

1. If X, Y are two discrete random variables, we define the *covariance of X and Y* in the following manner:

$$\text{Cov}(X, Y) = E((X - E(X))(Y - E(Y))).$$

(a)(2 points) Show that this formula can be simplified to

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y).$$

(b)(1 point) Show that if X and Y are independent, then $\text{Cov}(X, Y) = 0$.

2.(2 points) What is the expected number of runs of k heads in a string of n coin flips? Carefully prove your answer, labeling all “expected values properties” that you use.

Bonus (+2 points) We construct a *random graph* by flipping a fair coin to determine whether any given pair of vertices should be connected by an edge (i.e. Pick a pair of vertices and flip the coin. If the coin lands heads, then we draw an edge between them. If the coin lands tails, then we don't draw an edge between them. We repeat this experiment for all possible pairs of vertices). What is the expected number of edges in a random graph with n vertices? What is the expected number of triangles?