Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 5.1

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1. (1 pt)
Setup
After reviewing the textbook section on Sleuthing Galileo and experimenting with the applet, you should be ready to describe the kinds of mathematical facts you are looking for and the methods you expect to use. For this problem, you will study the mathematical issues and techniques involved in modeling the rolling ball experiment, verifying that Galileo's experimental data are consistent with the results of the theory. In the Interpretation and Summary question, you will discuss any inconsistencies or open questions, in particular the role of time measurements in the determination of g.
Thinking and Exploring
Part I: The Mathematical Model
This table shows Galileo's measurements of H, the release height of the ball above the table, and D, the horizontal distance traveled by the ball.

H	D
1000	1500
828	1340
800	1328
600	1172
300	800

Let s be the vertical drop of the ball whose velocity down the ramp is u. Then the gain in kinetic energy $\frac{mu^2}{2}$ equals the loss of potential energy mgs, where m is the mass of the ball and g is the acceleration due to gravity.

Using the equation $mgH = \frac{mu^2}{2}$, compute the speed at the bottom of the ramp u_T as a function of g for each of the release heights in the table.
H = 1000: ______________
H = 828: ______________
H = 800: ______________
H = 600: ______________
H = 300: ______________
Compute the horizontal and vertical components u_1 and u_2 of velocity u = 85 if the angle A made by the table is 0.1 radians. Refer to the textbook for information on the the position of the ball on the ramp.
$u_1 =$ ______________
$u_2 =$ ______________
If the terminal horizontal component of velocity at the bottom of the ramp equals 0, what is your interpretation of the configuration of the ramp? You may address this question in the Interpretation and Summary section.

The ball leaves the ramp with horizontal velocity equal to u_{1T}, and its horizontal velocity when it hits the floor is
___
A. faster than u_{1T}
B. equal to u_{1T}
C. slower than u_{1T}

because
___
A. the ball obeys Newton's First Law of Motion
B. gravity slows down the horizontal velocity of the ball
C. the potential energy of the ball is increasing
D. the terminal velocity of the ball when it leaves the ramp is nonzero
E. the force of gravity is constant

There are two equations that use t_f, the time the ball spends in the air after it leaves the ramp. The first describes the horizontal distance D traveled during t_f seconds, and the second describes the vertical distance L from the table to the floor (see textbook).
Write D as a function of terminal horizontal velocity v and air time t.
D = ______________
For each of the horizontal distances traveled by the ball as recorded in the table, compute the terminal horizontal velocity of the ball leaving the ramp if the time the ball spends in the air is 3.4 seconds.
D = 1500: ______________
D = 1340: ______________
D = 1328: ______________
D = 1172: ______________
D = 800: ______________
Express t_f as a function of L and g.
$t_f =$ ______________
Combine the equations involving flight time and express D as a function of H, L, and A.
D = ______________

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2. (1 pt)
Thinking and Exploring
Part II: Galileo's Data
Show that in our model, we can equally well regard H as a quadratic function of D, L, and A.
H = ______________
Let $k = 4L\sin^2(A)$. What is $\ln H$ in terms of D and k?
$\ln H =$ ______________
Using the tools you have, such as Maple, an applet, or another computer program, graph the $(\ln D, \ln H)$ data and the best-fitting line $y = ax + b$. Are the data consistent with the model?

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If the data are consistent with the model, what are k and A?

k = ____________________

A = ____________________

What are some reasonable choices of ramp angle A and table height L? For each of these values of L, calculate the resulting angle A.

L = 800, A = ____________________

L = 1200, A = ____________________

L = 20000, A = ____________________

What is the range of possible values of A? Enter **infinity** or **-infinity** for ∞ or $-\infty$.

between __________ and __________

What is the range of possible values of L? Enter **infinity** or **-infinity** for ∞ or $-\infty$.

between __________ and __________

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3. (1 pt)

Thinking and Exploring

Part III: The Total Travel Time and g

Imagine you have a stopwatch that you start at the instant you release the ball on the ramp, and stop when you see or hear it hit the floor. Then you can determine g. Here are the steps.

Recall that you developed a formula for t_f, the time the ball is in the air. What is it?

$t_f =$ ____________________

To find the time t_r on the ramp, refer to the sketch from the textbook showing the relationship between the distance s_r the ball rolls on the ramp and the corresponding vertical drop s. If A = 0.1 radians and s_r = 420 units, what is s?

s = ____________________

Recall that u equals the velocity of the ball down the ramp. In problem 2, you calculated the speed at the bottom of the ramp as a function of g and H for various values of H. The relationship between u and s defines a differential equation, since $u = \frac{ds_r}{dt}$.

Express u as a function of s_r and use the initial condition $s_r(0) = 0$ to solve the differential equation. What are the units of g in the solution $s(t)$?

___

A. units of time divided by units of distance
B. units of distance divided by units of time
C. units of distance divided by units of time squared
D. g is dimensionless and has no units
E. units of velocity time units of time

What is t_r in terms of A, H, and g?

$t_r =$ ____________________

What is $T = t_f + t_r$?

$T =$ ____________________

What happens to T as A $\to$ 0? as A $\to \pi/2$? Include your answer to this question in the Interpretation and Summary section.

If H = L, find a formula for g in terms of A, H, and T.

g = ____________________

If you were to set up an experiment to measure g as Galileo did, with H = L, and recorded the following total travel times, give the experimental values of g that you would calculate. Assume the angle of the ramp is set at A = 0.1.

for H = 1000, T = 0.878: g = ______________

for H = 828, T = 0.799: g = ______________

for H = 800, T = 0.785: g = ______________

for H = 600, T = 0.646: g = ______________

for H = 300, T = 0.481: g = ______________

If you assume g is constant, which of the above measured times would you suspect of experimental error?

T = ____________________

Let A = $\pi/4$. If you assume the value of g in punti per second squared is equal to what you found in the previous question, what travel time would you measure in the lab when H = L = 900 punti? (see textbook)

T = ____________________ seconds

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4. (1 pt)

Thinking and Exploring

Part IV: The Path of the Ball

Now that you have derived the equations of motion of the ball on the ramp and experimented with formulas for g and the travel time, you are ready to complete the picture by describing the x and y coordinates of the ball as a function of time, and to graph the path of the ball.

The function $x(t)$ can be written as piecewise functions $x_r(t)$ and $x_f(t)$, where $x_r(t)$ is the x coordinate of the ball when it is on the ramp, and $x_f(t)$ is the x coordinate of the ball when it is in the air. Likewise, $y_r(t)$ and $y_f(t)$ give the y coordinate of the ball.

Derive formulas for $w = u_1$ and $z = u_2$, the horizontal and vertical components of velocity when the ball is on the ramp. What are $x_r(t)$ and $y_r(t)$ in terms of w and z?

$x_r(t) =$ ____________________

$y_r(t) =$ ____________________

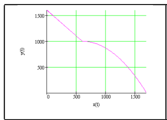
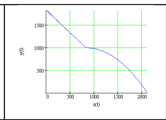
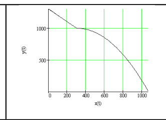
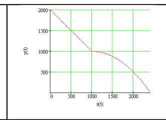
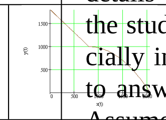
Explain why the following piecewise representations of x(t) and y(t) make sense:

$$x(t) = \begin{cases} \frac{1}{4}gt^2 \sin 2A & \text{if } t \le t_r \\ \sqrt{2gH}(t - t_r)\sin A + B & t > t_r \end{cases}$$

$$y(t) = \begin{cases} H + L - \frac{1}{2}gt^2 \cos^2 A & \text{if } t \le t_r \\ L - \frac{1}{2}g(t - t_r)^2 & t > t_r \end{cases}$$

Address this question in the Interpretation and Summary section.

Using Maple, the applet, or another computer program, graph the path of the ball. Begin by specifying values of A and L, and plot the path of the ball for the 5 values of H in Galileo's data. Compare the resulting values of D in the plots and the data. Identify each of the graphs below with the value of H in Galileo's table, and give the resulting value of D. A is chosen in each case to be $\frac{\pi}{4}$. You should be able to determine L by examining the graphs.

				
A	B	C	D	E

A. D = _______________
B. D = _______________
C. D = _______________
D. D = _______________
E. D = _______________

For the same values of A and L, fix a value of H. Plot the path for values of g equal to 10, 20, 100, 1000. What happens to the (x(t), y(t)) graph when g is increased? Address this question in the Interpretation and Summary section.

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5. (1 pt)

Interpretation and Summary

Now that you have derived the formulas for the path of the ball and calculated experimental values for the force of gravity g, it is time to interpret and summarize the mathematical results in terms of the original objective.

Pretend that your synopsis is going to appear in the next issue of a magazine such as Scientific American. Include enough details so that a reader would learn what the major issues of the study are, and how you went about addressing them, especially in relation to the stated goals of the Objective. Be sure to answer the questions asked in earlier WebWorK problems. Assume that readers will not see your answers above. Be sure to write in complete sentences using correct rules of standard English grammar. **Express yourself in a page or less.**

To submit your answer, use the Feedback button below. Include your email address, and enter your report in the Your comments box. When you are satisfied with your composition, click the Submit Your Comments button.

When you are done, return to this screen and complete the Affirmation below.

Affirmation: Even though I may have discussed the CSC project with other people, I have written up this CSC report by myself and on my own. No sharing of electronic files or notes has been involved.

Please submit your name in the answer box, just as it appears in the WebWorK database and on the problem list screen.

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