Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel Exercises for Section 3.1

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1. $(1 \mathrm{pt})$

Which of the following is a solution to the differential equation $y^{\prime \prime}+y=0$ ?
$\qquad$
A. $\frac{5}{6}$
B. $x^{3}-6 x$
C. $C e^{x^{2}}$
D. $-13 \sin (x)+13 \cos (x)$

## 2. (1 pt)

Which of the following is a solution to the differential equation $y^{\prime}=-16 x y$ ?
A. $7 \sin (x)+\frac{7}{8} \cos (x)$
B. $\frac{1}{-16 x}$
C. $C e^{-8 x^{2}}$
D. 1
3. $(1 \mathrm{pt})$

Solve the following differential equation by separation of variables. Express your answer in terms of variables $\mathrm{x}, \mathrm{y}$, and C .

$$
\frac{d y}{d x}=\frac{x^{6}}{y^{3}}
$$

4. (1 pt)

Solve the following differential equation by separation of variables. Express your answer in terms of variables $\mathrm{x}, \mathrm{y}$, and C .

$$
\frac{d y}{d x}=\frac{8+\frac{1}{10} y}{e^{x}}
$$

5. ( 1 pt )

Rule of 72 This rule of thumb states that to find the approximate doubling time for an investment that earns $x$ percent interest compounded per period, divide $x$ into 72 to find the number of periods. So, if an investment earns 7 percent per year, it would take about $\frac{72}{7}=10.29$ years for the investment to double in value.

Give the formula for the doubling time as a function of $x$.
Doubling time $D(x)=$ $\qquad$
What is the exact time for the investment earning 7 percent annually to double?
years

## 6. $(1 \mathrm{pt})$

A scientist in a lab monitors the growth of a population of bacteria. Her observations on the size of the population are given in the following table.

| Time (hours) | Population (in thousands) |
| :---: | :---: |
| 0 | 1.000 |
| 3 | 1.059 |
| 5 | 1.100 |
| 6.5 | 1.131 |

Assume the population follows an exponential model of the form $a e^{k t}$. What is the growth constant $k$ ?
$k=$ $\qquad$
How big is the population of bacteria one hour after the last measurement in the table?

Population $=$ $\qquad$
7. (1 pt)

Match each of the slope fields with its differential equation.

$-\frac{d y}{d x}=\sin (x)$
$-\frac{d y}{d x}=x^{2}-2$
$-\frac{d y}{d x}=\frac{3 x^{2}}{2 y}$
$-\frac{d y}{d x}=\frac{1}{x}$
8. (1 pt)

This problem assumes you can run the Slope Field applet, or another application that displays slope fields and families of solution curves for a differential equation.

The following graph shows a family of solution curves for the differential equation $\frac{d y}{d x}=y-x$.


Consider the solution curve passing through the point $(-2,1)$. How would you describe this curve?
A. The solution curve is concave up
B. The solution curve has a maximum
C. The solution curve is a straight line
D. There are infinitely many solution curves
E. The solution curve is periodic
9. $(1 \mathrm{pt})$

A scientist in a lab monitors the decrease of a population of bacteria after a toxic agent is introduced into the bacteria's environment. Her observations on the size of the population are given in the following table.

| Time (hours) | Population (in thousands) |
| :---: | :---: |
| 0 | 4.000 |
| 8 | 2.109 |
| 11.5 | 1.594 |
| 13.5 | 1.358 |

Assume the population follows an exponential model of the form $a e^{k t}$. What is the decay constant $k$ ?
$k=$
At what time is the population of bacteria equal to one half its initial population?

Time =
10. (1 pt)

Which of the following differential equations are separable? Check all that apply.

- A. $\frac{d y}{d x}=\frac{x^{2}}{y+3}$
- B. $\frac{d y}{d x}=x-y$
- C. $\frac{d y}{d x}=\frac{\cos (4 x)}{\sqrt{y}}$
- D. $\frac{d y}{d x}=\sin (x+y)$
- E. $\frac{d y}{d x}=y^{2} \tan (x)$

