1 Determine which of the following series diverge, converge conditionally, and converge absolutely. Mention all tests you use. Remember that to show that a series converges *conditionally*, you show that $\sum a_n$ converges and also that $\sum |a_n|$ diverges (i.e., that the series does not converge absolutely).

a. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ **b.** $\displaystyle\sum_{n=1}^{\infty} 10^{10}\left(\frac{2}{3}\right)^n$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{e^n}$ **e.** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^{10}+2n-1}}{\sqrt{n^9+2n^2}}$ **f.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$

g. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n(\ln\ln n)}$ **h.** $\displaystyle\sum_{n=2} \frac{1}{n^{\ln\ln n}}$ **i.** $\displaystyle\sum_{n=2} \frac{1}{(\ln n)^{\ln n}}$

j. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n^4 + \sin n^5}{n^9}$ **k.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^3+2n+1)}{\sqrt{n^7-2n+10}}$ **l.** $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

2 Find the first two nonzero terms in the Maclaurin series for $f(x) = \tan x$.

3 Find the Maclaurin series for $f(x) = x\arctan(3x^3)$.

4 Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-5)^{n+2}(x-1)^n}{n^2}$.

5 Evaluate the following integrals.

c. $\displaystyle\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$ **d.** $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{x^2}{\sqrt{1-x^2}}\,dx$

e. $\displaystyle\int e^{3x}\cos x\,dx$ **f.** $\displaystyle\int x\ln x\,dx$

6 Find the point in which the line $x = 2-t$, $y = 1+3t$, $z = 4t$ intersects the plane $2x - y + z = 2$.

7 Determine whether the planes given by $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 3$ are parallel, perpendicular, or neither. If neither, find the angle between them.

8 Determine whether the planes given by $3x + 6z = 1$ and $2x + 2y - z = 3$ are parallel, perpendicular, or neither. If neither, find the angle between them.

9 Find an equation of the plane which contains the x-axis as well as the line given by the parametric equations $x = t$, $y = 2t$, $z = 3t$.

10 Find an equation of the plane which contains the origin and the line $x = 6t + 2$, $y = 2 - 4t$, $z = 9$.

11 Find the arc length of the curve $\mathbf{r}(t) = \cos^3 t\mathbf{j} + \sin^3 t\mathbf{k}$ from $t = 0$ to $t = 1$.

12 Suppose the gradient of $f(x, y, z)$ is

$$\nabla f = \langle 2xyz + 2e^z, x^2 z - \cos y, x^2 y + 2e^z \rangle,$$

and that $x = s^2 t$, $y = t^3$, and $z = e^s$. What is $\dfrac{\partial f}{\partial s}$? You need not simplify your answer, but it should not contain ∂ symbols.

13 Consider the function $f(x, y) = x^3 + y^2 - xy$. At the point $(1, 1)$, in what direction(s) is the rate of change of f equal to zero? Give your answer as one or more unit vectors.

14 Find the rate of change of the function $f(x, y) = \sqrt{24 - x^2 - y^2}$ at the point $(4, -2)$ in the direction given by $\theta = \pi/6$. In what direction does f attain its maximum rate of change at the point $(4, -2)$? (You need not specify this direction by an angle.)

15 Let $f(x, y, z) = ye^{-x^2} \sin z$. Find the equation of the tangent plane to the level syrface of f at the point $(0, 1, \pi/3)$.

16 A ball is placed at the point $(1, 2, 3)$ on the surface $z = y^2 - x^2$. Give the direction in the xy-plane that the ball will start to roll.

17 Find and classify all critical points of the function $f(x, y) = 3x - x^3 - 3xy^2$.

18 Find and classify all critical points of the function $f(x, y) = x^3 + y^4$.

19 Let $f(x, y) = x \sin y$.

a. Compute f_x, f_y, f_{xx}, f_{xy}, and f_{yy}.

b. What are the critical points of f?

c. Classify the critical points of f.

d. Find the absolute maximum and minimum of f on the region given by $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$.

20 Find the maximum and minimum of $f(x, y) = x^2 + 2x + y^2$ on the disk $x^2 + y^2 \le 4$.