Math 75 – Homework #3

posted April 11, 2014; due Monday, April 14, 2014

Exercises

1. Suppose F is a finite field with q elements and let $f(x) \in F[x]$ have degree d. Show that f is irreducible if and only if the monic gcd of $f(x)$ and $x^{q^j} - x$ is 1 for each $j < d$.

2. Suppose F is a finite field with q elements and let $f(x) \in F[x]$ have degree d. Show that f is irreducible if and only if $f(x) \mid x^{q^d} - x$ and the monic gcd of $f(x)$ and $x^{q^j} - x$ is 1 for each $j < d$ with $j \mid d$.

3. Suppose p is a prime and F is a finite field with p^{15} elements. We know that the polynomial $x^{p^{15}} - x$ has every element of F as a root. Let K be the set of roots in F of the polynomial $x^{p^3} - x$. Show that K is a subfield of F.

4. Let $F = \mathbf{Z}/(2)$. Set $K = F[x]/(x^3 + x + 1)$ and $L = F[x]/(x^3 + x^2 + 1)$. Then both K and L are 8-element fields, so must be isomorphic. Find an isomorphism from K to L.