Let (b_i) be a sequence of length n with $b_i = i^2 - (i-1)^2$. Prove that

$$\sum_{i=1}^{n} b_i = n^2.$$

1. Stare at this problem for a while before reading any farther. Think about how we did the proof for the sum of i's, and see if you can get started on this one. When you feel hopelessly stuck, read on.

2. Step 1: Start here:
$$\sum_{i=1}^{n} b_i =$$
and replace b_i by its alternative.

3. Step 2: The new sum is a difference of two terms. Hmmm... we have a property for the sum of two terms, not the difference. So rewrite the difference as a sum. (Don't remember how? Here you go: $a - b = a + (-b)$.) Then use the property to split the Σ into $\Sigma + \Sigma$.

4. Step 3: One of the sums should now look like:
$$\sum_{i=1}^{n} -(i-1)^2.$$
We don't have a property to work with this directly, so we need to use some algebra. Work through the steps to get $-(i-1)^2 = -i^2 + 2i - 1$.

5. Step 4: You can now make the appropriate replacement inside the troublesome sum (troublesum?). Use a property to split this up.

6. Step 5: There should be two sums which look exactly alike, except that one is the negative of the other. These cancel!

7. Step 6: The others are amenable to other properties. Use them. You should end up with no sigmas at all! Simplify until you arrive at the result. Congratulations! You've proven the theorem!