points equal to the number of candidates ranked below  $X$ . To get the group ranking one adds, for each candidate, the number of points thus awarded by each voter.

*Coombs:* Similar to single transferable vote, except that instead of eliminating the candidate with the smallest number of first-place votes, one eliminated the candidate with the largest number of last-place votes.

*Condorcet:* (In a sense this isn't really a method of voting.) If there is a candidate  $X$  such that for every other candidate a majority of the voters prefer  $X$  to that candidate, then  $X$  wins. (The problem is that there may not be such a candidate.)

## *Desirable Properties of a Voting System*

The following list contains some properties that should always hold (I've marked with an asterisk those I feel should always hold), but many that we regard as desirable but for which there are circumstances in which we would tolerate a violation.—the group ranking should have the following properties:

*\*Transitivity:* The group ranking should be transitive—that is, if it ranks A above B and B above C, then it should rank A above C.

*\*Pareto optimality:* If everyone ranks A above B, then the group ranking should place A above B.

*Condorcet winner:* If there is a candidate X such that for every other candidate a majority of the voters prefer X to that candidate, then X should win.

*Condorcet loser:* If there is a candidate X such that for every other candidate a majority of the voters prefer that candidate to X, then X should not win.

*Monotonicity:* If for some profile the voting system produces A as a winner, then if it is applied to another profile that differs from the first only in having A ranked higher by some voters, then A should still win. It is only with great restraint that I haven't marked this with an asterisk. It comes very close to getting one from me.

*\*Second choice can make a difference:* The voting system shouldn't produce a result that depends solely on voters' first choices. Thus if my first choice is A, but I like B better than all the other candidates, there are circumstances in which my ranking of B affects the outcome.

*Manipulability:* A voting system is manipulable if there are circumstances in which I can cast a vote that ranks the candidate differently from my preferences so as to increase the likelihood that the results of the election will be more in accord with my preferences. A system of voting should avoid manipulability. (Note: This property does not get an asterisk because it cannot be completely satisfied according to the Gibbard-Satterthwaite theorem (spelling not guaranteed!)).

**A note on transitivity:** If one extends the idea of transitivity to include the following statement concerning ties: If the result of the group ranking is that A and B are ranked equal and it ranks B above C, then it should rank A above C. I am not giving this extended notion of transitivity an asterisk. I could consider a violation of this if necessary.