Homework

15.7 # 12 $\quad f(x,y) = xy - x^2y - xy^2$

$$\left.\begin{aligned} f_x &= y - 2xy - y^2 \\ f_y &= x - x^2 - 2xy \end{aligned}\right\} \text{ Set } = 0$$

(1) $y(1-2x-y) = 0 \qquad$ (2) $x(1-x-2y) = 0$

From (1) either (a) $y=0$ or (b) $y = 1-2x$

(a) gives $x(1-x) = 0$ in (2) so $x=0$ or $x=1$

$\underline{\underline{(0,0)}}$ and $\underline{\underline{(1,0)}}$ are critical points

(b) $y = 1-2x$ gives $x(1-x-2(1-2x)) = 0$ in (2)

so $x=0$ or $3x-1$ (so $x = 1/3$)

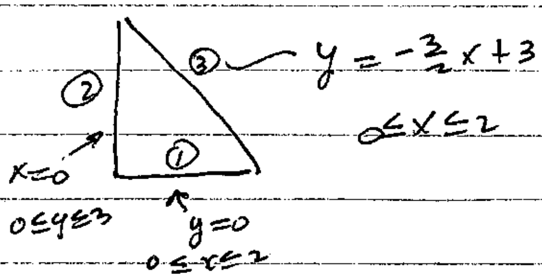
$\therefore$ $\underline{\underline{(0,1)}}$ and $\underline{\underline{(\frac{1}{3},\frac{1}{3})}}$ are critical point

Use 2nd derivative test to show $(0,0)$, $(1,0)$, $(0,1)$ are saddle points and $(\frac{1}{3},\frac{1}{3})$ is a local max.

15.7 # 27 $\quad f(x,y) = 1+4x-5y$

$f_x = 4$, $f_y = -5$ no critical points

Inside the triangle :



side ① $y=0$

$f(x,0) = 1+4x$

min $x=0$, max $x=2$

side ② $x=0$, $f(0,y) = 1-5y$ max $y=0$, min $y=3$

side ③ $y = -\frac{3}{2}x + 3$

$f = 1+4x-5(-\frac{3}{2}x+3) = -14 + 4x + \frac{15}{2}x =$

$\frac{23}{2}x - 14$ min $x=0$, max $x=2$

points to check $(0,0)$, $(2,0)$, $(0,3)$, the vertices of the triangle.

Show max value is at $(2,0) = 9$

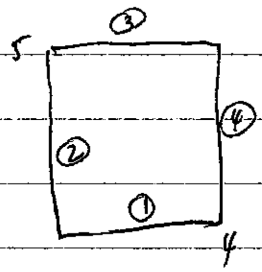
Show min value is at $(0,3) = -14$

16.7 : 30 $f(x,y) = 4x + 6y - x^2 - y^2$

$f_x = 4 - 2x$

$f_y = 6 - 2y$ $(2,3)$ critical point

On the sides of the rectangle



side ① $y=0$, $0 \le x \le 4$

$f(x,0) = 4x - x^2$

$f'(x,0) = 4 - 2x$ $x = 2$

point to check $(2,0)$ $(0,0)$, $(4,0)$

Similarly side ② additional points $(0,3)$, $(0,5)$.

Side ③ $y = 5$ $f(x,5) = 4x - x^2 + 5$

get $(2,5)$ and $(4,5)$

Side ④ $x = 4$ get $(4,3)$

Test all double underlined points for absolute max/min

$f(2,3) = 13$ abs. max

$f(0,0) = f(4,0) = 0$ abs. min

15.7 | 40 Minimize $\overset{w}{\overset{\shortparallel}{x^2+y^2+z^2}}$ where $x^2y^2z=1$

(Note: This problem can also be done with Lagrange Multipliers).

$$z = (x^2y^2)^{-1}$$

$$\therefore \quad w = x^2 + y^2 + (x^2y^2)^{-2}$$

$$w_x = 2x - 2(x^2y^2)^{-3}2xy^2 \qquad \text{Set} = 0$$

$$w_y = 2y - 2(x^2y^2)^{-3}(2x^2y)$$

$$\therefore \quad x = \frac{2xy^2}{x^6y^6} \qquad y = \frac{2x^2y}{x^6y^6}$$

$$x^6y^4 = 2 \qquad\qquad x^4y^6 = 2$$

$$\therefore x^6y^4 = x^4y^6 \qquad \therefore x^2 = y^2$$

$x = \pm y$ and from $x^6y^4 = 2$ we get $x^{10} = 2$

so $x = \pm 2^{1/10}$ $\quad y = \pm 2^{1/10}$ $\quad \therefore z = 2^{-4/5}$

The four points to check for absolute min. are

$(\pm 2^{1/10}, \pm 2^{1/10}, 2^{-4/5})$ These all give the same value in $x^2+y^2+z^2$ so they are all points on the surface closest to the origin.