Reading Assignment 17

Read Sect. 6.4 and 6.5

1. What do we seek in Section 6.4?
2. What does the Lemma in page 369 says? What does the proof say about the eigenvalues of T and T^*?
3. State Schur's theorem.
4. Give an outline of the proof of Schur's theorem.
5. What is a normal operator?
6. Give an example that shows that normal operators might not be diagonalizable.
7. Give examples to illustrate each of the parts (a) - (d) of Theorem 6.15.
8. What are the conditions so that a normal operator is diagonalizable.
9. Define self-adjoint and give an example of an operator that is self-adjoint.
10. What does the Lemma in page 374 says about self-adjoint operators?
11. State Theorem 6.17 and give an example that illustrates this theorem.
12. According to the introductory paragraph in Section 6.5, what operators will be studied in this section?
13. What can be said about the eigenvalues of operators that "preserve length"?
14. Define unitary and orthogonal operators.
15. If V is infinite dimensional, how do we call operators that preserve length in this case? What conditions must an operator T on V satisfy in addition to $\|T(x)\| = \|x\|$, for all $x \in V$, in order to be called unitary or orthogonal?
16. What are examples of orthogonal operators in R^2?

17. In Theorem 6.18, what does it mean to say that "the following statements are equivalent"?

18. State Theorem 6.18.

19. Read the proof of Theorem 6.18. How do we use the Lemma, in page 380, in the proof of Theorem 6.18?

20. What are the consequences (corollaries) of Theorem 6.18?

21. Define a reflection of R^2 about a one dimensional subspace L. Why do you think L was used to denote a one dimensional space in R^2?

22. Define orthogonal and unitary matrices and give an example of each.

23. When we say A is "unitarily equivalent" or "orthogonally equivalent" to D, what do we mean?

24. State Theorems 6.19 and 6.20. What new information not given in Theorems 6.16 and 6.17, respectively, is given in Theorems 6.19 and 6.20?

25. Theorem 6.21 gives the matrix form of Schur's theorem. State this theorem.

Practice Problems:
Section 6.4 # 1,2
Section 6.5 # 1, 2, 4, 5