Topology

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The candidate should know the following definitions and theorems and be able to apply them to draw conclusions about specific topological spaces and continuous maps.

Point Set Theory

Definitions: Hausdorff, regular, and normal spaces. Connected, locally connected and path connected spaces. Compact, locally compact and para-compact spaces. Metric spaces, uniform continuity, and the compact-open topology for function spaces. Products of spaces and quotients of spaces. Embeddings and homeomorphisms.

Key Ideas and Theorems: Properties preserved by continuous maps, products, quotients and subspaces. Tychonoff's Theorem, Urysohn's Lemma and the Heine-Borel Theorem.

Algebraic Topology:

a. Basic Homotopy. Homotopy of maps, retracts, deforma-tion retracts, homotopy type. The fundamental group and covering spaces. Compu-tation of the fundamental group (Van Kampen's Theorem and the edge-path group).

b. Homology Theory. Definitions and basic properties of singular homology theory: the Eilenberg-Steenrod axioms. Computation of homology groups using CW-complexes. Basic cohomology theory.

c. The Algebra of Topology: Exact sequences, chain and cochain complexes, chain homotopy, introduction to categories and functors, the functors tensor, Hom, Tor and Ext. The universal coefficient theorems.

Theorems: The Mayer-Vietoris sequence, the Euler-Poincare formula, the Brouwer fixed point theorem.

Differential Topology

Definitions, basic properties, and examples of the following: Smooth manifolds and smooth maps. The tangent space and the differential of a smooth map. Embeddings, immersions diffeomorphisms. Submanifolds and product manifolds. The inverse image of a regular value. Partitions of unity. Orientability. Vector fields, integral curves and flows, the bracket operation. Definitions of Lie group and Lie algebra, examples of matrix Lie Groups. Differential Forms. Exterior differentiation and integration of differential forms. Riemannian metrics.

Theorems: (In each case the statement and simple applications but not proofs are required.) The inverse function theorem for manifolds, Sard's Theorem, Stokes' Theorem.

References

The student is not expected to read all the books on the list. The major references are indicated with an asterisk. The student should consult with a faculty member to determine which sources cover which material.

Point Set Topology

1. Munkres, *Topology: A First Course*
2. Willard, *General Topology*

Basic Homotopy

1. Massey, *Algebraic Topology: An Introduction* (Chapters 2, 4, 5)
2. Munkres, *Topology: A First Course* (Chapter 8)

Homology Theory and the Algebra of Topology

1. Massey, *Singular Homology Theory*
2. Rotman, *An Introduction to Algebraic Topology*
3. Spanier, *Algebraic Topology*

Differential Topology

1. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*
2. Bott and Tu, *Differential Forms in Algebraic Topology*
3. Guillemin and Pollak, *Differential Topology*
4. Spivak, *Calculus on Manifolds* (This should be read first.)
5. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1