Homework

15.8: 4 $\quad f(x,y) = 4x + 6y$

$g(x,y) = x^2 + y^2 = 13$ Use Lagrange

$4 = \lambda 2x \qquad x = \frac{2}{\lambda}$

$6 = \lambda 2y \qquad y = 3/\lambda$

$\left(\frac{2}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 = 13, \quad 13 = 13\lambda^2$

$\lambda^2 = 1 \qquad \lambda = \pm 1$

points to check $\underline{(2,3)}, \underline{(-2,-3)}$

$f(2,3) = 26$ abs. max

$f(-2,-3) = -26$ abs. min.

15.8: 10 $\quad f(x,y,z) = x^2 y^2 z^2$

$g(x,y,z) = x^2 + y^2 + z^2 = 1$ Lagrange Multipliers:

$2xy^2z^2 = \lambda 2x$ divide by 2, mult by x

$2x^2yz^2 = \lambda 2y$ ——— y

$2x^2y^2z = \lambda 2z$ ——— z

get $\lambda x^2 = \lambda y^2 = \lambda z^2$ If $\lambda \neq 0$,

$x^2 = y^2 = z^2$ so $3x^2 = 1$ (from $x^2 + y^2 + z^2 = 1$)

$\therefore x = \pm \frac{1}{\sqrt{3}}$ $\therefore$ points to check

$\underline{\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)}$. If $\lambda = 0$, one or two of the variables are 0 which gives f the value of 0. This is abs. min. At the other 8 points $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$ f has value $\frac{1}{27}$ which is abs. max.

15.8: 19 $\quad f(x,y) = e^{-xy} \qquad x^2 + 4y^2 \leq 1$

on $x^2 + 4y^2 < 1$ find critical pts.

$f_x = (-y)e^{-xy}, \quad f_y = (-x)e^{-xy}$ Set $= 0$

e to any power is positive

$\therefore$ $(0,0)$ is only critical point.

On boundary $g(x,y) = x^2 + 4y^2 = 1$ Use Lagrange

$(-y)\, e^{-xy} = \lambda 2x$

$(-x)\, e^{-xy} = \lambda 8y$

$e^{-xy} = -\dfrac{2\lambda x}{y}$

$(-x)\left(\dfrac{-2\lambda x}{y}\right) = \lambda \cdot 8y$

$\therefore 2\lambda x^2 = 8\lambda y^2$. Since $\lambda \neq 0$

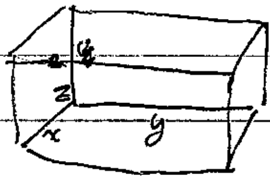
$x^2 = 4y^2$. From $x^2 + 4y^2 = 1$ we get

$2x^2 = 1$ $\quad x = \pm \dfrac{1}{\sqrt{2}}$, $\quad y = \pm \dfrac{1}{2\sqrt{2}}$

So check $\left(\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{2\sqrt{2}}\right)$ Check 5 ~~cases~~ doubly

underlined points to see which is abs. min and which abs. max.

p. 1013. 64.



$y + 2x + 2z \leq 108$

on interior $y + 2x + 2z < 108$

$f(x,y,z) = xyz$ (volume)

$f_x = yz$, $f_y = xz$, $f_z = xy$. set $= 0$

Only solution $(0,0,0)$ (obviously gives min. volume)

Now on boundary $f(x,y,z) = xyz$ $\quad g(x,y,z) = y + 2x + 2z = 108$ Lagrange method

$yz = \lambda 2$, $xz = \lambda$, $xy = \lambda 2$

$yz = 2xz$ so $y = 2x$; $xy = yz$ so $x = z$

$\therefore y + 2x + 2z = 108$ becomes $2x + 2x + 2x = 108$

$\therefore 6x = 108$ $\quad x = 18$, $z = 18$, $y = 36$