# Lecture 22 Activity: Newton's Method and Antiderivatives

Ben Logsdon
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`math.dartmouth.edu/~blogsdon/activity22.pdf`

1. Use Newton's Method and a calculator to estimate $\sqrt[4]{2}$ correct to 4 decimal places. (Hint: Use $f(x) = x^4 - 2$ with starting approximation 1.)
2. Find general antiderivatives of the following functions.
   2.1 $x^4$
   2.2 $3x^2 + \sin x$
   2.3 $\frac{5}{1+x^2}$
3. Find the antiderivative of $2x + 1$ that passes through the point $(1, 4)$.

# Solution 1

We'll use a starting approximation of $x_0 = 1$. We have

$$\begin{aligned}
x_1 &= 1 - \frac{(1)^4 - 2}{4(1)^3} &&= 1.25 \\
x_2 &= 1.25 - \frac{1.25^4 - 2}{4(1.25)^3} &&= 1.1935 \\
x_3 &= 1.1935 - \frac{1.1935^4 - 2}{4(1.1935)^3} &&\approx 1.1892 \\
x_4 &\approx 1.1892 - \frac{1.1892^4 - 2}{4(1.1892)^3} &&\approx 1.1892.
\end{aligned}$$

Thus, 1.1892 is an approximation of $\sqrt[4]{2}$ correct to four decimal places.

# Solution 2.1

To find the antiderivative of $x^4$, we need the power rule, which gives us $\frac{x^5}{5} + C$.

# Solution 2.2

An antiderivative of $3x^2$ is $x^3$, and an antiderivative of $\sin x$ is $-\cos x$. Therefore, an antiderivative of $3x^2 + \sin x$ is $x^3 - \cos x$. The general antiderivative is $x^3 - \cos x + C$.

# Solution 2.3

The derivative of $\arctan x$ is $\frac{1}{1+x^2}$. The derivative of $5 \arctan x$ is $\frac{5}{1+x^2}$. Therefore, the general antiderivative of $\frac{5}{1+x^2}$ is $5 \arctan x + C$.

# Solution 3

The general antiderivative of $2x + 1$ is $x^2 + x + C$. To find the specific antiderivative passing through $(1, 4)$, we plug in 1 for $x$ and set the whole thing equal to 4:

$$4 = 1^2 + 1 + C.$$

Solving this equation leads to $C = 2$. So the antiderivative passing through $(1, 4)$ is $x^2 + x + 2$.