Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 3.4

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1. (1 pt)
Let y be the solution to the following initial value problem:
$\frac{dy}{dx} = 1 - 2xy; \quad y(-1) = -1$

PART I
By hand, use Euler's method with step size $h = 1$ to approximate $y(4)$.

The first row of blanks should be filled with the x and y values of the starting point, namely, -1 and -1. Fill the next row with the x and y values of the results of the first iteration. Continue until you reach the approximation to $y(4)$. Leave all subsequent blanks empty.

Starting Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)
Next Point: (________, ________)

PART II
Use the Euler's Method applet or a computer algebra system such as Maple to draw the slope field of the differential equation in the IVP above. Include a sketch of the Euler approximation. How well does the Euler approximation fit the slope field? In other words, how well does Euler's method with step size 1 approximate the graph of y from $x = -1$ to $x = 4$?

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A. Pretty well
B. Not very well

PART III
Implement Euler's method with step sizes $h = .5$ and then $h = .1$ to find better approximations to the graph of y from $x = -1$ to $x = 4$. Enter below your new approximations to $y(4)$.

$h = 0.5: \quad y(4) =$ ________
$h = 0.1: \quad y(4) =$ ________

Of the step sizes tested above, which gives the best approximation to the graph of y from $x = -1$ to $x = 4$?

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A. $h = 1$
B. $h = .5$
C. $h = .1$

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2. (1 pt)
Use Euler's Method with h = 0.5 to estimate y(3) if $y' = y$ and $y(2) = 2.1$.

$y(3) \approx$ ________

What is the exact value of y(3)?

$y(3) =$ ________

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3. (1 pt)
Use Euler's Method with h = 0.5 to estimate y(5) if $y' = \cos(x) - \sin(x)$ and $y(0) = 1$.

$y(5) \approx$ ________

What is the exact value of y(5)?

$y(5) =$ ________

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4. (1 pt)
Use Euler Method's to estimate $y(8)$ for the differential equation $xy' + y + x = e^x$ with step size 1 and $y(1) = e + 1$.

$y(8) \approx$ ________

The Improved Euler's Method makes a change to the original method in the following way: At each point (x_i, y_i), compute $x_{i+1} = x_i + h$ as before. Then the value $F(x_i, y_i) = z_{i+1}$ is calculated. The average of $F(x_i, y_i)$ and $F(x_{i+1}, z_{i+1})$ is used in place of $F(x_i, y_i)$ to find y_{i+1}.

$y_{i+1} = y_i + \frac{h}{2}[F(x_i, y_i) + F(x_{i+1}, z_{i+1})]$

What estimate for $y(8)$ do you get using the Improved Euler's Method?

$y(8) \approx$ ________

The exact value of $y(8)$ is 372.994748380216. What percent improvement is obtained by using the Improved Euler's Method?

________ percent

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5. (1 pt)
An object of mass m is dropped from an airplane. Assume the force of resistance due to air is directly proportional to the speed of the object. Let g be the force of gravity, v the velocity of the object, and k the constant of proportionality. What is the differential equation describing the motion of the object?

$\frac{dv}{dt} =$ ________

Assume the initial velocity of the falling object is 0. Use Euler's Method with stepsize h = 0.1 and k = 15.9 to find the speed of a 90 kg object 12 seconds after being dropped from the airplane.

Speed after 12 seconds = ________ meters per second

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6. (1 pt)
Use Euler Method's to estimate $y(5)$ for the differential equation $xy' - 3y = x^5$ with step size 1 and $y(1) = 1$.

$y(5) \approx$ ________

What estimate for $y(5)$ do you get using the Improved Euler's Method described in the previous problem?

$y(5) \approx$ ________

7. (1 pt)
We have seen that the Euler Method approximation to the actual solution curve for an initial value problem $y' = F(x,y)$, $y(x_0) = y_0$ improves when we decrease the step size h used in the approximation.

Let $y' = y$ and $y(0) = 1$, and use $h = \frac{1}{n}$. Fill in the rows below with the starting point and the results of the next three iterations.

Starting Point: (____________, ____________)
Next Point: (____________, ____________)
Next Point: (____________, ____________)
Next Point: (____________, ____________)

What estimate does this result in for $y(1)$?

____

What is the limit as $n \to \infty$?

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8. (1 pt)
Use Euler Method's to estimate $y(1)$ for the differential equation $\cos(y)y' = x + y$ with step size 0.01 and $y(0) = 0$.

$y(1) \approx$ ____________

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9. (1 pt)
Use Euler Method's to estimate $y(0.2)$ for the differential equation $xy' + y(x-1) = 0$ with step size 0.1 and $y(1) = \frac{1}{2}$.

$y(0.2) \approx$ ____________

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10. (1 pt)
Use Euler Method's to estimate $y(5)$ for the differential equation $y' = -2xy^2$ with step size 0.1 and $y(-5) = \frac{4}{101}$.

$y(5) \approx$ ____________