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IDEALS IN TRANSFORMATION-GROUP C^*-ALGEBRAS

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Abstract. We characterize the ideal of continuous-trace elements in a separable transformation-group C^*-algebra $C_0(X) \rtimes G$. In addition, we identify the largest Fell ideal, the largest liminal ideal and the largest postliminal ideal.

Keywords: *Transformation group, continuous-trace C^*-algebras, liminal and postliminal C^*-algebras.*

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1. INTRODUCTION

Let (G, X) be a locally compact Hausdorff transformation group: thus G is a locally compact Hausdorff group and X is a locally compact Hausdorff space together with a jointly continuous map $(s, x) \mapsto s \cdot x$ from $G \times X$ to X such that $s \cdot (t \cdot x) = st \cdot x$ and $e \cdot x = x$. The associated transformation-group C^*-algebra $C_0(X) \rtimes G$ is the C^*-algebra which is universal for the covariant representations of the C^*-dynamical system $(C_0(X), G, \alpha)$ in the sense of [20]. More concretely, $C_0(X) \rtimes G$ is the enveloping C^*-algebra of the Banach $*$-algebra $L^1(G, C_0(X))$ of functions $f : G \to C_0(X)$ which are Bochner integrable with respect to a fixed left Haar measure on G (cf. Section 7.6 from [18]). In the main results, we will always assume that G and X are second countable so that $C_0(X) \rtimes G$ is separable. In our main results, we assume either that G is abelian or that G acts freely.

It is natural to attempt to characterize properties of $C_0(X) \rtimes G$ in terms of the dynamics of the action of G on X, and there are a large number of results of this sort in the literature ([9], [13], [23], [24], [25], [15] and [16]). We were motivated by a particularly nice example due to Green (Corollary 18 of [13]) in which he was able to characterize the closure I of the ideal of continuous-trace elements in $C_0(X) \rtimes G$ in the case G acts freely and $C_0(X) \rtimes G$ is postliminal. (Since we'll be working exclusively with separable C^*-algebras, we will not distinguish between Type I and postliminal algebras.) There are three ingredients required for this sort of project. First, one needs a global characterization of algebras $C_0(X) \rtimes G$

which have continuous trace. Second, one needs to know that the ideal I is of the form $C_0(Y) \rtimes G$ for an open G-invariant set Y in X. And third, one wants a straightforward characterization of Y in terms of the dynamics. Assuming that G acts freely, Green showed that $C_0(X) \rtimes G$ has continuous trace if and only if every compact set $K \subset X$ is *wandering* in that

$$\{s \in G : s \cdot K \cap K \neq \emptyset\}$$

is relatively compact in G (Theorem 17 from [13]). If $C_0(X) \rtimes G$ is postliminal, then G acting freely implies every ideal is of the form $C_0(Y) \rtimes G$, and Green showed $I = C_0(Y) \rtimes G$ where

$$Y = \{y \in X : y \text{ has a compact wandering neighborhood } N \text{ such that } q(N) \text{ is closed and Hausdorff}\}, \tag{1.1}$$

where $q : X \to X/G$ is the quotient map. (The criteria in (1.1) are slightly different than those given by Green; unfortunately, the statement in Corollary 18 from [13] is not quite correct — see Remark 3.5.)

To extend Green's results to actions which are not necessarily free, we relied (i) on the second author's result (Theorem 5.1 from [24]) stating that if G is abelian then $C_0(X) \rtimes G$ has continuous trace if and only if the stability groups move continuously and every compact set is *G-wandering* as defined in Section 3, and (ii) on a result of N.C. Phillips which allows us to assume the ideal in question is of the form $C_0(Y) \rtimes G$. Our characterization is given in Theorem 3.10 and is valid for abelian groups, freely acting amenable groups, or freely acting groups for which $C_0(X) \rtimes G$ is postliminal.

For abelian groups or freely acting groups, Gootman showed that $C_0(X) \rtimes G$ is postliminal if and only if the orbit space X/G satisfies the T_0 axiom of separability (Theorem 3.3 of [9]). Similarly $C_0(X) \rtimes G$ is liminal if and only if each orbit is closed (Theorem 3.1 from [23]). Using these results, we give characterizations of the largest postliminal and liminal ideals in $C_0(X) \rtimes G$ in Theorems 3.16 and 3.14, respectively.

The set of $a \in A^+$ such that $\pi \mapsto \operatorname{tr} \pi(a)$ is bounded on $\widehat{A}$ is the positive part of a two-sided ideal $\mathcal{T}(A)$. If $\mathcal{T}(A)$ is dense in A, then A is said to have *bounded trace*. Such algebras are also *uniformly liminal* (Theorem 2.6, [2]). The first author has characterized when $C_0(X) \rtimes G$ has bounded trace (Theorem 4.9, [15]), and she has used this to find the largest bounded trace ideal in Theorem 5.8 from [15]. An intermediate condition between A being a continuous-trace C^*-algebra and an algebra with bounded trace is that A be a *Fell algebra*. A point $\pi \in \widehat{A}$ is called a *Fell point* of the spectrum if there is a neighborhood V of π and $a \in A^+$ such that $\rho(a)$ is a rank-one projection for all $\rho \in V$. Then A is a Fell algebra if every $\pi \in \widehat{A}$ is a Fell point, and a Fell algebra is a continuous-trace C^*-algebra if and only if $\widehat{A}$ is Hausdorff (cf. Section 5.14 of [22]). If G acts freely, then $C_0(X) \rtimes G$ is a Fell algebra if and only if X is a Cartan G-space ([16]), and we treat the case of continuously varying stabilizers below (Proposition 3.3). Using these results, we identify the largest Fell ideal in $C_0(X) \rtimes G$ when the stability groups vary continuously (Corollary 3.4).

Naturally our techniques depend on describing ideals in $C_0(X) \rtimes G$ in terms of the dynamics. To do this, we need to know that each primitive ideal in $C_0(X) \rtimes G$ is induced from a stability group (cf. Definition 4.12 from [23]). Cross products with this property are called *EH-regular*, and in the separable case it suffices for G to be amenable ([12]) or for the orbit space X/G to be T_0 (Proposition 20 from [14]). Therefore, if G is abelian then $C_0(X) \rtimes G$ is EH-regular. If G acts freely, then we will have to assume either that G is amenable or the orbit space is T_0. If the action is free and $C_0(X) \rtimes G$ is EH-regular, then ideals in $C_0(X) \rtimes G$ are in one-to-one correspondence with G-invariant open sets Y in X. If G does not act freely, then we must assume that G is abelian so that we can employ the dual action to conclude that the ideals we are interested in correspond to G-invariant open subsets of X.

2. INVARIANCE OF IDEALS UNDER THE DUAL ACTION

Although ideals in $C_0(X) \rtimes G$ can be difficult to describe in general, there is always an ideal associated to each G-invariant open subset Y of X. The closure of $C_c(G \times Y)$ (viewed as a subset of $C_c(G \times X)$) is an ideal in $C_0(X) \rtimes G$ which we can identify with $C_0(Y) \rtimes G$ (cf., e.g., Lemma 1 of [13]). When the action of G is free and $C_0(X) \rtimes G$ is EH-regular, Corollary 5.10 from [23] implies that $\mathrm{Prim}\big(C_0(X) \rtimes G\big)$ is homeomorphic to the quotient space $(X/G)^{\sim}$ of X/G where $G\cdot x$ is identified with $G\cdot y$ if $\overline{G\cdot x} = \overline{G\cdot y}$. It follows that every ideal of $C_0(X) \rtimes G$ is of the form $C_0(Y) \rtimes G$ for some G-invariant open set Y.

When G is abelian and does not necessarily act freely, we can distinguish those ideals of $C_0(X) \rtimes G$ of the form $C_0(Y) \rtimes G$ via the dual action. Indeed, let $\widehat{G}$ denote the Pontryagin dual of G. The *dual action* $\widehat{\alpha}$ of $\widehat{G}$ on $C_0(X) \rtimes G$ is given by

$$\widehat{\alpha}_\tau(f)(s) = \tau(s) f(s, \cdot) \quad \text{for } f \in C_c(G \times X) \text{ and } \tau \in \widehat{G}.$$

The induced action of $\widehat{G}$ on $(C_0(X) \rtimes G)^\wedge$ is $\tau \cdot \pi = \pi \circ \widehat{\alpha}_\tau^{-1}$, and this action is jointly continuous (cf., e.g., Lemma 7.1 of [22]). The importance of the dual action for us comes from the following lemma due to N.C. Phillips.

LEMMA 2.1. (Proposition 6.39, [19]) *Suppose that (G, X) is a second countable transformation group with G abelian. If I is a $\widehat{G}$-invariant ideal of $C_0(X) \rtimes G$, then there is an open G-invariant set Y in X such that $I = C_0(Y) \rtimes G$.*

As an example, note that it is easy to see that the set of Fell points of the spectrum is invariant under the dual action. If π is a Fell point, then by definition there exist $a \in A^+$ and an open neighborhood V of π in $\widehat{A}$ such that $\sigma(a)$ is a rank-one projection for all $\sigma \in V$. If $b = \widehat{\alpha}_\tau(a)$ then for every $\rho \in \tau \cdot V$ we have $\rho(b) = \sigma(a)$ for some $\sigma \in V$. Hence $\tau \cdot \pi$ is also a Fell point. Thus the largest Fell ideal must be of the form $C_0(Y) \rtimes G$.

Recall that a positive element a of a C^*-algebra A is a *continuous-trace element* if the function $\pi \mapsto \mathrm{tr}(\pi(a))$ is finite and continuous on $\widehat{A}$. The linear span $m(A)$ of these elements is an ideal in A, and A is a *continuous-trace C^*-algebra* if $m(A)$ is dense in A.

We want to prove that $\overline{m(A)}$ is invariant under the dual action. To do this, we need a lemma of Green which characterizes this ideal by determining its irreducible representations. Recall that if I is an ideal of a C^*-algebra A, then the spectrum $\widehat{I}$ of I is homeomorphic to the open set $\mathcal{O}_I := \{\rho \in \widehat{A} : \rho|_I \neq 0\}$ in $\widehat{A}$. We will also use that every C^*-algebra A has a dense hereditary ideal $\kappa(A)$ — called the *Pedersen ideal* of A — which is the smallest dense ideal in A (Theorem 5.6.1 of [18]). As Green's result is an essential ingredient in many of our proofs, we give the brief argument here. The key idea of the proof is that $\pi\big(m(A)\big) \neq \{0\}$ if and only if π has lots of *closed* neighborhoods in $\widehat{A}$.

LEMMA 2.2. (p. 96, [13]) *Let A be a C^*-algebra and $I = \overline{m(A)}$. Then $\pi \in \mathcal{O}_I$ if and only if*

(i) *there exists an ideal J of A which has continuous trace such that $\pi \in \mathcal{O}_J$; and*

(ii) *π has a neighborhood basis consisting of closed sets.*

Proof. Let $\pi \in \mathcal{O}_I$. There exists a positive element $a \in m(A)$ such that $\operatorname{tr}(\pi(a)) = 1$. It follows that the set

$$L = \Big\{\rho \in \widehat{A} : \operatorname{tr}(\rho(a)) \geqslant \frac{1}{2}\Big\}$$

is a closed neighborhood of π and $L \subset \mathcal{O}_I$. Let $\{F_\alpha\}$ be a compact neighborhood basis of π in $\widehat{A}$. Notice that L is Hausdorff since $\mathcal{O}_I$ is. Thus $F_\alpha \cap L$ is closed in L, and therefore in $\widehat{A}$ as well. It follows that $\{F_\alpha \cap L\}$ is a neighborhood basis of π consisting of closed sets. This proves item (ii). That item (i) holds is obvious (just take $J = I$).

Conversely, let $\pi \in \widehat{A}$ satisfy items (i) and (ii). Then there exists an ideal $J_0 \subset J$ of A such that $\pi \in \mathcal{O}_{J_0}$ and $\overline{\mathcal{O}}_{J_0} \subset \mathcal{O}_J$. Let a be a positive element of the Pedersen ideal $\kappa(J_0)$ of J_0. Then $\rho \mapsto \operatorname{tr}(\rho(a))$ is continuous on $\mathcal{O}_J$ because $\kappa(J_0) \subset \kappa(J) \subset m(J)$. Since $\rho \mapsto \operatorname{tr}(\rho(a))$ vanishes off of $\mathcal{O}_{J_0}$, it is continuous on all of $\widehat{A}$. Thus $\kappa(J_0) \subset m(A) \subset I$, whence $J_0 \subset I$, and $\pi \in \mathcal{O}_I$. ∎

PROPOSITION 2.3. *Let (G, X) be a second countable transformation group with G abelian. Then $I = \overline{m(C_0(X) \rtimes G)}$ is $\widehat{G}$-invariant, and $I = C_0(Y) \rtimes G$ for some open G-invariant subset Y of X.*

Proof. We use Lemma 2.2 to show that $\tau \cdot \pi \in \mathcal{O}_I$ whenever $\pi \in \mathcal{O}_I$ and $\tau \in \widehat{G}$. If $\pi \in \mathcal{O}_I$ then there exists an ideal J of A with continuous trace such that $\pi \in \mathcal{O}_J$. Note that $\tau \cdot \pi \in \tau \cdot \mathcal{O}_J = \mathcal{O}_{\tau \cdot J}$, where $\tau \cdot J = \widehat{\alpha}_\tau(J)$. Since J has continuous trace each element ρ of $\mathcal{O}_J$ is a Fell point and $\mathcal{O}_J$ is Hausdorff. Thus $\tau \cdot \mathcal{O}_J$ is also Hausdorff, and each point $\tau \cdot \rho$ in $\tau \cdot \mathcal{O}_J$ is a Fell point. It follows that $\tau \cdot J$ is an ideal of A with continuous trace and $\tau \cdot \pi \in \mathcal{O}_{\tau \cdot J}$.

Finally, if $\{F_\alpha\}$ is a neighborhood basis of π consisting of closed sets then $\{\tau \cdot F_\alpha\}$ is a neighborhood basis of $\tau \cdot \pi$ with the same properties. Thus $\tau \cdot \pi \in \mathcal{O}_I$ by Lemma 2.2.

We have shown that $\mathcal{O}_I$ and hence I are $\widehat{G}$-invariant. The final assertion follows from Lemma 2.1. ∎

More generally, for an amenable C^*-dynamical system (A, G, α), an ideal I of $A \rtimes_\alpha G$ is invariant under the dual coaction if and only if $I = J \rtimes_\alpha G$ for some unique, α-invariant ideal in J of A (Theorem 3.4 from [10]). Since we use a representation theoretic approach to identify $\overline{m(C_0(X) \rtimes G)}$ there are two obstacles to extending our techniques to non-abelian groups. First, there is no notion of induced coaction on $(C_0(X) \rtimes G)^\wedge$, and second, we do not have a concrete description of $(C_0(X) \rtimes G)^\wedge$ in terms of X and G.

If G is abelian, consider the quotient space obtained from $X \times \widehat{G}$ where

$$(x, \omega) \sim (y, \tau) \quad \text{if and only if} \quad \overline{G \cdot x} = \overline{G \cdot y} \text{ and } \omega|_{S_x} = \tau|_{S_y}.$$

This identification makes sense because $\overline{G \cdot x} = \overline{G \cdot y}$ implies $S_x = S_y$ for abelian groups. Since we're assuming (G, X) is second countable, Theorem 5.3 from [23] implies that

$$[(x, \omega)] \mapsto \ker \Big(\operatorname{Ind}_{(x, S_x)}^G (\omega|_{S_x}) \Big)$$

is a homeomorphism of $X \times \widehat{G}/\sim$ onto $\operatorname{Prim}(C_0(X) \rtimes G)$. We write $\pi_x(\omega)$ for $\operatorname{Ind}_{(x,S_x)}^G(\omega|_{S_x})$. As noted in the paragraph following the proof of Theorem 5.3 from [23], the map sending (x, ω) to $\ker \pi_x(\omega)$ is open from $X \times \widehat{G}$ onto $\operatorname{Prim}\big(C_0(X) \rtimes G\big)$. In particular, sets of the form $U \times V/\sim$, with U and V open in X and $\widehat{G}$, respectively, form a basis for the topology on $\operatorname{Prim}\big(C_0(X) \rtimes G\big)$.

Let $\Sigma(G)$ denote the space of closed subgroups of G endowed with the compact Hausdorff topology from [7]. The stability subgroups S_x are said to *vary continuously* if the map $\sigma : X \to \Sigma(G) : x \mapsto S_x$ is continuous.

If A is a Fell algebra and $\pi \in \widehat{A}$ then π has an open Hausdorff neighborhood in $\widehat{A}$ (Corollary 3.4 from [1]). We want to be able to choose this neighborhood to be $\widehat{G}$-invariant.

LEMMA 2.4. *Suppose (G, X) is a second countable transformation group with G abelian and with continuously varying stability groups. If $C_0(X) \rtimes G$ is a Fell algebra, then every irreducible representation of $C_0(X) \rtimes G$ has an open $\widehat{G}$-invariant Hausdorff neighborhood in $(C_0(X) \rtimes G)^\wedge$.*

Proof. Since $C_0(X) \rtimes G$ is postliminal, we can identify $\operatorname{Prim}\big(C_0(X) \rtimes G\big)$ and $(C_0(X) \rtimes G)^\wedge$. We can view $(C_0(X) \rtimes G)^\wedge$ as the appropriate quotient of $X/G \times \widehat{G}$, and then the map $(G \cdot x, \omega) \mapsto [\pi_x(\omega)]$ is an open surjection onto $(C_0(X) \rtimes G)^\wedge$ (Theorem 5.3 of [23]). In particular, (the class of) $\pi := \pi_x(\omega)$ is a typical element of $(C_0(X) \rtimes G)^\wedge$. Since A is a Fell algebra, π has an open Hausdorff neighborhood ([1]) which is of the form $\mathcal{O}_J$ for some closed ideal J of A. We can shrink J a bit if need be, and assume that there are open neighborhoods U of $G \cdot x$ in X/G and V of ω in $\widehat{G}$ such that $U \times V/\sim$ is homeomorphic to $\mathcal{O}_J$. Suppose that $G \cdot x$ and $G \cdot y$ are distinct points in U. Note that each orbit is closed in X because $C_0(X) \rtimes G$ is liminal (Theorem 3.1 of [23]). Thus, for each $\omega \in V$, the points $[G \cdot x, \omega]$ and $[G \cdot y, \omega]$ are distinct in $U \times V/\sim$. Since $\mathcal{O}_J = U \times V/\sim$ is Hausdorff and $z \mapsto [G \cdot z, \omega]$ is continuous, we can separate $G \cdot x$ and $G \cdot y$ by G-invariant open sets and it follows that U is Hausdorff. Thus,

$$\mathcal{O} := U \times \widehat{G}/\sim$$

is a $\widehat{G}$-invariant neighborhood of π which is Hausdorff because U is Hausdorff and the stability subgroups vary continuously ([25]). ∎

3. IDENTIFYING IDEALS IN $C_0(X) \rtimes G$

Let (G, X) be a transformation group with continuously varying stability groups. Define an equivalence relation on $X \times G$ by

$$(x, s) \sim (y, t) \text{ if and only if } x = y \text{ and } s^{-1}t \in S_x.$$

The continuity of the map σ sending $x \mapsto S_x$ implies that $X \times G/\sim$ is locally compact Hausdorff and that the quotient map $\delta : X \times G \to X \times G/\sim$ is open (Lemma 2.3 from [24]). The action of G on X is *σ-proper* if the map $[(x, s)] \mapsto (x, s \cdot x)$ of $X \times G/\sim$ into $X \times X$ is proper (Definition 4.1 of [21]). It is not hard to see that the action is σ-proper if and only if, given any compact subset K of X, the image in $X \times G/\sim$ of

$$\{(x, s) \in X \times G : x \in K \text{ and } s \cdot x \in K\} \tag{3.1}$$

is relatively compact. Any set K for which the image of (3.1) is relatively compact is called *G-wandering* (p. 406 in [21]). If the action is free, then the notions of σ-properness and G-wandering reduce to the standard notions of properness and wandering, respectively.

LEMMA 3.1. *Let (G, X) be a (not necessarily second countable) transformation group with continuously varying stability groups. If U is an open G-wandering neighborhood of X then the action of G on $G \cdot U$ is σ-proper.*

Proof. Let K be a compact set in $G \cdot U$ and choose $t_1, \ldots, t_n \in G$ such that $K \subset \bigcup_{i=1}^{n} t_i \cdot U$. It suffices to show that for each i and j,

$$\delta\big(\{(y, w) \in G \cdot U \times G : y \in K \cap t_i \cdot U \text{ and } w \cdot y \in K \cap t_j \cdot U\}\big) \tag{3.2}$$

is relatively compact in $(G \cdot U \times G)/\sim$.

Let $[(y_\alpha, w_\alpha)]$ be a net in the set described in Equation 3.2. It will suffice to find a convergent subnet. Since δ is open, we can pass to a subnet, relabel, and assume that this net lifts to a net (y_α, s_α) in $G \cdot U \times G$ with $s_\alpha^{-1} w_\alpha \in S_{y_\alpha}$. Now $y_\alpha \in K \cap t_i \cdot U$ and $s_\alpha \cdot y_\alpha \in K \cap t_j \cdot U$, so that $y_\alpha = t_i \cdot x_\alpha$ for some $x_\alpha \in U$ and $s_\alpha t_i \cdot x_\alpha = s_\alpha \cdot y_\alpha \in K \cap t_j \cdot U$, that is, $t_j^{-1} s_\alpha t_i \cdot x_\alpha \in U$.

Now $\{(x_\alpha, t_j^{-1} s_\alpha t_i)\}$ is a net in $\{(y, w) : y \in U \text{ and } w \cdot y \in U\}$. Since U is G-wandering $\delta\big(\{(y, w) : y \in U \text{ and } w \cdot y \in U\}\big)$ is relatively compact. By passing to a subnet and relabeling, we may assume that for some $n_\alpha \in S_{x_\alpha}$ the net $\{(x_\alpha, t_j^{-1} s_\alpha t_i n_\alpha)\}$ converges in $X \times G$. Since t_i and t_j are fixed,

$$\{(t_i \cdot x_\alpha, s_\alpha t_i n_\alpha t_i^{-1})\} = \{(y_\alpha, s_\alpha t_i n_\alpha t_i^{-1})\}$$

also converges. Since $t_i n_\alpha t_i^{-1} \in S_{y_\alpha}$ and $s_\alpha^{-1} w_\alpha \in S_{y_\alpha}$, we conclude that $\{[(y_\alpha, w_\alpha)]\}$ converges. ∎

In [16] the first author showed that if the action of G on X is free then $C_0(X) \rtimes G$ is a Fell algebra if and only if X is a Cartan G-space (that is, each point of X has a wandering neighborhood). If the stability subgroups vary continuously, we can prove a similar result using the following generalization of Proposition 1.1.4 from [17].

LEMMA 3.2. *Suppose that (G, X) is a (not necessarily second countable) transformation group with G abelian and with continuously varying stability groups. If each point of X has a G-wandering neighborhood, then $G \cdot x$ is closed in X for all $x \in X$.*

Proof. Suppose that $y \in \overline{G \cdot x}$. Let U be a G-wandering neighborhood of y. Then there are $s_\alpha \in G$ such that $s_\alpha \cdot x \to y$ and $s_\alpha \cdot x \in U$ for all α. We may replace x by $s_{\alpha_0} \cdot x$ for some $s_{\alpha_0} \in G$, and assume that $x \in U$. Then

$$\{(x, s_\alpha)\} \subset \{(z, s) \in X \times G : z \in U \text{ and } s \cdot z \in U\}. \tag{3.3}$$

Since the right-hand side of (3.3) has relatively compact image in $X \times G/{\sim}$ and δ is open, we can pass to a subnet and relabel so that there are $t_\alpha \in S_x$ such that $s_\alpha t_\alpha \to s$ in G. Then $y = s \cdot x$ and $G \cdot x$ is closed. ∎

PROPOSITION 3.3. *Let (G, X) be a second countable transformation group. Suppose that either G acts freely, or that G is abelian and that the stability groups vary continuously. Then $C_0(X) \rtimes G$ is Fell algebra if and only if each point of X has a G-wandering neighborhood.*

Proof. The free case is treated in [16]. Now suppose that G is abelian, that the stability groups vary continuously and that $C_0(X) \rtimes G$ is a Fell algebra. Fix $x \in X$ and let $\pi = \pi_x(1) \in (C_0(X) \rtimes G)^\wedge$. By Lemma 2.4, π has an open Hausdorff $\widehat{G}$-invariant neighborhood $\mathcal{O}_J$, where J is an ideal of A. Thus $J = C_0(Y) \rtimes G$ for some G-invariant open subset Y of X, and J has continuous trace. The action of G on Y is σ-proper by Theorem 5.1 of [24]. Note that $x \in Y$, and let N be a neighborhood of y which is compact in Y. Then N is G-wandering relative to Y, and since Y is G-invariant N is also G-wandering relative to X.

Conversely, assume each point in X has a G-wandering neighborhood. Then Lemma 3.2 implies that the orbits are closed, and $C_0(X) \rtimes G$ is postliminal ([9]; even liminal ([23])). In particular, each $\pi \in (C_0(X) \rtimes G)^\wedge$ is of the form $\pi_x(\omega)$ for some $x \in X$ and $\omega \in \widehat{G}$. Let U be a G-wandering open neighborhood of x. By Lemma 3.1 the action of G on $G \cdot U$ is σ-proper. Since the stability subgroups vary continuously it follows from Theorem 5.1 of [24] that $J = C_0(G \cdot U) \rtimes G$ is an ideal of A which has continuous trace. Thus $\pi_x(\omega)$ is a Fell point of $\widehat{J}$, whence it is also a Fell point of $\mathcal{O}_J \subset \widehat{A}$. ∎

Corollary 3.4. *Let (G,X) be a second countable transformation group. Suppose that either G acts freely and $C_0(X)\rtimes G$ is EH-regular, or that G is abelian and that the stability groups vary continuously. Then the largest Fell ideal of $C_0(X)\rtimes G$ is $C_0(W)\rtimes G$ where W is the open G-invariant subset*

$$W=\{w\in X : w \text{ has a } G\text{-wandering neighborhood in } X\}.$$

Proof. Again, the free case is dealt with in [16]. In any event, the largest Fell ideal of $C_0(X)\rtimes G$ is J where $\mathcal{O}_J=\{\pi\in(C_0(X)\rtimes G)^\wedge : \pi \text{ is a Fell point of } (C_0(X)\rtimes G)^\wedge\}$. Since $\mathcal{O}_J$ is invariant under the dual action, it follows that $J=C_0(W)\rtimes G$ for some open G-invariant subset W of X. Now apply Proposition 3.3. ∎

Remark 3.5. When the action of G on X is free and $C_0(X)\rtimes G$ is postliminal, Green (Corollary 18 from [13]) characterized the ideal $\overline{m(C_0(X)\rtimes G)}$ as $C_0(Y')\rtimes G$ where

$$Y'=\{x\in X : x \text{ has a compact wandering neighborhood } N \text{ such that } G\cdot N \text{ is closed in } X\}; \tag{3.4}$$

the following example shows that this is not quite correct. The correct statement is contained in Theorem 3.10 below and says that the open subset Y of X corresponding to $\overline{m(C_0(X)\rtimes G)}$ is given by equation (1.1) in Section 1.

Example 3.6. Consider the transformation group described by Palais in p. 298 of [17], where X is the strip $\{(x,y): -1\leqslant x\leqslant 1 \text{ and } y\in\mathbb{R}\}$ and the group action is by $G=\mathbb{R}$. Beyond the strip $-1<x<1$ the action moves a point according to

$$t\cdot(1,y)=(1,y+t) \quad\text{and}\quad t\cdot(-1,y)=(-1,y-t).$$

If $(x_0,y_0)\in\operatorname{int}(X)$ let $C_{(x_0,y_0)}$ be the vertical translate of the graph of $y=\frac{x^2}{1-x^2}$ which passes through (x_0,y_0). Define $t\cdot(x_0,y_0)$ to be the point (x,y) on $C_{(x_0,y_0)}$ such that the length of the arc of $C_{(x_0,y_0)}$ between (x_0,y_0) and (x,y) is $|t|$, and $x-x_0$ has the same sign as t. That is, (x_0,y_0) moves counter-clockwise along $C_{(x_0,y_0)}$ at unit speed.

Palais states that a compact set is wandering if and only if it meets at most one of the lines $x=1$ and $x=-1$; this is only partially correct. Certainly, if a compact set meets at most one of the boundary lines then it is wandering. However, $N=[0,1]\times[-1,1]\cup\{(-1,0)\}$ is an example of a wandering compact set meeting both boundary lines; moreover, $G\cdot N$ is closed in X, and N is a neighborhood of $(1,y)$ for all $y\in(-1,1)$. One sees from these examples that for this transformation group, the set Y' described in (3.4) is all of X whence $C_0(X)\rtimes G$ should have continuous trace. But this is impossible because $X/G\cong(C_0(X)\rtimes G)^\wedge$ is not Hausdorff: for example, $G\cdot(-1+1/n,0)$ is a sequence which converges to the distinct orbits $G\cdot(-1,0)$ and $G\cdot(1,0)$. Alternatively, note that not every compact set is wandering which contradicts Theorem 17 of [13].

Remark 3.7. In Theorem 3.10, we want to consider sets $K\subset X$ which are G-wandering even though we definitely are not assuming that the stabilizer map σ is continuous on all of X. To make sense of this, we have to assume that σ is at least continuous on $G \cdot K$, and then it makes sense to ask if K is G-wandering in $G \cdot K$ (or, equivalently, in any G-invariant set Z which contains K and on which σ is continuous). If K is open, it is not hard to see that σ is continuous on $G \cdot K$ if and only if σ is continuous on K. However, in general the continuity of σ on K does not imply that σ is continuous on $G \cdot K$. The next lemma will allow us to ignore this difficulty when applying the theorem.

LEMMA 3.8. *Suppose that (G, X) is a (not necessarily second countable) locally compact transformation group with G abelian and with stabilizer map σ. Let $q : X \to X/G$ be the quotient map. If σ is continuous on a compact set K and if $q(K)$ is Hausdorff, then σ is continuous on $G \cdot K$.*

Proof. Suppose that $r_\alpha \cdot x_\alpha \to r \cdot x$ for r_α, $r \in G$ and $x_\alpha, x \in K$. We want to show that $S_{r_\alpha \cdot x_\alpha} = S_{x_\alpha}$ converges to $S_{r\cdot x} = S_x$. Since this happens if and only if every subnet converges to S_x, we can pass to some convergent subnet (by the compactness of K), relabel and assume that $x_\alpha \to y \in K$. But now $G \cdot x_\alpha$ converges to both $G \cdot x$ and $G \cdot y$, and since $q(K)$ is Hausdorff, $y = s \cdot x$ for some $s \in G$. Thus by assumption, $S_{r_\alpha \cdot x_\alpha} = S_{x_\alpha}$ converges to $S_y = S_x$. ∎

REMARK 3.9. Up until this point, our work here has concentrated on the case in which G is abelian, and we have relied on results from [16] to handle free actions by nonabelian groups. Hereafter, we'll have to treat both cases.

THEOREM 3.10. *Let (G, X) be a second countable transformation group, and let σ be the stabilizer map sending $x \mapsto S_x$. Assume either that G acts freely and $C_0(X) \rtimes G$ is EH-regular, or that G is abelian. Let $I := \overline{m\big(C_0(X) \rtimes G\big)}$. Then $I = C_0(Y) \rtimes G$, where Y is the open G-invariant subset*

$$Y = \{\, y \in X : \sigma \text{ is continuous on a } G\text{-wandering compact neighborhood } N \text{ of } y \text{ such that } q(N) \text{ is closed and Hausdorff}\}, \tag{3.5}$$

where $q : X \to X/G$ is the quotient map.

Proof. Our proof is modeled on the proof of Corollary 18 from [13]. Here we'll give the proof for G abelian and remark that the free case follows from the same sort of argument together with the following observation. If the action is free, then EH-regularity implies that $\operatorname{Prim}\big(C_0(X) \rtimes G\big)$ is homeomorphic to the T_0-ization $(X/G)^\sim$ of X/G (Corollary 5.10 from [23]). It follows that the map $Y \mapsto C_0(Y) \rtimes G$ from the set of G-invariant open subsets of X to the set of ideals of $C_0(X) \rtimes G$ is a bijection.

By Proposition 2.3, $I = C_0(Z) \rtimes G$ where Z is an open G-invariant subset of X. Let Y be as in (3.5). Suppose that $\pi \in \mathcal{O}_I$. Since I has continuous trace, it is certainly postliminal, and $\pi = \pi_x(\omega)$ for $x \in Z$ and $\omega \in \widehat{G}$. Furthermore, Theorem 5.1 from [24] implies that the stabilizer map σ is continuous on Z and that the action of G on Z is σ-proper. Let N be a compact neighborhood of x in Z. Then N is G-wandering relative to Z, and since Z is G-invariant, N is also G-wandering relative to X.

Let $q : X \to X/G$ be the quotient map. We claim there is a closed neighborhood V of $G \cdot x$ in X/G such that $V \subset q(N)$. To prove the claim, we identify $\operatorname{Prim}\big(C_0(X) \rtimes G\big)$ with $X \times \widehat{G}/\!\sim$. Then Lemma 2.2 implies $\ker \pi_x(\omega)$ has a closed neighborhood $W \subset (N \times \widehat{G})/\sim$. The map $y \mapsto \ker \pi_y(\omega)$ is continuous by Lemma 4.9 of 23, and factors through X/G by Corollary 4.8 of [23]. Thus we get a continuous map $s_\omega : X/G \to \operatorname{Prim}(C_0(X) \rtimes G)$. Let $V := s_\omega^{-1}(W)$. Then V is a closed neighborhood of $G \cdot x$. To prove the claim, it remains to see that $V \subset q(N)$. But if $G \cdot y \in V$, then there is a $(z, \gamma) \in N \times \widehat{G}$ such that $(y, \omega) \sim (z, \gamma)$. In particular, $\overline{G \cdot y} = \overline{G \cdot z}$. Since Z is open and G-invariant, it follows that $y \in Z$. (We have $s_\alpha \cdot y \to z$ for $s_\alpha \in G$.) Thus $G \cdot y$ and $G \cdot z$ have the same closures in Z. But $C_0(Z) \rtimes G$ is liminal and each orbit must be closed in Z (Proposition 4.17 of [23]). Thus $G \cdot y = G \cdot z \in q(N)$ as claimed.

With V as above, set $N' = q^{-1}(V) \cap N$. Note that N' is compact and G-wandering and $G \cdot N' = q^{-1}(V)$ is closed. Finally, $G \cdot N'/G$ is Hausdorff because $G \cdot N' \subset Z$, and Z/G is Hausdorff since $C_0(Z) \rtimes G$ has continuous trace [25]. This implies that $x \in Y$. Therefore $Z \subset Y$, and $I = C_0(Z) \rtimes G \subset C_0(Y) \rtimes G$.

To prove the reverse implication notice that $C_0(Y) \rtimes G$ is a Fell algebra by Proposition 3.3. In particular, it is postliminal, and every irreducible representation of $C_0(Y) \rtimes G$ is of the form $\pi = \pi_y(\omega)$ for $y \in Y$ and $\omega \in \widehat{G}$. We will show that $\pi \in \mathcal{O}_I$ by verifying items (i) and (ii) of Lemma 2.2. Since $C_0(Y) \rtimes G$ is a Fell algebra π has a Hausdorff open neighborhood $\mathcal{O}_J$, where J is a closed ideal of $C_0(X) \rtimes G$ (Corollary 3.4 from [1]). Note that J is a Fell algebra with Hausdorff spectrum. Hence J has continuous trace. This establishes item (i) of Lemma 2.2.

Let N be a compact G-wandering neighborhood of y as in (3.5). We identify $(C_0(X) \rtimes G)^\wedge$ with $X \times \widehat{G}/\sim$. Note that $V = G \cdot N \times \widehat{G}/\sim$ is a closed neighborhood of π (first consider the complement and recall that the quotient map is open). That V is Hausdorff follows from [25] because $G \cdot N/G$ is Hausdorff and the stability subgroups vary continuously on $G \cdot N$ by Lemma 3.8. Let $\{F_\alpha\}$ be a neighborhood basis of π in $(C_0(X) \rtimes G)^\wedge$ consisting of compact sets. Since a compact subset of a Hausdorff space is closed, $\{F_\alpha \cap V\}$ is a neighborhood basis of π in $(C_0(X) \rtimes G)^\wedge$ consisting of closed sets. This establishes item (ii). Since π was an arbitrary irreducible representation of $C_0(Y) \rtimes G$, we must have $C_0(Y) \rtimes G \subset I = C_0(Z) \rtimes G$. Therefore $Z = Y$ and we're done. ∎

Example 3.11. If $A = C_0(X) \rtimes \mathbb{R}$ is the transformation group in Example 3.6, then $I = \overline{m(A)}$ corresponds to the open strip $Y = \{(x, y) : -1 < x < 1\}$.

Example 3.12. Let $G = \mathbb{R}^+$ act on $X = \mathbb{R}^2$ by $t \cdot (x, y) = (x/t, y/t)$. The orbits are rays emanating from the origin together with the origin which is a fixed point. Each orbit is locally closed so $C_0(X) \rtimes G$ is postliminal ([9]). The stability subgroups do not vary continuously on any neighborhood of $(0, 0)$. If U is any G-wandering (hence wandering) neighborhood of $(x, y) \neq (0, 0)$ then $(0, 0) \in \overline{G \cdot U}$ so that $G \cdot U$ is not closed in X. Thus Theorem 3.10 implies that $m(C_0(X) \rtimes G) = \{0\}$. Note that the action of G on $W := X \setminus \{(0, 0)\}$ is free and proper so that $C_0(W) \rtimes G$ is an essential ideal of $C_0(X) \rtimes G$ with continuous trace.

It should be pointed out that even for liminal algebras A, it is possible that $m(A) = \{0\}$. To see this, recall that a point x of a topological space X is *separated* if for any point y of X not in the closure of $\{x\}$, the points x and y admit a pair of disjoint neighborhoods. If A is a separable C^*-algebra, then the set $\mathcal{S}$ of separated points of the spectrum $\widehat{A}$ is a dense G_δ ([4], 3.9.4).

LEMMA 3.13. *Let A be a C^*-algebra and $I := \overline{m(A)}$. Then $\mathcal{O}_I$ is contained in the interior of the separated points $\mathcal{S}$ of $\widehat{A}$.*

Proof. Let $\pi \in \mathcal{O}_I$, and $\rho \in \widehat{A}$ such that $\rho \notin \overline{\{\pi\}}$. If $\rho \in \mathcal{O}_I$ then ρ and π can be separated by disjoint relative open subsets of $\widehat{A}$ because $\mathcal{O}_I$ is Hausdorff. Since $\mathcal{O}_I$ is open these relative open sets are open. Now suppose that $\rho \notin \mathcal{O}_I$. Fix a positive element a of $m(A)$ such that $\operatorname{tr}(\pi(a)) > 1$ and let $f : \widehat{A} \to [0,\infty)$ be the (continuous) map $\sigma \mapsto \operatorname{tr}(\sigma(a))$. Note that $\rho(a) = 0$. Now $f^{-1}((1,\infty))$ and $f^{-1}([0,\frac{1}{2}))$ are disjoint open neighborhoods of π and ρ, respectively. Thus $\mathcal{O}_I \subset \mathcal{S}$ and since $\mathcal{O}_I$ is open we have $\mathcal{O}_I \subset \operatorname{int} \mathcal{S}$. ∎

Dixmier has given an example of a separable liminal C^*-algebra A such that the interior of the separated points in $\widehat{A}$ is empty (Proposition 4 of [3]). Thus $m(A) = \{0\}$ for this algebra.

THEOREM 3.14. *Let (G,X) be a second countable transformation group. Suppose that either G acts freely and $C_0(X) \rtimes G$ is EH-regular, or that G is abelian. Then the largest liminal ideal of $C_0(X) \rtimes G$ is $C_0(Z) \rtimes G$ where Z is the open G-invariant subset*

$$Z = \{\, x \in X : x \text{ has a neighborhood } U \text{ such that } G \cdot z \text{ is closed in } G \cdot U \text{ for each } z \in U \,\}. \tag{3.6}$$

Proof. If J is the largest liminal ideal then $\mathcal{O}_J = \{\pi \in \widehat{A} : \pi\big(C_0(X) \rtimes G\big) = \mathcal{K}(\mathcal{H}_\pi)\}$. If G is abelian then $\mathcal{O}_J$ is invariant under the dual action, and we have $J = C_0(Y) \rtimes G$ for some open G-invariant subset Y of X. This follows from our EH-regularity assumption in the free case. Let Z be as in (3.6). Note that every $y \in Y$ has a neighborhood U (namely Y) such that $G \cdot z$ is closed in $G \cdot U$ for every $z \in U$ by Theorem 3.1 of [23], so $Y \subset Z$.

Let $x \in Z \setminus Y$. Let V be an open neighborhood of x such that $G \cdot z$ is closed in $G \cdot V$ for each $z \in V$. Not every orbit in $Y' = Y \cup G \cdot V$ can be closed in Y' because $C_0(Y) \rtimes G$ is the largest liminal ideal. Suppose that $G \cdot z$ is not closed in Y'. Then there exists $s_\alpha \in G$ and $w \in Y'$ such that $s_\alpha \cdot z \to w \notin G \cdot z$.

Since $w \in Y'$, w has a neighborhood W such that $G \cdot u$ is closed in $G \cdot W$ for all $u \in W$. But we can assume that $s_{\alpha_0} \cdot z \in W$ for some s_{α_0} and then $G \cdot s_{\alpha_0} \cdot z = G \cdot z$ must be closed in $G \cdot W$. Thus $w \in G \cdot z$, and this is a contradiction. Hence $Z = Y$ and we are done. ∎

Every C^*-algebra A has a largest postliminal ideal I, and this ideal I is the smallest ideal such that the corresponding quotient is anti-liminal (Proposition 4.3.6 in [4]). When $A = C_0(X) \rtimes G$ and G is abelian, it is clear that I is invariant under the dual action: for every $\tau \in \widehat{G}$ the ideal $\widehat{\alpha}_\tau(I)$ is postliminal and $A/\widehat{\alpha}_\tau(I)$ is antiliminal, hence $\widehat{\alpha}_\tau(I) \subset I$. If G is abelian or G acts freely then $C_0(X) \rtimes G$ is Type I if and only if X/G is T_0 (Theorem 3.3 from [9]). Effros and Glimm have given a number of conditions on a second countable locally compact transformation group (G,X) which are equivalent to X/G being T_0 (see [8], Theorems 2.1 and 2.6 from [5] and [6]). For example, X/G is T_0 if and only if each orbit is *regular*: the map $sS_x \mapsto s \cdot x$ is a homeomorphism of G/S_x onto $G \cdot x$. (The term regular is borrowed from the definition on p. 223 of [14].) Using the Effros-Glimm results, we have the following.

LEMMA 3.15. (Effros-Glimm) *Suppose that (G, X) is a second countable locally compact transformation group and that U is a neighborhood of $x \in X$. Then the following are equivalent:*

(i) *$G \cdot U/G$ is T_0 in the quotient topology;*
(ii) *$G \cdot y$ is regular for each $y \in U$;*
(iii) *$G \cdot y$ is a G_δ subset of X for each $y \in U$;*
(iv) *$G \cdot y$ is locally closed in X for each $y \in U$;*
(v) *$G \cdot y$ is second category in itself for each $y \in U$.*

THEOREM 3.16. *Let (G, X) be a second countable transformation group. Suppose that either G acts freely and $C_0(X) \rtimes G$ is EH-regular, or that G is abelian. Then the largest postliminal ideal of $C_0(X) \rtimes G$ equals $C_0(Z) \rtimes G$ where Z is the G-invariant subset*

$$Z = \{x \in X : x \text{ has a neighborhood } U \text{ such that } G \cdot U/G \text{ is } \mathrm{T}_0\}. \tag{3.7}$$

REMARK 3.17. The set Z can be realized as the set of points with neighborhoods satisfying any of the equivalent conditions of Lemma 3.15.

Proof. If G is abelian, the largest postliminal ideal of $C_0(X) \rtimes G$ is invariant under the dual action, so equals $C_0(Y) \rtimes G$ for some G-invariant open subset Y of X. Let Z be as in (3.7). Every $y \in Y$ has an open G-invariant neighborhood U (namely Y) such that $G \cdot U/G$ is T_0 by Theorem 3.3 from [9]. Thus $Y \subset Z$.

Let $x \in Z \setminus Y$ and V an open neighborhood of x such that $G \cdot V/G$ is T_0. Note that $T := (G \cdot V \cup Y)/G$ cannot be T_0 by the maximality of $C_0(Y) \rtimes G$. Choose distinct points $G \cdot z_1$ and $G \cdot z_2$ in T such that every open neighborhood U_1 of $G \cdot z_1$ contains $G \cdot z_2$ and every open neighborhood U_2 of $G \cdot z_2$ contains $G \cdot z_1$.

If $G \cdot z_1 \in T \setminus (G \cdot V/G)$ and $G \cdot z_2 \in T \setminus (Y/G)$ then $G \cdot V/G$ is an open neighborhood of $G \cdot z_2$ which does not contain $G \cdot z_1$, which is a contradiction.

If $G \cdot z_1$ and $G \cdot z_2$ both belong to Y/G or if $G \cdot z_1$ and $G \cdot z_2$ both belong to $G \cdot V/G$ then we get an immediate contradiction because Y/G and $G \cdot V/G$ are open and T_0. Hence $Y = Z$. ∎

REMARK 3.18. Let (A, G, α) be a C^*-dynamical system with G compact (but not necessarily abelian). It follows from Propositions 2.3 and 2.5 of [11] that the largest liminal and postliminal ideals in $A \rtimes_\alpha G$ are of the form $J \rtimes_\alpha G$ where J is an α-invariant ideal of A. This is trivial if $A = C_0(X)$, because $C_0(X) \rtimes G$ has bounded trace (hence is liminal) when G is compact (Proposition 3.4 from [15]).

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