NAME : _____

SECTION : (circle one) 12:30–1:35 1:45–2:50

# Math 8

Practice
Midterm 1

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all problems.

- ***Print*** your name in the space provided and circle your instructor's name.
- Mark your multiple choice answers on the final page of *this booklet*. The multiple choice booklet *will not* be collected.
- Sign the FERPA release on the next page *only if* you wish your exam returned in lecture.
- Calculators or other computing devices are not allowed.
- Use the blank page at the end of the exam for scratch work.
- **Except in the multiple choice section, you must show all work and give a reason (or reasons) for your answer. A CORRECT ANSWER WITH INCORRECT WORK WILL BE CONSIDERED WRONG.**

1. (10) Compute the Taylor polynomial of degree 2 centered at $x = 0$ for the function $f(x) = e^{x^2}$.

(b) Use the Remainder Theorem to give a bound on the error involved in using this Taylor polynomial to approximate $f(x)$ at $x = 1$.

2. (10) Find the sum of the series

$$\sum_{n=3}^{\infty} \frac{47^n - 3^n}{7^{2n}}.$$

You do *not* need to simplify your answer.

3. (10) Does the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)(\ln \ln n)}$$

converge absolutely, converge conditionally, or diverge? You should mention any tests you apply, and make sure that the series satisfies the conditions of those tests.

4. (10) Does the series

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2 + 7}$$

converge absolutely, converge conditionally, or diverge? You should mention any tests you apply, and make sure that the series satisfies the conditions of those tests.

5. (10) Find the interval of convergence of the power series

$$\sum_{n=3}^{\infty} \frac{(2-2x)^n}{8^n \ln n}.$$

6. (10) Express the integral

$$\int \frac{\sin x - x}{x}\, dx$$

as an infinite series.

(b) Where does this series converge?

7. (4) Suppose $\sum_{n=1}^{\infty} a_n = 3$ and $s_n$ is the $n$th partial sum of the series. Which of the following statements is true?

**A**. $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} s_n$ need not exist

**B**. $\lim_{n\to\infty} a_n < 1$ and $\lim_{n\to\infty} s_n = 0$

**C**. $\lim_{n\to\infty} a_n = 3$ and $\lim_{n\to\infty} s_n = \infty$

**D**. $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} s_n = 3$

**E**. $\lim_{n\to\infty} a_n$ need not exist and $\lim_{n\to\infty} s_n = 3$

8. (4) Consider the series $\sum_{n=10}^{\infty} \frac{1}{n^{\ln\ln n}}$. Which of the following arguments is correct?

**A**. this series diverges by the Test for Divergence

**B**. this series diverges by comparison to $\sum 1/n$

**C**. this series converges by comparison to $\sum 1/n$

**D**. this series diverges by comparison to $\sum 1/n^2$

**E**. this series converges by comparison to $\sum 1/n^2$

9. (4) Which of the following series could the Integral Test be applied to?

| $I$ | $II$ | $III$ |
|---|---|---|
| $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2}$ | $\sum_{n=1}^{\infty} \frac{17}{n(\ln n)^{13}}$ | $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n}$ |

**A**. None

**B**. $I$ only

**C**. $II$ only

**D**. $III$ only

**E**. $I$ and $II$ only

**F**. $I$ and $III$ only

**G**. $II$ and $III$ only

**H**. $I$, $II$, and $III$

10. (4) Which of the following series converge (either absolutely or conditionally)?

| $I$ | $II$ | $III$ |
|---|---|---|
| $\sum_{n=1}^{\infty} (-1)^n \frac{2n-4}{\sqrt{4n^3+n}}$ | $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ | $\sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}}$ |

**A**. None

**B**. $I$ only

**C**. $II$ only

**D**. $III$ only

**E**. $I$ and $II$ only

**F**. $I$ and $III$ only

**G**. $II$ and $III$ only

**H**. $I$, $II$, and $III$

11. (4) Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{\sin n}{n}$?

| | |
|---|---|
| $I$ | The series diverges by the Integral Test. |
| $II$ | The series diverges by comparison to the harmonic series. |
| $III$ | The series diverges by the Test for Divergence. |

**A**. None

**B**. $I$ only

**C**. $II$ only

**D**. $III$ only

**E**. $I$ and $II$ only

**F**. $I$ and $III$ only

**G**. $II$ and $III$ only

**H**. $I$, $II$, and $III$

12. (4) Suppose that $f(x) = \sum_{n=0}^{\infty} \frac{n^2(x-3)^n}{2^n}$ for $|x-3| < 2$. What is $f^{(38)}(3)$? (The 38th derivative of $f$ at 3.)

**A**. $\frac{38^2}{2^{38}\,38!}$

**B**. $\frac{38^2}{2^{38}}$

**C**. $\frac{38^2\,38!}{2^{38}}$

**D**. $\frac{38}{2^{38}}$

**E**. $\frac{38!}{2^{38}}$

13. (4) What is $\lim_{n\to\infty} \frac{n!}{n^n}$?

**A**. 0

**B**. $1/e$

**C**. 1

**D**. $e$

**E**. $\infty$

14. (4) What is the value of the improper integral $\int_e^\infty \frac{dx}{x(\ln x)^2}$?

**A**. 0

**B**. $1/e$

**C**. 1

**D**. $e$

**E**. The integral diverges

15. (4) Which of the following is the Taylor series centered at $x = 0$ for $f(x) = \cos 3x^3$?

**A**. $\displaystyle\sum_{n=0}^{\infty} 9^n \frac{x^{6n}}{(2n)!}$

**B**. $\displaystyle\sum_{n=0}^{\infty} 9^n \frac{x^{2n+3}}{(2n)!}$

**C**. $\displaystyle\sum_{n=0}^{\infty} (-9)^n \frac{x^{2n+3}}{(2n)!}$

**D**. $\displaystyle\sum_{n=0}^{\infty} (-9)^n \frac{x^{6n}}{(2n)!}$

**E**. None of the above

16. (4) Which of the following series could be rearranged to sum to 3?

| $I$ | $II$ | $III$ |
|---|---|---|
| $\displaystyle\sum_{n=1}^{\infty} (-1)^n$ | $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ | $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^n}$ |

**A**. None

**B**. $I$ only

**C**. $II$ only

**D**. $III$ only

**E**. $I$ and $II$ only

**F**. $I$ and $III$ only

**G**. $II$ and $III$ only

**H**. $I$, $II$, and $III$