

## Reading Assignment 13

### Read Sect. 5.2

1. What is the objective of Sect. 5.2?
2. What can you say about the eigenvectors corresponding to distinct eigenvalues? See Theorem 5.5
3. How is Theorem 5.5. proved? What method of proof is used?
4. In the proof of Theorem 5.5 after we apply the operator  $T - \lambda_i I$  to both sides of equation (1), we obtain an equation that does not involve the  $k$ -th term. Can you write in detail the steps skipped in the proof and show me what happened?
5. (Review) How do we prove that a set  $S = \{v_1, v_2, \dots, v_k\}$  is linearly independent?
6. What can you conclude if  $T : V \rightarrow V$  is linear and  $V$  is  $n$ -dimensional and you are told that  $T$  has  $n$  distinct eigenvalues? Explain how this follows from Theorem 5.5.
7. Is it true that every diagonalizable operator  $T : V \rightarrow V$ , where  $V$  is  $n$ -dimensional has  $n$  distinct eigenvalues? State the theorem to support your claim or give a counterexample.
8. Is it true that every linear operator has at least one eigenvalue? Why or why not?
9. What is the definition of splits over  $F$ ? Give an example of a polynomial that splits over  $R$  and one that doesn't split over  $R$ .
10. What does Theorem 5.6 say about the characteristic polynomial of diagonalizable operators?
11. Is it true that if the characteristic polynomial splits, then the operator is diagonalizable? If true write the theorem that supports this statement and if false give a counterexample.

12. Define algebraic multiplicity of an eigenvalue  $\lambda \in F$ .
13. In the last paragraph of page 263 an explanation about the relationship of multiplicity and diagonalizable operators is given. What is this relationship?
14. Define Eigenspace, be precise.
15. Prove that for any eigenvalue  $\lambda$  of  $T : V \rightarrow V$ ,  $E_\lambda$  is a subspace of  $V$ .
16. What is the dimension of  $E_\lambda$  according to Theorem 5.7?
17. In your own words, what is Theorem 5.8 saying?
18. Theorem 5.9 gives the main result of this section, what does it say? Be precise.
19. How do we test if a linear operator is diagonalizable?
20. State and give an example of Theorem 5.11.

**Practice Problems:** Sect. 5.2 #1,2,3