Lecture 24 Activity: Definite Integrals

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math.dartmouth.edu/~blogsdon/activity24.pdf

1. Calculate the integral $\int_0^1 x^3 dx$ using the definition. You may use the formula $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$. (The answer is 1/4).
2. Calculate $\int_{-1}^{1} \sqrt{1-x^2}$. **Hint:** Use geometry for this one, not the definition. What shape is this function?
3. What is $\int_0^{2\pi} \sin x \, dx$? **Hint:** Draw the graph of $\sin x$ and make an educated guess.
4. Suppose $\int_0^5 f(x)dx = 3$, $\int_0^5 g(x)dx = -2$, $\int_0^3 h(x)dx = 10$.
   - 4.1 What is $\int_0^5 (f(x) - 3g(x))dx$?
   - 4.2 If $\int_0^5 (f(x) + h(x))dx = 5$, what is $\int_3^5 h(x)dx$?
5. **Challenge problem:** The formula we used to define $\int_a^b f(x)dx$ used Riemann sums with the right endpoint approximation. What would the formula look like if we used a left endpoint approximation instead? What about a midpoint approximation? Do you think the value of the definite integral changes based on which version we use?