Teaching Introductory Combinatorics by Guided Group Discovery

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A commonplace complaint among mathematics instructors is that, even in courses for majors, a distressingly small proportion of the class really seems to "get it." The goal of this project is to produce materials for teaching a first undergraduate course in combinatorics in which a large majority of the students learn a large majority of the material covered. The materials are based on a sequence of problems designed to lead students to understand the processes of combinatorial mathematics, abstract these processes to general principles, and apply the general principles. In addition to the intended audience of math majors, the materials developed should be ideal for the preparation of grade 8-12 mathematics teachers, because some of their content matches material taught in grades 8-12 and the instructional methods should be exemplary of effective instruction.

The PI, the evaluator, and other instructors who have used these materials agree that a large majority of the students do learn the large majority of the material. The notes are, however, a work in progress, because the evaluation demonstrates that adding appropriate summary material to the course materials will further benefit students and respond to the one almost unanimous student concern about the course, a concern that they may "see the individual trees, but not the forest."

There is an advisory board with eight members whose affiliations range from highly selective research universities to regional universities, engineering schools, selective liberal arts colleges, and regional liberal arts colleges. Members are Marc Lipman from Indiana University/Purdue University, Fort Wayne, Karen Collins from Wesleyan University, Fred McMorris from Illinois Institute of Technology, Louis Shapiro from Howard University, Victor Reiner from University of Minesota, Elizabeth McMahon from Lafayette College, Mark Miller from Marietta College, and Rosa Orellana from Dartmouth. The role of the advisory board is to ensure that the materials are usable at a wide variety of institutions. Members of the advisory board advised on the selection of topics, critiqued the problems used and the text in which the problems are embedded, and most have tested (or arranged for a test of) materials in an appropriate venue at their own institutions.

The materials developed consist of problems and text with the vast majority of the intellectual content in the problems themselves. The purpose of the text is to provide definitions, explain common themes, provide outlines to help students summarize what they have learned, and to help students understand why they are being asked to do what they are asked to do. The materials are divided into sections as a textbook is. There will likely be a separate section of hints for problems.

These hints now form the basis of an experimental online hint system. The materials will be published as a slim book by a commercial publisher. There will be an instructor's version of the book which contains complete solutions to the problems as well as protocols for using the materials. In order to answer the criticism of strong students that they could have learned more, an instructor will be able to use two protocols in the same class, one for students who want to progress quickly and one for students who know only the prerequisite material. The prerequisites are comfort with sets, functions and algebraic notation (including summation notation), some experience with reading (and perhaps doing) proofs, and, ideally, a modest exposure to mathematical induction. Two appendices are designed to met the needs of students weak in these areas. A third appendix, on exponential generating functions, allows the course to provide ample challenge to students who enter the course with a significantly stronger background. If possible, hints and perhaps solutions will be on the publisher's web page for students to download once the instructor chooses to allow them to do so.

Although the materials described could be used in an individual way with careful instructor feedback on student solutions, the intent is to develop protocols for use of student working groups. Students in a group are responsible for

- Making sure that other students in their group understand what the problems are asking for.
- Working in concert with other students in their group to develop "believable" solutions to the problems, understood by all in the group.

Most students who have taken courses from these materials have found that group activities have given them new perspectives on how to solve problems and more complete understanding than they usually feel they have in a course.

In addition to commercial distribution of the materials developed, the project sponsored a workshop designed to train approximately twenty additional faculty in the use of these materials and methods. The principal investigator will continue to report at national meetings on both the course and the innovative teaching methods. He will publish articles documenting the student experience and the evaluation findings. The workshop, the dissemination activities at national meetings and the promotion surrounding the publication of the materials should help make faculty aware of and hopefully interested in the innovative approaches to teaching being developed.

The current version of the notes is available in PDF at
www.math.dartmouth.edu/∼kpbogart.
Faculty members and publishers interested in the version with solutions or the online hints should contact Professor Bogart at
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