Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 4.3

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1. (1 pt)
The following sum can be interpreted as a sum of areas of rectangles which approximate the area of a plane region R. By taking the limit of this sum, approximate the area of R.

$$S_n = \sum_{i=1}^{n} \frac{2}{n}\left(1+\frac{2i}{n}\right)$$

Area of R = ________ square units

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2. (1 pt)
Express the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n} \ln\left(1+\frac{6i}{n}\right)$$

as a definite integral $\int_0^b f(x)dx$ by finding b and $f(x)$.

$b =$ ________

$f(x) =$ ________

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3. (1 pt)
Simplify the following expression.

$$7\int_a^b f(x)dx + 7\int_b^c f(x)dx - 5\int_c^a f(x)dx$$

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- A. $7\int_a^c f(x)dx + 5\int_c^a f(x)dx$
- B. $12\int_a^c f(x)dx$
- C. $24\int_a^c f(x)dx$
- D. $12\int_a^b f(x)dx$
- E. $2\int_a^c f(x)dx$

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4. (1 pt)
Evaluate the following integral using properties of definite integrals and interpreting integrals as areas:

$$\int_{-1}^{6} (4x-2)dx$$

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5. (1 pt)
Evaluate the following integral using properties of definite integrals and interpreting integrals as areas.

$$\int_{-4}^{0} \sqrt{16-x^2}\, dx$$

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6. (1 pt)
Evaluate the following integral using properties of definite integrals and interpreting integrals as areas.

$$\int_{-\pi/4}^{\pi/4} \sin x\, dx$$

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7. (1 pt)
Evaluate the following integral using properties of definite integrals and interpreting integrals as areas.

$$\int_{-2}^{2} \left(5u^3 - 5u^9 + \frac{\pi}{2}\right) du$$

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8. (1 pt)
Given $\int_0^a x^2 dx = \frac{a^3}{3}$
evaluate

$$\int_0^2 \left(3v^2+4v+6\right) dv$$

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- A. π
- B. 56
- C. 0
- D. 28
- E. 9.3

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9. (1 pt)
Find $\int_0^2 g(x)dx$, where

$$g(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ x^9, & 1 < x \le 2 \end{cases}$$

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10. (1 pt)
Consider the function $f(x) = x$.

Which of the following statements is true? Check all that apply.

- A. $f(x)$ is an even function.
- B. There exists c in $[a,b]$ such that $\int_a^b f(x)dx = (b-a)f(c)$.
- C. $f(x)$ has the property $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.
- D. $f(x)$ is an odd function.
- E. $f(x)$ has the property $\int_{-a}^{a} f(x)dx = 0$.

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11. (1 pt)
What is $\int_{-13}^{13} \sqrt{169-x^2}\, dx$?

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12. (1 pt)
Which of the following integrals are negative?

- A. $\int_0^3 -x^2+3x+4\, dx$
- B. $\int_0^1 x-\sin(x)\, dx$
- C. $\int_{-3}^{6} -|x|\, dx$
- D. $\int_{-\pi-1}^{\pi} \sin(x)\, dx$
- E. $\int_{-3\pi/2}^{\pi} \cos(x)\, dx$
- F. All of the above
- G. None of the above

13. (1 pt)
What is
$$\int_{\frac{5\pi}{2}}^{\frac{9\pi}{2}} \cos(x)+\sqrt{\pi^2-\left(x-\frac{7\pi}{2}\right)^2}+5x\,dx?$$
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What is the average value of this function over the interval $\left[\frac{5\pi}{2},\frac{9\pi}{2}\right]$?
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14. (1 pt)
Write the expression
$$\lim_{n\to\infty}\sum_{i=0}^{n-1}\left(\left(1+\frac{5i}{n}\right)^2+3\left(1+\frac{5i}{n}\right)+16\right)\frac{5}{n}$$
as a definite integral, $\int_a^b f(x)dx$.
a = ______________
b = ______________
f(x) = ______________

15. (1 pt)
What is $\int_{9}^{25}\sqrt{64-(x-17)^2}+10\,dx$?
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What is the average value of this function over the interval $[9,25]$?
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16. (1 pt)
Write the expression
$$\lim_{n\to\infty}\sum_{i=0}^{n-1}\sin\left(\left(\frac{4\pi i}{n}\right)^2\right)\frac{4\pi}{n}$$
as a definite integral, $\int_a^b f(x)dx$.
a = ______________
b = ______________
f(x) = ______________

17. (1 pt)
Write the expression
$$\lim_{n\to\infty}\sum_{i=0}^{n-1}\frac{i}{n^2}$$
as a definite integral, $\int_a^b f(x)dx$.
a = ______________
b = ______________
f(x) = ______________

18. (1 pt)
Write the expression
$$\lim_{n\to\infty}\sum_{i=0}^{n-1}\frac{8\ln(4(2+\frac{8i}{n}))}{3n}$$
as a definite integral, $\int_a^b f(x)dx$.
a = ______________
b = ______________
f(x) = ______________

19. (1 pt)
What is $\int_{-\frac{3\pi}{2}}^{\frac{\pi}{2}}|x|+\cos(x)\,dx$?
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