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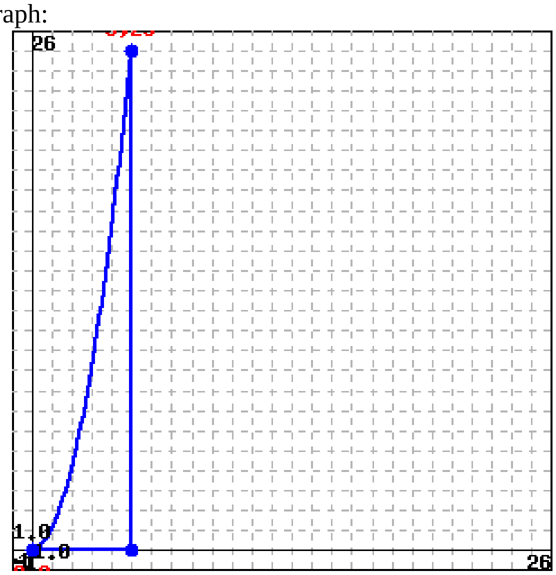
1. (1 pt)
Setup

The purpose of this section is for you to write a clear statement of the mathematics that will be applied to the problem. What is the nature of the information that is available? What are the steps that you should follow to analyze the information? What tools will you bring to bear in the analysis? The setup phase in tackling a scientific problem is often called the modeling phase.

Once the physical problem has been thus expressed in mathematical terms one can apply the theories and techniques of mathematics toward finding solutions. Be sure to consider what you are doing as you go along. You should show that you understand very clearly what is the physical problem, what is the mathematical model, and what is the connection between the two.

In this exercise, you will use the concepts of center of mass and Riemann sum to find the centroid of a common region.

Consider the area under a parabola, as shown in the following graph:



Suppose you estimate the area of the region with a Riemann sum using 11 rectangles.

What is the x-coordinate of the center of the first rectangle?
x = ____________________

What is the y-coordinate of the center of the first rectangle?
y = ____________________

What is the x-coordinate of the center of the 10th rectangle?
x = ____________________

What is the y-coordinate of the center of the 10th rectangle?
y = ____________________

Write, in terms of k, formulas for the x- and y-coordinates of the kth rectangle.
x = ____________________
y = ____________________

Find, in terms of k, the moments with respect to the x- and y-axes of the kth rectangle.

Moment with respect to the x-axis = ____________________
Moment with respect to the y-axis = ____________________

Which of the following belong in the integrals

$x_0 = c\int_a^b f(x)dx$

$y_0 = d\int_a^b g(x)dx$

to correctly compute the center of mass of the region?
___

A. a = 0, b = 5, c = $\frac{1}{41.667}$, d = $\frac{1}{83.333}$, $f(x) = x^3$, $g(x) = x^4$
B. a = 0, b = 11, c = $\frac{1}{41.667}$, d = $\frac{1}{41.667}$, $f(x) = x^3$, $g(x) = x^4$
C. a = 0, b = 5, c = $\frac{1}{41.667}$, d = $\frac{1}{83.333}$, $f(x) = x^4$, $g(x) = x^3$
D. a = 0, b = 11, c = $\frac{1}{83.333}$, d = $\frac{1}{41.667}$, $f(x) = x^3$, $g(x) = x^4$

Evaluate the integrals with their correct components to find the coordinates of the center of mass of the region.

$x_0 =$ ____________________
$y_0 =$ ____________________

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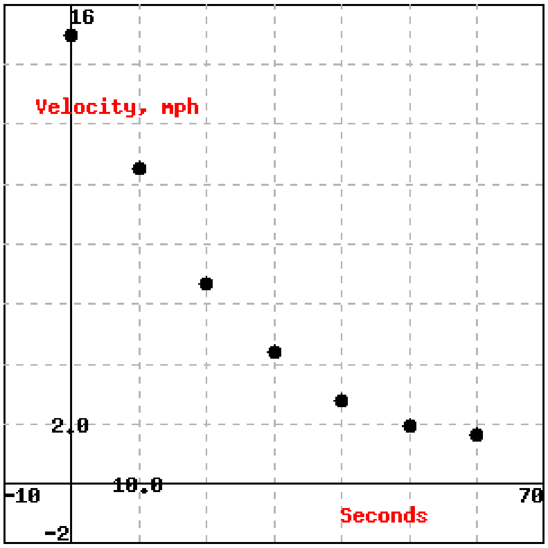
2. (1 pt)
Braking Problem

If we wanted to model the braking of a good driver, it would be reasonable to assume that he or she applies force to the brake in such a way that the deceleration at time t is proportional to the velocity of the car at that time. Here are some data that represent at specific instants of time t (in seconds) the velocity v(t) (in miles per hour) of a car being braked to a stop by a good driver:

t = [0, 10, 20, 30, 40, 50, 60]
v = [14.95, 10.5, 6.65, 4.4, 2.8, 1.95, 1.65]



Find the distance required for the driver to bring the car to a stop from the initial speed by approximating the distance directly from the time-and-velocity data by constructing a lower Riemann sum with 6 rectangles.

Stopping distance = ____________________ feet

Assume the data can be modeled by an exponential function of the form

$v(t) = v_0 e^{rt}$

What are v_0 and r?

v_0 = ____________________

r = ____________________

What is the stopping distance found by integrating the exponential?

Stopping distance = ____________________ feet

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3. (1 pt)

Thinking and Exploring

Here are some suggested activities for you to carry out to develop an understanding of the difference between the two hydrographs of the Gorge River, Gorge1 and Gorge2 in the Flood Watch applet.

You should answer the following questions. In your answers, consider how to evaluate any integrals that arise using the Fundamental Theorem of Calculus, or how to approximate the integral using a Riemann Sum. Justify all approaches and conclusions both in terms of mathematics and in terms of the geological problem you are addressing.

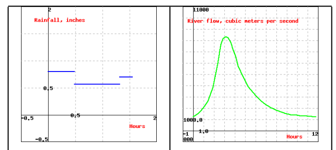
Part A

Consider first the rainfall and flow data that comprise Gorge1 hydrograph for the Gorge River. The rainfall is measured in units of inches per hour, the discharge in units of cubic meters per second, and the horizontal time line x in hours.

Rainfall:

$$\begin{cases} 0 & x \le 0 \\ 0.8 & 0 < x \le 0.49 \\ 0.56 & 0.49 < x \le 1.32 \\ 0.7 & 1.32 < x \le 1.55 \\ 0 & x > 1.55 \end{cases}$$

Graphs of the rainfall and flow over time are shown below. Go to the applet or Maple worksheet containing the flow data and type in the rainfall data given above that is somewhat different from that for Gorge1 in the applet. You then can use the applet to get the information you will need to do the analysis and answer the following questions.



Rainfall | Discharge

What is the Baseflow Discharge Rate?

Baseflow Discharge Rate = ____________________ cubic meters per second

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What is the earliest time at which the Total Discharge rate for the Gorge River is at a maximum?

____________________ hours

What is the maximum Total Discharge rate?

Maximum discharge = ____________________ cubic meters per second

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What is the volume due to Baseflow discharge over the 12-hour observation time?

Discharge, Baseflow = ____________________ cubic meters

What is the volume due to the Rainfall discharge over the 12-hour observation time?

Discharge, rain = ____________________ cubic meters

What is the Total Discharge volume over the 12-hour observation time?

Total discharge = ____________________ cubic meters

About how long did the rain storm last?

____________________ hours

What is the centroid of the precipitation?

x = ____________________ hours,

y = ____________________ inches per hour

What is the centroid of the Rainfall Discharge?

x = ____________________ hours,

y = ____________________ cubic meters per second

What is the lag time?

____________________ hours

What is the area of the watershed?
Area = ____________________ square miles

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4. (1 pt)

Thinking and Exploring

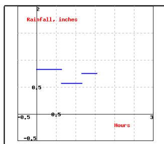
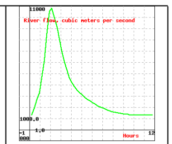
Part B

Now consider new data for rainfall and flow (Gorge2 in the Flood Watch applet), a second hydrograph for the Gorge River recorded 10 years later using the same units of measure.

Rainfall:

$$\begin{cases} 0 & x \le 0 \\ 0.82 & 0 < x \le 0.64 \\ 0.56 & 0.64 < x \le 1.16 \\ 0.75 & 1.16 < x \le 1.55 \\ 0 & x > 1.55 \end{cases}$$

Graphs of the rainfall and flow over time are shown below. Go to the applet or Maple worksheet containing the flow data and type in the rainfall data given above that is somewhat different from that for Gorge2 in the applet. You then can use the applet to get the information you will need to do the analysis and answer the following questions.

	
Rainfall	Discharge

What are the amount and duration of the second rainstorm?
Amount, second rainstorm = ____________________ inches
Duration, second rainstorm = ____________________ hours

What is the maximum Total Discharge rate for the more recent hydrograph?
Maximum discharge = ____________________ cubic meters per second

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What is the Baseflow Discharge Rate in the second hydrograph and how does it compare to that of the first?
Baseflow Discharge Rate, second hydrograph = ____________________ cubic meters per second
This is ___

A. greater than the first Baseflow Discharge Rate
B. less than the first Baseflow Discharge Rate
C. the same as the first Baseflow Discharge Rate

What is the Total Discharge volume in the second hydrograph and how does it compare to that of the first?
Total Discharge volume, second hydrograph = ____________________ cubic meters
This is ___

A. greater than the first Total Discharge volume
B. less than the first Total Discharge volume
C. the same as the first Total Discharge volume

What is the lag time in the second hydrograph and how does it compare to that of the first?
Lag time, second hydrograph = ____________________ hours
This is ___

A. greater than the first lag time
B. less than the first lag time
C. the same as the first lag time

What is the area of the watershed for the second storm?
Area = ____________________ square miles

Is the Gorge River more likely to flood than it was 10 years earlier?

___

A. no
B. yes

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5. (1 pt)

Interpretation and Summary

Now that you have (1) studied the two hydrographs of the Gorge River and (2) thought about and derived mathematical facts, it is time to interpret and summarize the mathematical results in terms of the original objective.

You should be sure to provide answers to the following questions:

What changes in the watershed might have occurred in the 10 years between the hydrographs to account for the different pattern of the more recent hydrograph relative to the earlier one?

How might these changes influence the likelihood of the Gorge River flooding?

Pretend that you are reporting on this project for a publication such as Scientific American, and that your Interpretation and Summary is going to head the report. Include enough details so that a reader would learn what the major issues of the report are, how you went about addressing them, and the most important interpretations and conclusions. What will you want to tell readers about your success with regard to the original stated objective of the investigation? Assume that readers will not see your answers above. Be sure to write in complete sentences using correct rules of standard English grammar. **Express yourself in a page or less.**

To submit your answer, use the Email instructor button below. Include your email address, and enter your report in the Feedback box. When you are satisfied with your composition, click the Send Feedback button.

When you are done, return to this screen and complete the Affirmation below.

Affirmation: Even though I may have discussed the CSC project with other people, I have written up this CSC report by myself and on my own. No sharing of electronic files or notes has been involved.

Please type your name in the answer box, just as it appears in the WebWorK database and on the problem list screen, and click the Check Answers button.

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6. (1 pt)
In example 2 on the Web site, say the base flow is 164 $\frac{m^3}{sec}$, i.e. the rainfall discharge between time 0 and 5 hours is 164 $\frac{m^3}{sec}$ and the rainfall discharge after time 20 hours is 164 $\frac{m^3}{sec}$. Assuming the rest of the conditions are the same, calculate the new rainfall discharge volume.

volume = ____________________ m^3

Now calculate the lag time between the rainfall event and the rainfall discharge.

lag time = ____________________ hrs

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7. (1 pt)
Say in example 2 on the Web site the rainfall is measured in centimeters per hour, rather than in inches per hour. Assuming all other data remain the same, what is the new area of the rainfall plot? (Use the conversion 1 meter = 39.37 inches.) What is the lag time?

Rainfall plot area = ____________________inch-hour

Lag time = ____________________ hrs

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8. (1 pt)
Say in example 2 on the Web site the rainfall discharge (on the y axis) is in cubic meters per minute instead of cubic meters per second. Assuming all other conditions remain the same, what is the total volume of rainfall discharge? What is the lag time?

Rainfall discharge = ____________________ m^3

Lag time = ____________________ hrs

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9. (1 pt)
What is the centroid of the region between the curves $y^2 = 2x$ and $x - y = 4$ for $-2 \le y \le 2$?

$x_0 =$ ____________________

$y_0 =$ ____________________

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10. (1 pt)
What is the centroid of the region between the curve $y = -(x-3)^2 + 3$ and the x axis?

$x_0 =$ ____________________

$y_0 =$ ____________________

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11. (1 pt)
Calculate the centroid of the region between the curves $f(y) = \sin(y) + 10$ and $g(y) = \cos(y) + 4$ for $1 \le y \le 13$.

$x_0 =$ ____________________

$y_0 =$ ____________________

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12. (1 pt)
Fertilizer is applied to a sunflower, following the same time pattern as in example 3 on the Web site, and the flower grows at a rate of $e^x - 1$ inches per month for 3 months (where x is in months). Assuming the sunflower is initially of height 0, what is its height at the end of 3 months?

____________________ inches

What is the lag time between the application of fertilizer and the flower's growth?

____________________ months

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13. (1 pt)
Say the flower in example 3 on the Web site grows for 2 years at the same growth rate ($4x^2$ inches per month), with all other conditions the same. What is the lag time between the application of fertilizer and the flower's growth?

____________________ months

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14. (1 pt)
Say the flower in example 3 on the Web site grows for 2 months at a growth rate of $e^x - 1$ inches per month, with x in months, but also shrinks at the rate of $e^x - 1$ inches per year (where x is in years) due to lack of water. Assuming all other conditions the same, what is the lag time between the application of fertilizer and the flower's growth?

____________________ months

What is the height of the flower after 2 months?

____________________ inches