

## Reading Assignment 12

The main objective of these assignments is to help you in reading the book, making sure that you understand the main ideas and the important theorems. A secondary objective is for you to write down **precise** mathematical concepts. Please do not be lax when writing these assignments, write full sentences and try to be as thorough as possible. You will get more out of these assignments if you do this. I give you a few token extra credit points as an incentive, but my hope is that your true gain will be when you perform better in exams and better in future courses that require you to write mathematics.

### Read Sect. 4.3, 4.4

1. What is the determinant of elementary matrices of type (1), (2) and (3)?
2. Give an example that illustrates Theorem 4.7.
3. Write an outline of the main ideas in the proof of Theorem 4.7.
4. What is the determinant of the inverse of an invertible matrix?
5. Theorem 4.8 says that for any square matrix  $A$ ,  $\det(A) = \det(A^t)$ , what is a consequence of this theorem?
6. What is the geometric interpretation of a of the determinant of a  $3 \times 3$  matrix? Does this generalize to  $n \times n$  matrices?
7. Section 4.4 gives a summary of everything you learned in Sec. 4.2 and 4.3. List the main results summarized in 4.4.
8. (Review) How do you prove that a subset  $W$  is a subspace of a vector space  $V$ ?
9. (Review) How do you prove that the elements of a subset  $S$  of a vector space  $V$  are linearly independent?

**Practice Problems:** Sect. 4.3 # 1

Sect. 4.4 # 1, 2, 3, 4,

## Read Section 5.1

1. What questions do we seek to answer in Chapter 5?
2. Define a diagonalizable operator and a diagonalizable matrix.
3. Give an example of a diagonalizable operator.
4. Define the mathematical terms: eigenvector and eigenvalue for both matrices and linear operators.
5. State Theorem 5.1
6. How do we compute eigenvalues?
7. Define the characteristic polynomial of a matrix?
8. Give an example of a  $3 \times 3$  matrix and compute its characteristic polynomial.
9. What does your book say about the characteristic polynomial of similar matrices?
10. How is the characteristic polynomial of a linear operator defined? How does your answer to question (9) makes this definition well-defined?
11. What does Theorem 5.3 say? Be precise.
12. Give an example to illustrate Theorem 5.4.
13. How do we compute the eigenvalues of a linear operator?
14. Summarize in a few sentences the geometric description given in pages 254 and 255.

**Practice Problems:** 1, 2, 3, 4