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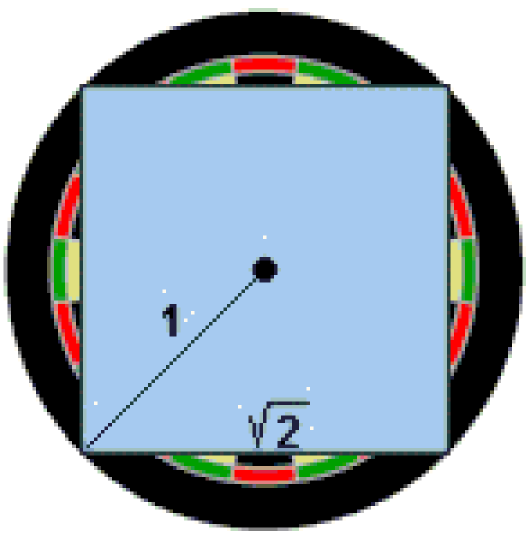
1. (1 pt)
There is a long history of methods for computing π, the value of the area of a circle of radius 1. Archimedes Method of inscribed and circumscribed polygons is described in the book. This problem asks you to look at another method using random number behavior.

Throwing Darts

Imagine that you have a circular dart board of radius 1 unit, with an unknown area that you will call π, which you wish to estimate. Tacked onto the board is a square piece of paper with sides of length $\sqrt{2}$, centered on the board.



You throw darts at the board and record for each dart whether it hit inside the square or not. Assume that every dart lands on the board, and that you are not aiming for any part of the board in particular, so that each spot of the board has an equal probability of being hit.

What is the area of the square?

__________________ square units

Suppose you have thrown 148 darts and have recorded the results with "S" for square and "C" for circle. The results are:

Number of S's: 97
Number of C's: 51

What estimate do these results give for the value of π?

$\pi \approx$ __________________

Viete's product

Another method of estimating π was discovered by French mathematician Francois Viete in 1579.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

What approximation do you get for π by computing the product of 2 terms?

____

5 terms? __________________

How many terms do you need before the estimated value of π is correct to 8 decimal places?

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2. (1 pt)
What is the area of a triangle whose boundaries are the points (6, 0), (13, 0), (13, 8)?

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What is the formula for the area of a right triangle whose base sits on the x-axis and has length L, and whose diagonal is formed by the line $y = \frac{8}{7}x - \frac{48}{7}$? Give your answer in terms of L.

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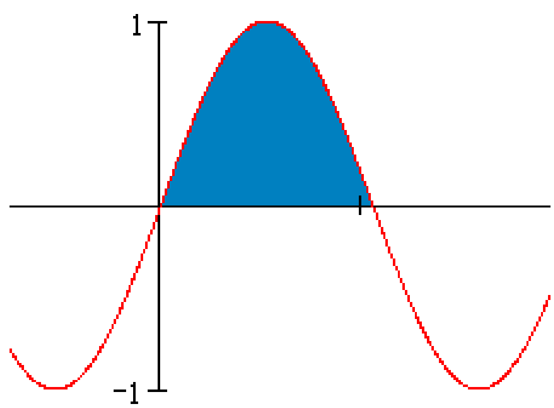
3. (1 pt)



Estimate the area under a single loop of the sine curve $\sin(4x)$ by using 4 inscribed rectangles of equal width.

Area = __________________

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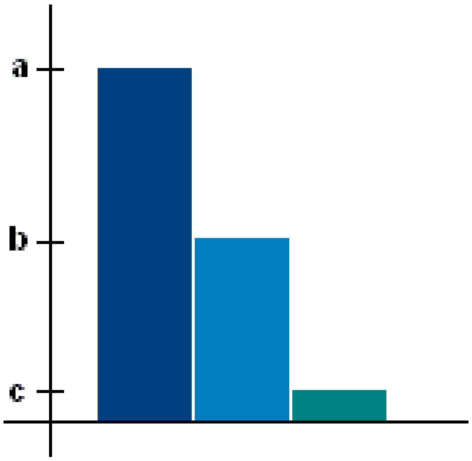
4. (1 pt)
An ice cream lover's Satisfaction Index (SI) is the measure of pleasure received from eating an ice cream cone. Consider the following SI for someone eating 3 ice cream cones.



The satisfaction received from the first cone is a = 10 units, from the second is b = 5 units, and from the third is c = 1 units.

What is the average satisfaction per cone consumed?

__________________ units

If the average falls to 2.5 after 4 cones are consumed, what is the SI for the fourth ice cream cone?

____

Which image below correctly shows the SI for four ice cream cones?

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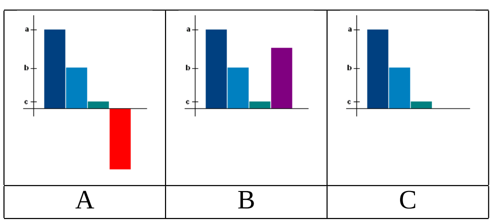

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5. (1 pt)
A hiker traveling south on the Appalachian Trail records his progress over a period of 2 weeks in a table. Note that some of the days are not recorded.

Day	Daily distance	Total distance
1	7.00	7.00
2	9.26	16.26
3	10.00	26.26
4	1.55	27.81
6	10.55	46.68
7	3.07	49.76
10	4.53	65.17
11	5.98	71.16
12	0.00	71.16
14	4.62	81.68

What is the minimum distance he could have traveled on day 8? What is the maximum?

minimum = ______________ miles

maximum = ______________ miles

What is the hiker's average distance traveled per day?

__________________ miles

If he maintains this average, how long will it take him to hike the entire 2160 miles of the Trail?

__________________ days

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6. (1 pt)
A manmade pool is created in the shape of an isoceles right triangle. Its short sides are length a = 8 feet, so it is convenient to label its corners with coordinates (0, 0), (8, 0), (0, 8). The depth D of the pool varies according to the formula $D(x,y) = xy$.

To determine the volume of the pool, make several estimates in the following way:

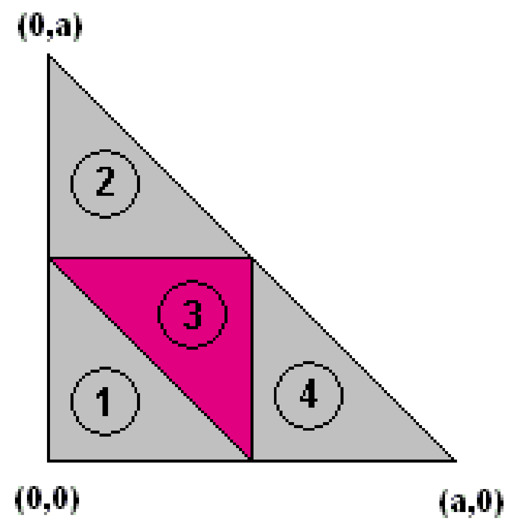
First estimate Choose a single sample point in the pool and assume it represents the average depth. Suppose the point is (4, 4). What is D(4, 4)?

__________________ feet

What is the estimate of the volume of the pool?

______________ cubic feet

Second estimate To improve the estimate, divide the triangle into 4 smaller triangles of equal area, like this:



Choose a sample point in each of the smaller triangles and use it to estimate the volume.

1: D(3, 1) = __________________

2: D(0, 4) = __________________

3: D(0.5, 4) = __________________

4: D(5, 1) = __________________

What is the resulting estimate of the volume of the pool?

__________________ cubic feet

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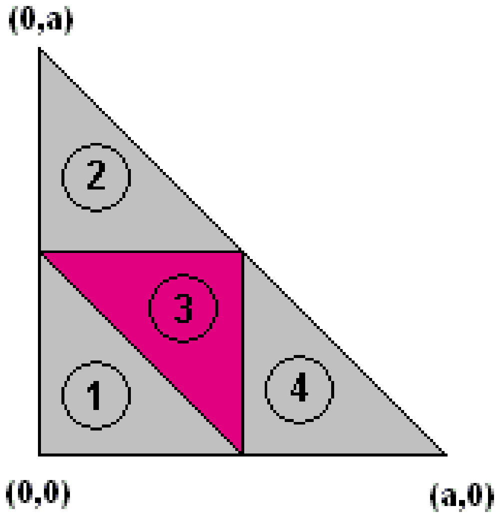
7. (1 pt)
A manmade pool is created in the shape of an isoceles right triangle. Its short sides are length a = 20 feet, so it is convenient to label its corners with coordinates (0, 0), (20, 0), (0, 20). The depth D of the pool varies according to the formula $D(x,y) = x + y$.

To approximate the volume of the pool, divide the triangle into 4 smaller triangles of equal area, like this:



Choose a sample point in each of the smaller triangles and use it to estimate the volume.

1: D(9, 0) = ____________________

2: D(7.5, 10.5) = ____________________

3: D(8.5, 5) = ____________________

4: D(17, 3) = ____________________

What is the estimate of the volume?

____________________ cubic feet

What is the maximum estimate for the volume of the pool using this method?

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What is the minimum?

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8. (1 pt)
A remarkably good approximation for π was given by the mathematician Ramanujan in 1914:

$$\pi \approx \sqrt[4]{9^2 + \frac{19^2}{22}}$$

If this approximation is used to find the circumference of a circle of radius 4010 miles, what is the difference from the exact circumference?

____________________ inches

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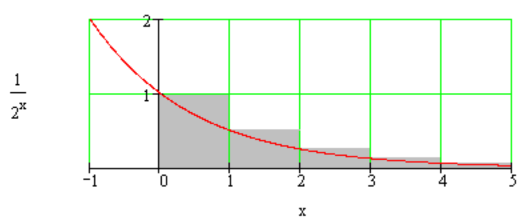
9. (1 pt)
Consider the graph of the function $\frac{1}{2^x}$ as shown in the following picture.



Each rectangle has width 1, and meets the curve at its upper left corner.

What is the area of the largest rectangle?

____

What is the sum of the areas of the 5 rectangles shown? Simplify your answer.

____

Which of the following is a formula for the sum of n such rectangles? Check all correct formulas.

- A. $1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{n-1}}$
- B. $\frac{2^n-1}{2^{n-1}}$
- C. $\frac{2^{n-1}}{2^n}$
- D. $\frac{2}{n}$

Based on the calculations in this problem, what is an upper bound for the area bounded by the x axis, the curve, and the y axis?

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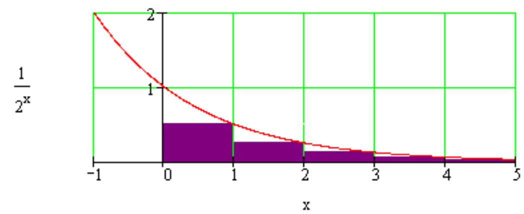
10. (1 pt)
Consider the graph of the function $\frac{1}{2^x}$ as shown in the following picture.



Each rectangle has width 1, and meets the curve at its upper right corner.

What is the area of the largest rectangle?

____

What is the sum of the areas of the 5 rectangles shown?

____

Give a formula for the sum of the areas of the first n rectangles.

____

Based on the calculations in this problem, what is a lower bound for the area bounded by the x axis, the curve, and the y axis?

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11. (1 pt)
Let E be the ellipse given by the equation

$$\frac{x^2}{16}+\frac{y^2}{49}=1$$

Approximate the area of E using 4 circumscribed rectangles in the first quadrant.

Area of E = ____________________

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12. (1 pt)
Approximate the area of the half circle of radius 11 centered at the origin using 11 inscribed rectangles above the x-axis (in the first and second quadrants).

Area = ____________________

13. (1 pt)

The area of an ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

Approximate the area of the ellipse given by the equation $\frac{x^2}{16}+\frac{y^2}{9}=1$ using 3 inscribed horizontal rectangles in the first quadrant.

Area of E = ________________

What is the difference between this approximation and the exact area?

____

14. (1 pt)

Consider the two ellipses with equations $\frac{x^2}{169}+\frac{y^2}{196}=1$ and $\frac{(x-8)^2}{169}+\frac{y^2}{196}=1$. Approximate the entire area of their intersection using 18 circumscribed rectangles chosen suitably in the first quadrant to approximate one fourth of the area.

Area = ________________

15. (1 pt)

Approximate the area of the section of the circle of radius 14 from the x-axis to the angle $\frac{5\pi}{6}$ by first approximating the area of the entire circle using 14 inscribed rectangles in the first quadrant.

Area = ________________

16. (1 pt)

The rate of flow of water through a pipe over a 1.5-hour period is the graph of the curve $y=(\tan x)^5$. Units of time are in hours, units of rate of flow are in $\frac{m^3}{sec}$. Use the method of accumulation to approximate the total volume of flow over the 1.5 hours, assuming the rate of flow is measured every 15 minutes starting at time 0.

Volume of flow = ________________ cubic meters

17. (1 pt)

What is the difference between the area approximation of the quarter circle of radius 7 using 10 circumscribed rectangles and the area approximation of the same quarter circle using 14 circumscribed rectangles?

____

18. (1 pt)

Water drains out of a pond at the rate $y=\frac{6}{x}$ where rate of flow is measured in cubic meters per minute and time is measured in hours, from time = 12 minutes to time = 1 hour. Assuming the pond began with 6000 cubic meters of water, approximate how much water is left in the pond at the end of the hour, assuming rate of flow is measured every 12 minutes beginning at time = 12 minutes.

Water left = ________________ cubic meters