Minority Opinion and the Vaccination Game

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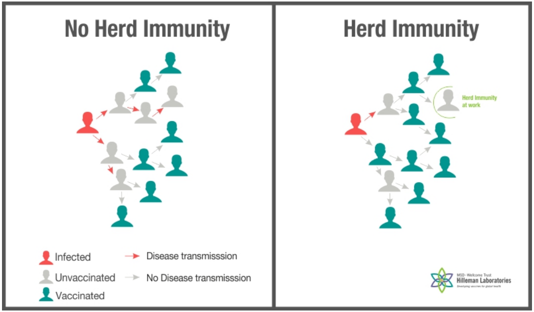
Outline

- Background
- Vaccine Game
- Deterministic Model
- Stochastic Model
- Cellular Automaton and Network Dynamics
- Results and Discussion

Vaccination and Herd Immunity

- Direct vs. indirect protection from disease
  - Direct: vaccinated individuals have immunity against disease
  - Indirect: susceptible individuals are sheltered by the immunity of others
- Elimination of smallpox & eradication of polio, measles, etc.
- As vaccination compliance increases, unvaccinated members are less motivated to vaccinate



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Finally, we impose lattice-like neighborhoods to visualize the spread of the biological and social contagions

Assumptions

1. Well-mixed
2. Population remains constant
3. Vaccine grants perfect immunity
4. Individuals do not alter their vaccination strategy in the midst of an epidemic

Game Setup

Let a denote the proportion of the population that belongs to G_1.

There are three potential outcomes to this game:

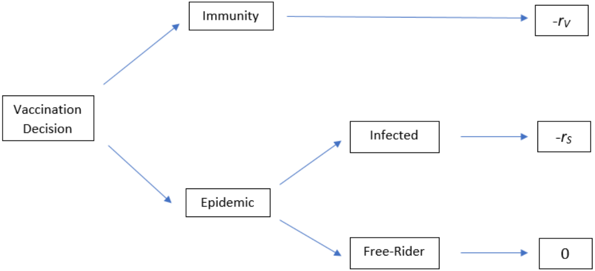

Figure 1: Vaccination Game Flow Chart

Expected Payoffs

Expected Payoffs:

$$\begin{aligned} E(V_i) &= -r_{vi} \\ E(NV) &= -\pi_p \times r_s + (1 - \pi_p) \times 0 \end{aligned}$$

where π_p is the probability of infection.

Nash Equilibria

To find the Nash Equilibria[1] for this game, we set

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[1] Fu, Rosenbloom, Wang, Nowak 2011

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R(\infty) &= -\frac{\log(1-r_i)}{R_0} \\
x^*{}_i &= 1 + \frac{\log(1-r_i)}{R_0 \times r_i}
\end{aligned}$$

The optimal strategy for each subgroup depends on their perceived risk ratio, r_i.

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[1] Fu, Rosenbloom, Wang, Nowak 2011

Initial Conditions

- $r_s = 1$ (both groups)
- $r_{v1} = 1/100$
- $r_{v2} = 1/20$
- We vary *a* to observe how the size of the minority group affects our population.

SIR Model

Kermack-McKendrick Epidemic Model[2]

$$\begin{aligned}
\frac{dS}{dt} &= -\beta SI + \mu(1-V) - \mu S \\
\frac{dI}{dt} &= \beta SI - \gamma I - \mu I \\
\frac{dR}{dt} &= \gamma I + \mu V - \mu R
\end{aligned}$$

Parameters: β, γ, μ

[2] Kermack, McKendrick 1927

Vaccination Dynamics

The state of the epidemic affects the change in the vaccination compliance:

$$\frac{dV_1}{dt} = V_1(1 - V_1)(-r_1 + I)$$
$$\frac{dV_2}{dt} = V_2(1 - V_2)(-r_2 + I)$$

Results

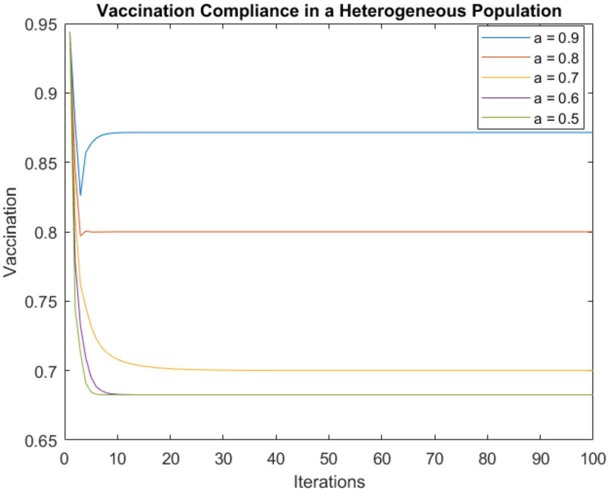


Figure 2: Vaccination compliance at varying levels of a.

Results

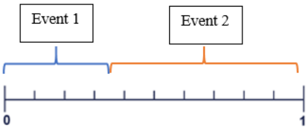
a	iter = 1	iter = 100
0.9	0.9439	0.8715
0.8	0.9436	0.8000
0.7	0.9434	0.7000
0.6	0.9431	0.6826
0.5	0.9428	0.6826

Table 1: Vaccination rates at varying levels of a

Transition to Stochastic Model

- Stochastic models incorporate an element of randomness typical of biological processes
- Gillespie Algorithm (Stochastic Simulation Algorithm)[3] [4]
  1. Initialization
  2. Time Component

  3. Event Component



  4. Iterate and Repeat

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[3] Martinez-Urreaga, Mira, Gonzalez-Fernandez 2003

[4] Regoes, Schafroth

Stochastic Model

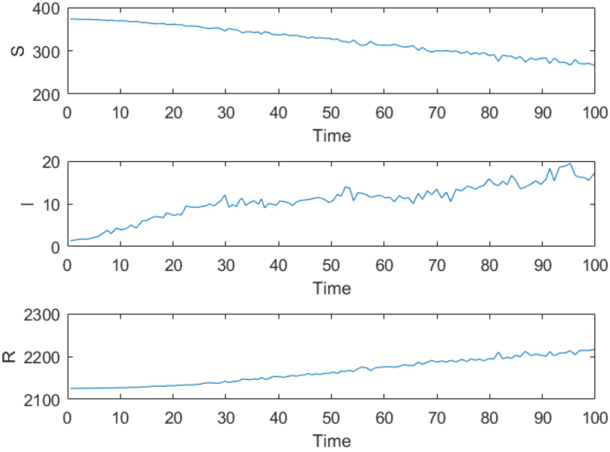


Figure 3: Stochastic SIR Model

Cellular Automaton & Neighborhood Dynamics

- Consider our "well-mixed" assumption from earlier
- We use the lattice structure of cellular automaton to simulate an individuals' interactions with neighbors
- Neighbors transmit infections and serve as role models in the vaccination game

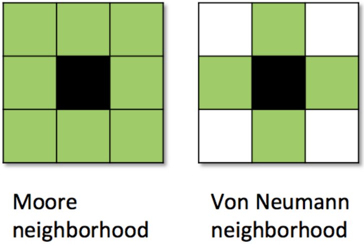


Figure 4: Types of 2-dimensional neighborhoods

Cellular Automaton & Neighborhood Dynamics

- Cyclic model has two stages:
  1. Vaccination Decision
     - Individuals evaluate the payoffs of their neighbors and "switch" to that strategy with probability [5]

$$1 - \frac{1}{1 + e^{-k(f_n - f_m)}}$$

  2. Epidemic
- SIR Demonstration

[5] Fu, Rosenbloom, Wang, Nowak 2011

Cellular Automaton & Neighborhood Dynamics

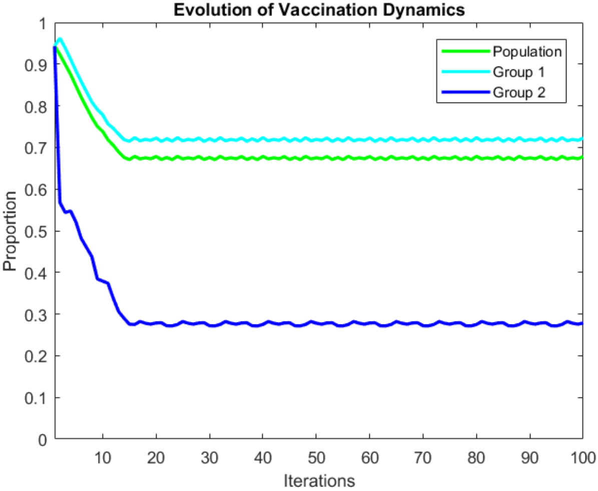


Figure 5: Vaccine Adherence with a = 0.9

Cellular Automaton & Neighborhood Dynamics

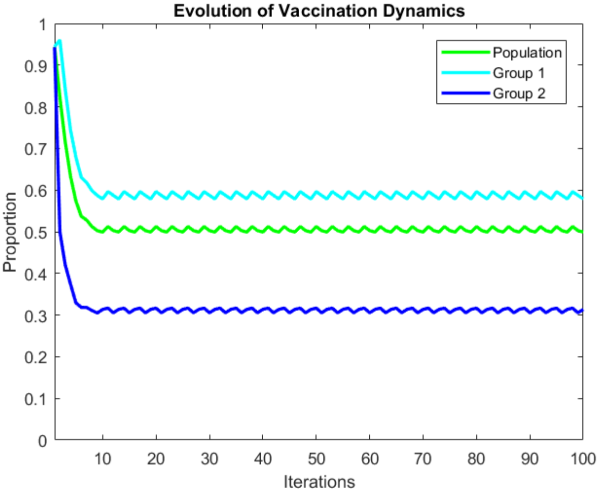
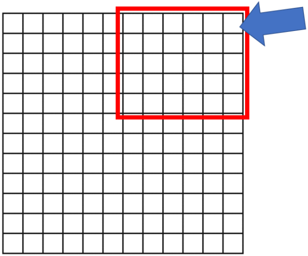


Figure 6: Vaccine Adherence with a $= 0.7$

Regionalization

- Before, we assumed that members of G_1 and G_2 were scattered randomly through the population
- Now, we look at a model where the groups are kept separate



Regionalization

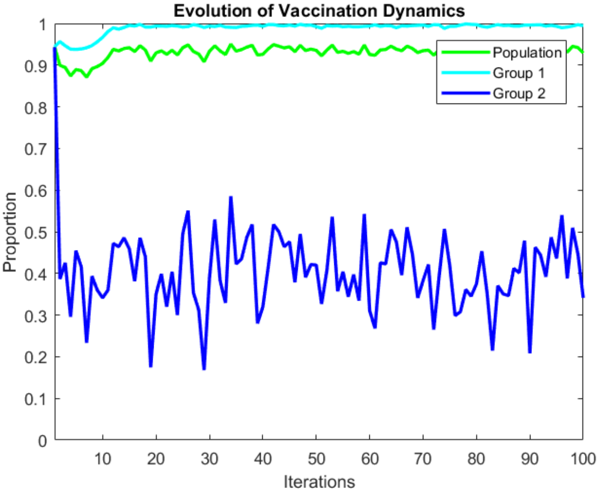


Figure 7: Regional Vaccine Compliance at a = 0.9

Regionalization

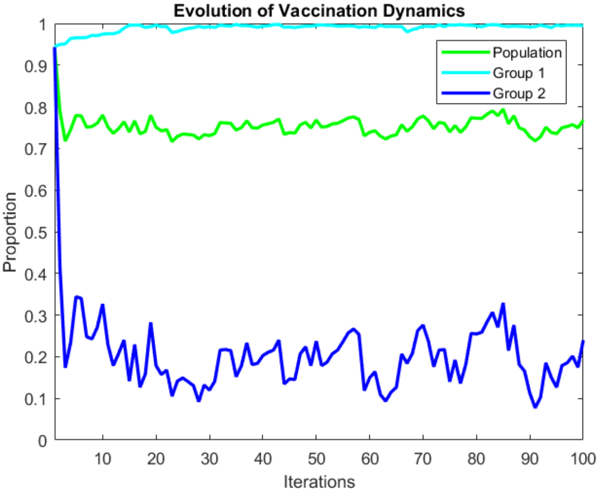


Figure 8: Regional Vaccine Compliance at a $= 0.7$

Results and Discussion

- Comparison of deterministic and stochastic CA models
- Effects of minority opinion
  - G_2 are less likely to vaccinate, causes G_1 to compensate on their behalf (deterministic)
  - G_2 are more likely to attempt free-riding, act as "bad influences" for G_1 neighbors
  - When we separate G_2 from the majority, vaccination rates for G_1 increased significantly
- Ideas for further research