Lecture 8 Activity: Product and Quotient Rules

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1. Compute derivatives of the following functions.
   1.1 $x^2 e^x$.
   1.2 $\frac{x-1}{x+1}$.
   1.3 $\frac{x^2+x-1}{x^3}$.
   1.4 $\frac{xe^x}{x^2+1}$.
2. What is the tangent line to $\frac{x-1}{x+1}$ at $x = 1$?
3. Suppose $f(1) = 3$, $f'(1) = -1$, $g(1) = 5$, $g'(1) = 2$, and $h'(1) = 3$. Which of the following can be determined from this information, and why?
   3.1 $(f + g)'(1)$ (the derivative of $f(x) + g(x)$ at $x = 1$).
   3.2 $(g - h)'(1)$ (the derivative of $g(x) - h(x)$ at $x = 1$).
   3.3 $(fg)'(1)$ (the derivative of $f(x)g(x)$ at $x = 1$).
   3.4 $(fh)'(1)$ (the derivative of $f(x)h(x)$ at $x = 1$).
4. Find a function $f(x)$ such that $f'(x) = xe^x$. (**Hint:** It looks like $axe^x + be^x$ for some constants a and b.)
5. **Challenge Problem:** Use the limit definition of the derivative to prove the product rule. (**Hint:** You'll start with $f(x + h)g(x + h) - f(x)g(x)$ in the numerator of the limit. Use algebra to change this to $(f(x + h) - f(x))g(x) + f(x)(g(x + h) - g(x))$.)