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1. (1 pt)

Setup

The purpose of this problem is for you to write a clear statement of the mathematics that will be applied to the problem. What is the nature of the information that is available? What are the steps that you should follow to analyze the information? What tools will you bring to bear in the analysis? The setup phase in tackling a scientific problem is often called the modeling phase.

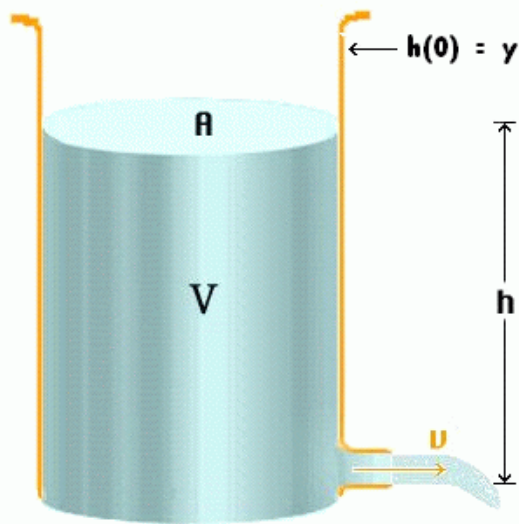
Consider the above questions as you develop the steps to arrive at the differential equation

$$\frac{dh}{dt} = -\frac{b\sqrt{2gh}}{A}$$

where A is the cross-sectional area of a cylindrical tank, b is the area of the outlet hole, $h = h(t)$ is the height of the water above the outlet hole at time t , and $y = h(0)$ is the initial height of water in the tank.

Toricelli's Law states the following principle:

Water in an open tank will flow out through a small hole in the bottom with the velocity it would acquire in falling freely from the water level to the hole.



tank cross section area = A
outlet hole area = b

Review the textbook material on the CSC, which shows that the motion of a falling object is described by the differential equation

$$\frac{ds}{dt} = -\sqrt{2gs(t)}$$

In this equation, $s(t)$ is the height of the object above the ground at time t , and g is the acceleration due to gravity. The derivative $\frac{ds}{dt}$ is the velocity of the object at time t .

Using only the symbols A, b, h, y , answer the following:

What is the initial volume $V(0)$ of the tank?

$$V(0) = \underline{\hspace{2cm}}$$

What is the volume of the tank at any time t ?

$$V(t) = \underline{\hspace{2cm}}$$

By Toricelli's Law, the velocity of a drop of water leaving the outlet hole is from above $\sqrt{2gh}$ (see textbook). For the expression $\frac{dh}{dt}$, use D below. Express the rate of change in the volume of the tank in two ways:

As a function of A and D .

$$\frac{dV}{dt} = \underline{\hspace{2cm}}$$

As a function of b, g , and h .

$$\frac{dV}{dt} = \underline{\hspace{2cm}}$$

Equate these two expressions for $\frac{dV}{dt}$ and solve the resulting equation for $D = \frac{dh}{dt}$, expressing it in terms of A, b, g , and h .

$$\frac{dh}{dt} = \underline{\hspace{2cm}}$$

2. (1 pt)

Thinking and Exploring

$$\frac{dh}{dt} = -\frac{b\sqrt{2gh}}{A}$$

Recall from the previous problem that A is the cross-sectional area of a cylindrical tank, b is the area of the outlet hole, $h = h(t)$ is the height of the water above the outlet hole at time t , and $y = h(0)$ is the initial height of water in the tank.

Verify that $h(t) = (kt + C)^2$, where k and C are constants, is a solution of the differential equation above by answering the following:

Let $P(t) = (kt + C)^2$. What is $\frac{dP}{dt}$?

$$\frac{dP}{dt} = \underline{\hspace{2cm}}$$

Rewrite the above derivative in terms of P .

$$\frac{dP}{dt} = \underline{\hspace{2cm}}$$

Determine in general what must be the values of the constants k and C in terms of the initial conditions A, b , and y , and the acceleration due to gravity g . What is k ?

$$k = \underline{\hspace{2cm}}$$

What is C ?

$$C = \underline{\hspace{2cm}}$$

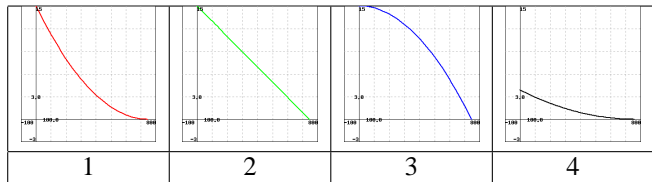
3. (1 pt)

Apply the result

How long would it take a tank to empty if the tank has a diameter of 25 feet, a height of 15 feet, and an outlet hole of diameter 10.8 inches? Use the value 32.174 ft/s/s for g .

_____ seconds

For the specific dimensions of the tank just described, which of the following graphs shows the solution function?



4. (1 pt)

Interpretation and Summary

Now that you have (1) modeled the barrel (tank) problem and (2) thought about the model and derived mathematical facts, it is time to interpret and summarize the mathematical results in terms of the original objective.

Pretend that you are reporting on this project for a publication such as *Scientific American*, and that your Interpretation and Summary is going to head the report. Include enough details so that a reader would learn what the major issues of the report are, how you went about addressing them, and the most important interpretations and conclusions. What will you want

to tell readers about your success with regard to the original stated objective of the investigation? Assume that readers of this summary will not see your work in the previous WebWorK problems of the case study, and that they can follow a math discussion. Be sure to write in complete sentences using correct rules of standard English grammar.

Your report should describe What you did, What you learned, and How it all was related to the original objective. Be specific when you state your findings. **Express yourself in a page or less.**

To submit your answer, use the Email instructor button below. Include your email address, and enter your report in the Feedback box. When you are satisfied with your composition, click the Send Feedback button.

When you are done, return to this screen and complete the Affirmation below.

Affirmation: Even though I may have discussed the CSC project with other people, I have written up this CSC report by myself and on my own. No sharing of electronic files or notes has been involved.

Please type your name in the answer box, just as it appears in the WebWorK database and on the problem list screen, and click the Check Answers button.
