Teaching Introductory Combinatorics by Guided Group Discovery

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Project Summary

A commonplace complaint among mathematics instructors is that, even in courses for majors, a distressingly small proportion of the class really seems to "get it." The goal of the proposed project is to produce materials for teaching a first undergraduate course in combinatorics in which a large majority of the students learn a large majority of the material covered. The materials will be based on a sequence of problems designed to lead students to understand the processes of combinatorial mathematics, abstract these processes to general principles, and apply the general principles. The materials developed should be ideal for the preparation of 9-12 mathematics teachers, because some of their content matches material taught in 9-12 and the instructional methods should be exemplary of effective instruction.

In a prototype course at a higher level, such materials have engaged every student who has taken the course in two offerings. Further, all students in the prototype course seem to have achieved at or near their potential.

There is an advisory board with eight members whose affiliations range from highly selective research universities to regional universities, engineering schools, selective liberal arts colleges, and regional liberal arts colleges. Members are Marc Lipman from Oakland University, Karen Collins from Wesleyan University, Fred McMorris from IIT, Louis Shapiro from Howard University, Victor Reiner from University of Minesota, Elizabeth McMahon from Lafayette College, Mark Miller from Marietta College, and Rosa Orellana from Dartmouth. The role of the advisory board will be to ensure that the materials are usable at a wide variety of institutions. Members of the advisory board will advise on the selection of topics, will critique the problems used, will critique the text in which the problems are embedded, will test (or arrange for a test of) materials in an appropriate venue at their own institutions, and will help to develop and test the methods used to introduce the materials to the community at large.

The materials developed will consist of problems and text with the vast majority of the intellectual content in the problems themselves. The purpose of the text will be to provide definitions, explain common themes, provide outlines to help students summarize what they have learned, and to help students understand why they are being asked to do what they are asked to do. The materials will be divided into sections as a textbook might. There will likely be a separate section of hints for problems. The materials will be published as a slim book by a commercial publisher. There will be an instructor's version of the book which contains complete solutions to the problems as well as protocols for using the materials. Included in those protocols will be a suggested list of problems on which instructors ask

students to rewrite their solutions until they are satisfactory. If possible, hints and solutions will be on the publisher's web page for students to download once the instructor chooses to allow them to do so.

Although the materials described could be used in an individual way with careful instructor feedback on student solutions, the intent is to develop protocols for use of student working groups. Students in a group will be responsible for

1. Making sure that other students in their group understand what the problems are asking for.

2. Working in concert with other students in their group to develop "believable" solutions to the problems.

3. On problems not designated by the instructor for rewrites until satisfactory, reading the solutions that other students in their group have written and giving feedback on readability and mathematical soundness of these written solutions.

4. Working with other students in the group to ensure that their revisions to solutions are responsive to instructor feedback.

While there is a cognitive basis to all four group functions, the third and fourth function are also intended to allow courses based on these notes to be the size of ordinary college courses in combinatorics rather than being limited to the number of students to whom one instructor can provide complete supervision and mentorship.

In addition to commercial distribution of the materials developed, the project will conclude with a workshop designed to train approximately thirty additional faculty in the use of these materials and methods. The principal investigator will report at national meetings on both the course and the innovative teaching methods. The workshop, the dissemination activities at national meetings and the promotion surrounding the publication of the materials will help make faculty aware of and hopefully interested in the innovative approaches to teaching being developed.

The current version of the notes is available in PDF at

www.math.dartmouth.edu/∼kpbogart .

The materials used to teach the course in the winter term, 2002 are available at

www.math.dartmouth.edu/∼m28w02 .

Anyone who would like to do a comparative beta test, teaching combinatorics in the standard way the next time he/she teaches it and using these materials the following time is encouraged to contact k.p.bogart@dartmouth.edu.