

Reading Assignment 3

Read Sect. 1.6

In the following questions, if I don't say anything assume every set mentioned in a question is contained in a vector space V over some field F .

1. What is a basis? Give an example of a vector space and a basis.
2. What is a very important property of bases?
3. What can you conclude if V has a finite generating set?
4. Describe a way to obtain a basis given a finite generating set S that is not linearly independent.
5. From the Replacement Theorem, what can you conclude if you are given two subsets S_1 and S_2 of a vectors space V and you are told that S_1 generates V and S_2 is linearly independent?
6. The Replacement Theorem is very important because of its consequences (corollaries). State the consequences of the Replacement Theorem.
7. How do we define dimension?
8. How large can a linearly independent set $S \subseteq V$ be if the dimension of V is n ?
9. If S is a generating set of V is there an upper bound on its size? Is there a lower bound?
10. T/F: If a subset S of V has $\dim(V)$ elements, then S is a basis. Justify your answer by proving it if it true or giving a counterexample if it is false.
11. Let W be a subspace of V . What is the relationship between the dimension of W and the dimention of V ?

Practice Problems: # 1, 2, 3, 4, 5, 6 (careful with the field).