Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 3.7

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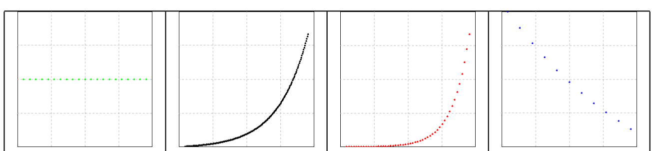
1. (1 pt)
Setup (Malthus)
The purpose of this section is for you to write a clear statement of the mathematics that will be applied to the problem. What is the nature of the information that is available? What are the steps that you should follow to analyze the information? What tools will you bring to bear in the analysis? The setup phase in tackling a scientific problem is often called the modeling phase.

Start by writing the initial-value problem that models the U.S. population beginning in the year 1790, assuming exponential growth. The initial condition should reflect the actual U.S. population in the year 1790, which for the sake of this investigation we take to be 3.9 million people.

Part 1

Each graph below shows the progression of a population's growth over the period 1790 to 1990. By experimenting on your own with different values of k, number of steps, and beginning and ending values, match the graph with the appropriate set of conditions. In each graph the beginning population is 3.9 million.



1	2	3	4

Number of steps = 50
Ending population = 5482.2 million
What is k? _____
Which graph displays these values? ___

Number of steps = 10
Ending population = 1.7 million
What is k? _____
Which graph displays these values? ___

Number of steps = 20
Ending population = 3.9 million
What is k? _____
Which graph displays these values? ___

Number of steps = 200
Ending population = 250.0 million
What is k? _____
Which graph displays these values? ___

Part 2

Continue experimenting with growth models by changing the number of steps for the same beginning population of 3.9 million and ending population of 250 million.

Number of steps = 20
What is k? __________

Number of steps = 50
What is k? __________

Number of steps = 200
What is k? __________

Number of steps = 1000
What is k? __________

Given the above results, if you increase the number of steps used in the estimate, how should you adjust k?

___

A. increase k
B. decrease k
C. no change in k

From the four cases of the same beginning and ending populations with different step sizes, use the best value for k and the same stepsize you used in finding this k to predict the U.S. population in the year 2100.

__________ million people

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2. (1 pt)
Thinking and Exploring (Malthus)
Note: For the rest of the CSC, use the Euler Population Predictions applet from the section 3.7 Web page.

The same problem of modeling population growth can be handled algebraically, using the mathematical solution of the initial-value problem.

With initial population 3.9 million in 1790 and 250 million in 1990, what value of k do you find using the algebraic methods? Give your answer accurate to 6 decimal places.

____

What population do you project in the year 2350 using the algebraic methods?

____________________ million people

How do the results of the algebraic methods compare with the values obtained experimentally using Eulers method?

___

A. population growth is faster with Euler's method
B. population growth is about the same
C. population growth is slower with Euler's method

3. (1 pt)

Thinking and Exploring (Verhulst) Part 1

The Verhulst model assumes that the growth rate declines, from a value k when conditions are very favorable, to the value 0 when the population has increased to the maximum value M that the environment can support. Specifically, it replaces the growth constant k by the expression

$$k\frac{M-Q(t)}{M}$$

Note that when Q(t) is small (relative to M) this expression is close to k, and as Q(t) approaches M it becomes close to 0. This leads to the differential equation

$$\frac{dQ}{dt} = k\frac{M-Q}{M}Q$$

Working with the applet, Maple, or another program, carry out the experimentation of running the program with a stepsize of 2, carrying capacity of 750 million, and different values of the growth constant k. Your objective should be to match the known U.S. population of 250 million in the year 1990.

Show your last point of the experiment here.

Final point:

year: x = _____

population: y = _____ million people

What value of k do you determine?

k = ____________________

Next, using the value of k found above, still using a stepsize of 2, project the U.S. population to the year 2100.

Year 2100 population = ____________________ million people

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4. (1 pt)

Thinking and Exploring (Verhulst) Part 2

Show how the same problem can be handled algebraically, using the mathematical solution of the initial-value problem. **(Hint: Use separation of variables. See the textbook, section 3.7.)**

What value of k do you find using the algebraic methods?

k = ____________________

What population do you project in the year 2080 using the algebraic methods?

Year 2080 population = ____________________ million people

How do the results of the algebraic methods compare with the values obtained experimentally using Eulers method approximation?

___

A. population growth is faster with Euler's method
B. population growth is about the same
C. population growth is slower with Euler's method

Comment, in particular, about the difference in the values of k for the Malthus and Verhulst models. Which is larger?

___

A. Malthus model gives a larger value of k
B. Verhulst model gives a larger value of k
C. values of k are the same

Give a short answer explaining why. This answer should go in your Interpretation and Summary report in the next problem.

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5. (1 pt)

Interpretation and Summary

Now that you have derived mathematical facts, based on the Malthus and Verhulst models, it is time to interpret and summarize the mathematical results. This should be done in terms of the original objective – to project the U.S. population into the future, based on the two models, and to learn as much as possible from the exercise that might guide public policy in the coming decades.

You will probably want to comment on how realistic the Malthus and Verhulst models seem to be. Explain why you think the models do, or do not, realistically predict the U.S. population very far into the future? What environmental or social factors might influence the growth of the population, diminishing the usefulness of the Malthus model? What advice do you have for public policy makers?

Pretend that you are reporting on this project for a publication such as Scientific American, and that your Interpretation and Summary is going to head the report. Include enough details so that a reader would learn what the major issues of the report are, how you went about addressing them, and the most important interpretations and conclusions. What will you want to tell readers about your success with regard to the original stated objective of the investigation? Assume that readers will not see your answers above. Be sure to write in complete sentences using correct rules of standard English grammar.

Your report should describe What you did, What you learned, and How it all was related to the original objective. **Express yourself in a page or less.**

To submit your answer, use the Email instructor button below. Include your email address, and enter your report in the Feedback box. When you are satisfied with your composition, click the Send Feedback button.

When you are done, return to this screen and complete the Affirmation below.

Affirmation: Even though I may have discussed the CSC project with other people, I have written up this CSC report by myself and on my own. No sharing of electronic files or notes has been involved.

Please type your name in the answer box, just as it appears in the WebWorK database and on the problem list screen, and click the Check Answers button.

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