Lecture 7 Activity: Basic Derivative Rules

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Math 3, Fall 2023

September 25, 2023

`math.dartmouth.edu/~blogsdon/activity7solns.pdf`

1. Compute derivatives of the following functions.
   1.1 $3x^{20} + 5e^x$.
   1.2 $-5x^2 - 2x + 1$.
   1.3 $\sqrt[3]{x}$. (Remember: $\sqrt[3]{x} = x^{1/3}$.)
   1.4 $1/x$. (Remember: $1/x = x^{-1}$.)
   1.5 $(5/x^2) + 6x$. (Remember: $1/x^2 = x^{-2}$.)
2. Suppose $f(x) = ax^2 + bx + c$ and that $f(0) = 5$, $f'(0) = 10$, and $f''(0) = -2$.
   2.1 What are a, b, and c? (**Hint:** First, figure out how to write f' and f'' in terms of a, b, and c.)
   2.2 What are the x and y coordinates of the highest point on this parabola? (**Hint:** The highest point is where the slope is 0. Write down the equation $f'(x) = 0$ and solve for x.)
3. How many functions have a derivative of 0? A derivative of x? A derivative of x^2?
4. **Challenge problem:** Use a graphing calculator to graph the function $f(x) = x^3 - 4x^2 + 4x - 1$. Notice that it has a local minimum and a local maximum. What are the x-coordinates of these two points?

Solutions

Solution 1.

1. $60x^{19} + 5e^x$.
2. $-10x - 2$.
3. $$\frac{d}{dx}\left(\sqrt[3]{x}\right) = \frac{d}{dx}\left(x^{1/3}\right) = \frac{1}{3} \cdot x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{\sqrt[3]{x}}.$$
4. $$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}x^{-1} = -1 \cdot x^{-2} = \frac{-1}{x^2}.$$
5. $\frac{-10}{x^3} + 6$.

Solutions

Solution 2.

1. First, let's calculate the first and second derivatives: $f'(x) = 2ax + b$ and $f''(x) = 2a$. So $-2 = f''(0) = 2a$, which means $a = -1$. Therefore, $f'(x) = -2x + b$. Since $10 = f'(0) = -2(0) + b$, this means $b = 10$. Finally, since $f(x) = -x^2 + 10x + c$, we have $5 = f(0) = c$.
2. The highest point will be where the slope is 0. Thus, we want to find the x such that $f'(x) = 0$. Above, we figured that $f'(x) = -2x + 10$, which means that we want to solve $0 = -2x + 10$. The solution to this equation is $x = 5$. To get the y-coordinate, we plug back into the original function: $f(5) = -5^2 + 10(5) + 5 = 30$.

Solutions

Solution 3.

For any constant c, $\frac{d}{dx}(c) = 0$. Similarly, $\frac{d}{dx}\left(\frac{x^2}{2} + c\right) = x$ and $\frac{d}{dx}\left(\frac{x^3}{3} + c\right) = x^2$. So in each case, the answer is "infinitely many," since there are infinitely many constants.

Solutions

Solution 4.

Similarly to in question 2.2, the local minimum and maximum are where $f'(x) = 0$. First, compute $f'(x) = 3x^2 - 8x + 4$. We can use the quadratic formula to get solutions to this, which wind up being $\frac{2}{3}$ and 2. These are the answers.