Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 4.6

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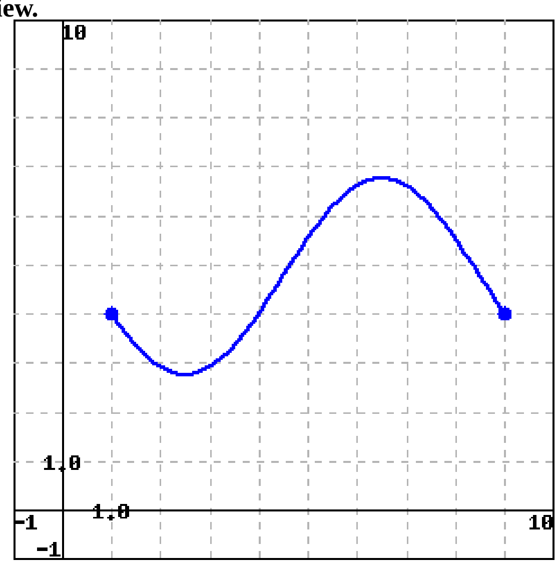
1. (1 pt)
Below is the graph of a function f. **Click on image for a larger view.**



Using the values from the graph, find the Trapezoid Rule estimates T_4 and T_8 for $\int_1^9 f(x)\,dx$. You may round the values from the graph to the nearest 0.5.

$T_4 =$ ____________________

$T_8 =$ ____________________

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2. (1 pt)
Consider the Trapezoid Rule:

$$T_n = \frac{h}{2}\left(y_0 + 2\sum_{j=1}^{n-1} y_j + y_n\right)$$

PART I
Write out the trapezoidal approximation T_5 to the integral $\int_0^1 e^{-x^4}\,dx$.

What is y_2?

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PART II
Give a decimal approximation to T_5. You may use Maple, an applet, or another computer algebra program to calculate the answer if you wish.

$T_5 =$ ____________________

Approximate the integral to four decimal places by evaluating T_n for values of n greater than 5.

$\int_0^1 e^{-x^4}\,dx \approx$ ____________________

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3. (1 pt)
Use the Trapezoid Rule to approximate the definite integral below for n = 4.

$\int_0^4 \frac{1}{1+x^2}dx =$ ____________________

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4. (1 pt)
Use the Trapezoid Rule to approximate the definite integral below for n = 8.

$\int_2^6 \sqrt{1+x^3}dx =$ ____________________

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5. (1 pt)
Use the Trapezoid Rule to approximate the definite integral below for n = 14.

$\int_0^1 x\sin(x)dx =$ ____________________

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6. (1 pt)
Use the Trapezoid Rule to approximate the definite integral below for n = 10.

$\int_1^e \frac{1}{x}dx =$ ____________________

What is the smallest n that results in the value of this integral equal to 1 to 3 decimal places?

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7. (1 pt)

x	y
2	4.02
2.25	4.4
2.5	5.06
2.75	5.6
3	6.05
3.25	6.57
3.5	7.07
3.75	7.42
4	8.02

The table indicates a relationship between two variables x and y. Assume $y = f(x)$ where f is continuous, and approximate $\int_2^4 f(x)dx$ using the Trapezoid Rule.

$\int_2^4 f(x)dx =$ ____________________

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8. (1 pt)

x	y
0	-0.060
0.1	0.190
0.2	0.299
0.3	0.346
0.4	0.349
0.5	0.389
0.6	0.475
0.7	0.724
0.8	0.737
0.9	0.863
1	0.901
1.1	0.811
1.2	0.842

The table indicates a relationship between two variables x and y. Assume $y = f(x)$ where f is continuous, and approximate $\int_0^{1.2} f(x)dx$ using the Trapezoid Rule.

$\int_0^{1.2} f(x)dx =$ ______________

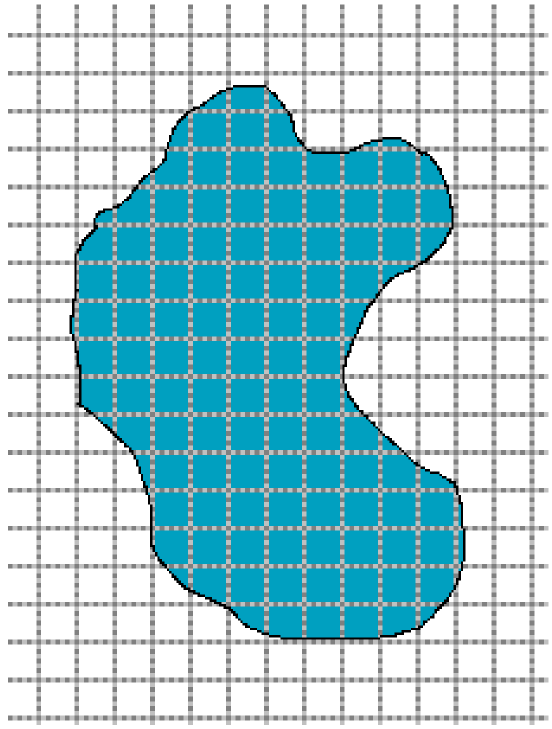
9. (1 pt)

x	y
5	2.74
5.5	2.85
6	1.95
6.5	3.55
7	2.15
7.5	3.74
8	3.33
8.5	3.92
9	2.00
9.5	2.08
10	3.66

The table indicates a relationship between two variables x and y. Assume $y = f(x)$ where f is continuous, and approximate $\int_5^{10} f(x)dx$ using the Trapezoid Rule.

$\int_5^{10} f(x)dx =$ ______________

10. (1 pt)
A map of a pond is shown below.



Assume each little square is 10 feet on each side. Use the Trapezoid Rule to estimate the area of the pond to the nearest 100 square feet.

Area = ______________

11. (1 pt)
What is the 9th subinterval trapezoid approximation of $\int_0^{20} \frac{14x}{\sqrt{x+1}}\,dx$?

What is the difference (in absolute value) between the approximation and the exact value of the integral?

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12. (1 pt)
What is the difference between the 7th subinterval trapezoid approximation and the 10th subinterval trapezoid approximation of $\int_{-5}^{8} x^3 + 3x\,dx$; that is, what is $T_7 - T_{10}$?

13. (1 pt)
Estimate $\int_0^5 x^{\frac{3}{2}}\,dx$ using 5 trapezoids.

What is the exact value of this integral?

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14. (1 pt)
Approximate $\int_9^{11} \ln(x)\,dx$ using 9 trapezoids.

According to the **Web site example**, what is the maximum difference between this approximation and the exact value of the integral?

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15. (1 pt)
The trapezoid approximation of an integral is often more accurate than an equivalent Riemann sum.

Approximate $\int_9^{15} x^6 - 23x - 27\,dx$

Using 4 trapezoids: ____________________

Using 4 circumscribed rectangles: ____________________

What is the exact value of the integral?

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16. (1 pt)
Approximate $\displaystyle\int_6^{11} x^{1/2}\,dx$ using 7 trapezoids.

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According to the **Web site example**, what is the maximum difference between this approximation and the exact value of the integral?

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What is the actual difference (in absolute value) between the approximation and the exact value of the integral?

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17. (1 pt)
Match the integrals to the values of their trapezoid approximations.

___1. $\int_4^{17} 1+\cos(x)\,dx$, 7 trapezoids
___2. $\int_2^{16} x^2+5x+9\,dx$, 16 trapezoids
___3. $\int_{10}^{18} \sqrt{x}\,dx$, 9 trapezoids
___4. $\int_4^{11} \ln(x)\,dx$, 9 trapezoids
___5. $\int_2^{20} \sin(x)+\tan(x)\,dx$, 4 trapezoids
___6. $\int_{10}^{16} e^x + x\,dx$, 10 trapezoids

A. 12.8578926740226
B. 2124.31275720165
C. -1015.12261897343
D. 29.82718692231
E. 13.8236654949988
F. 9128502.5952647

18. (1 pt)
Consider $\int_4^{16} \ln(x)x^{\frac{5}{2}}$. Refer to example 3 in this section on the Web site. Without evaluating the integral or computing a trapezoid approximation, determine the minimum number of trapezoids needed to guarantee that the trapezoid approximation of this integral is within .1 of the real value.

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19. (1 pt)
Consider $\int_5^{10} 4^x$. Refer to example 3 in this section on the Web site. Without evaluating the integral or computing a trapezoid approximation, determine the number of trapezoids needed to guarantee that the trapezoid approximation of this integral is within 10 of the real value.

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