

Reading Assignment 8

Read Sect. 3.1 and 3.2

1. State the three row [column] operations on an $n \times m$ matrix and give an example of each operation different from the examples in your book.
2. Give examples of elementary matrices for each type.
3. For the examples of the matrices in question (2) show that Theorem 3.1 is true for each type.
4. Can you write in your own words what Theorem 3.1 is saying?
5. For the example in (2), write the inverse for the three matrices and verify that the inverse of each matrix in (2) is an elementary matrix.
6. How is the rank of a matrix defined?
7. What is the rank of an invertible $n \times n$ matrix?
8. What is the relationship between the rank of a linear transformation $T : V \rightarrow W$, where V and W are finite dimensional vector spaces, and its matrix representation?
9. Give an example of a linear transformation from R^3 to R^3 and find its matrix representation with respect to the standard ordered basis of R^3 . Verify for your example that the rank of the linear transformation is the same as the rank of its matrix representation.
10. Choose a 3×3 matrix: A .
 - (a) Apply an elementary operation of type (1) to A to get a matrix B , show that the dimension of the columns of the subspace spanned by the columns of B is the same as the dimension of the subspace spanned by the columns of A .
 - (b) Repeat (a) for type (2) elementary operations.
 - (c) Repeat (a) for type (3) elementary operation.

11. Take the matrix in exercise 2(e) and apply both row and column operations until you get a matrix that looks like the matrix in Theorem 3.6, the process should be just as in example 3 in your book.
12. Corollary 3 says that every invertible matrix is a product of elementary matrices. The matrix in Example 5 is invertible. What are the elementary matrices that you have to multiply to get A ?
13. Write a 3×3 matrix and find its rank.
14. Write a 3×4 matrix A and compute A^t , show that they have the same rank.

Practice Problems: Section 3.1 # 1, 3

Section 3.2 # 1, 2, 4, 5, 6, 7