CHANCE News 13.02
Feb. 10, 2004 to March 9, 2004

Prepared by J. Laurie Snell, Bill Peterson, Jeanne Albert, Charles Grinstead, and Myles McLeod with help from Fuxing Hou and Joan Snell. We are now using a listserv to send out notices that a new Chance News has been posted on the Chance Website. You can sign on or off or change your address at this Chance listserv. This listserv is used only for this posting and not for comments on Chance News. We do appreciate comments and suggestions for new articles. Please send these to jlsnell@dartmouth.edu. Chance News is based on current news articles described briefly in Chance News Lite.

The current and previous issues of Chance News and other materials for teaching a Chance course are available from the Chance web site.

Statistician: A man who believes figures don't lie,
but admits that under analysis some of them won't
stand up either.

Evan Esar (1899 - 1995)

Contents of Chance News 13.02

Here are two Forsooth! items from the January and March, 2004 *RSS News*:

> Scottish Executive this week published its Cities Review and gave Glasgow almost half the money allocated for six cities: Edinburgh ended up with £24m over three years--a whopping two thirds less than the Weegies' £40m.
>
> *Evening News* (Edinburgh)
> 21 January 2003

(A Weegie is an inhabitant of Glasgow)

Olympic countdown-how the nine rivals rate as the countdown gets under way:

1.	Paris	chances 9/10
2.	London	chances 8/10
3.	Rio	chances 6/10
4.	Madrid	chances 6/10
5.	New York	chances 5/10
6.	Istanbul	chances 3/10
7.	Leipzig	chances 2/10
8.	Moscow	chances 3/10
9.	Havana	chances 0/10

Daily Mail
14 January 2004

Myles McLeod suggested a forsooth with a possible explanation for how it happened.

Russia boasts of new technology.
The technology will enable Russia to outwit any defence system.
Russia says it has developed a new ballistic missile technology that can beat any defence system.
***BBC News* world edition, last updated 19 Feb. 2004**

This BBC article reported the announcement of a new Russian nuclear missile system with one

questionable capability, or more likely an editorial snafu.

In 2002, the U.S. withdrew from the 1972 Anti-Ballistic Missile Treaty, the Cold War pact with Russia that limited both countries' ability to field missile nuclear missile systems.

Russian Colonel-General Yuri Baluyevsky announced this week that Russia has developed a new missile system that renders any anti-ballistic missile system defenseless. The missile can supposedly:

- Maneuver in orbit to evade anti-ballistic missile targeting efforts.
- Travel at five times the speed of light.

Ballistic missiles are called "ballistic" because they fly direct trajectories, from point A to point B at high speed. Once over a target country, these weapons deploy many smaller warheads. The multiple independent reentry vehicles (MIRVs) rely on the principle of strength in numbers and satellite targeting to ensure their survivability. The Russians have made the U.S. targeting problem much more difficult with the addition of maneuverability. Intercept vehicles must now work harder and with a lower probability of success against such missiles.

Now for the questionable claim that the missile can travel at five times the speed of light. In all likelihood, this is an editorial error, but for the sake of conversation, assume it is not. Light travels at 299,792,458 meters per second in a vacuum. That is equivalent to 186,282 miles per second. Thanks to Greek astronomer Eratosthenes (c. 276-196 B.C.), we know that the earth's circumference measures about 25000 miles. Therefore, a body moving at five times the speed of light would circumnavigate the earth over 37 times every second!

$$\text{Trips around the earth} = (5)(186{,}282 \text{ mi/sec})/(25{,}000 \text{ mi/trip}) = 37.25 \text{ trips}$$

According to some trekkie, the correct terminology for five times the speed of light is *Warp Factor Five*.

The Earth and Mars travel elliptical orbits around the sun. The distance between the earth and the sun is roughly 93,000,000 miles, or one astronomical unit (AU). Earth and Mars are separated by 0.5 to 1.5 AUs depending on their positions in their respective orbits. So, at warp factor five, travel time to Mars ranges between 49.9 and 149.8 seconds. If such short trips were a reality, current concerns about manned space flight, such as bone mass deterioration due to extended periods of weightlessness would no longer apply.

The more likely scenario, however, is that someone typed "speed of light" when they meant "speed of sound."

Editor's note: Myles is correct. In the 19 Feb. update, BBC changed "the speed of light" to "the speed of sound". This was also noted by Jeremy Keith, reported here along with a screen shot of the original statement to prove it really happened.

Mathematics at the 2004 AAAS Meeting.
American Mathematical Society
Claudia Clark, Mark Breen

This is an account of the mathematics talks at this year's annual meeting of the *American Association for the Advancement of Science*. The first item shows pictures of John Paulos receiving the 2003 AAAS Award for Public Understanding of Science and Technology. John is an old friend of Chance News readers who will be pleased that he received this prestigious award.

In John's most recent "Who's Counting" column for ABCNews.com, he explains why we can trace our maternal ancestor much further back than our paternal one as he reviews a new book *The Journey of Man*, by Spenser Wells.

Our next two articles also based on talks given at the 2004 AAAS meeting.

The not so random coin toss.
***National Public Radio*, All things considered, 24 February, 2004**
David Kestenbaum interviews Perci Diaconis

Heads or Tails?
***New Science, News Mathtrek,* Week of Feb. 28, 2004**
Ivars Peterson

The statistical behavior of coin tossing has long been a favorite topic in statistics classes. In Chance News 11.02 we gave the following summaries of historical and current experiments relating to flips, spins, and tips of a coin. For flips you toss the coin in the usual way, for spins you spin the coin on a table and for tips you stand the coins on edge on a table and hit the table to make them tip over.

Results of historical coin tossing

	Number of trials	Number of heads	Proportion of heads	Standard deviations from mean for fair coin	95% Confidence interval for proportion of heads
Buffon	4,040	2,048	.5070	.881	(.491, .522)
Pearson	24,000	12,012	.5005	.155	(.494, .507)
Kerrich	10,000	5,067	.5067	1.34	(.477, .517)

Results of Robin Lock's students

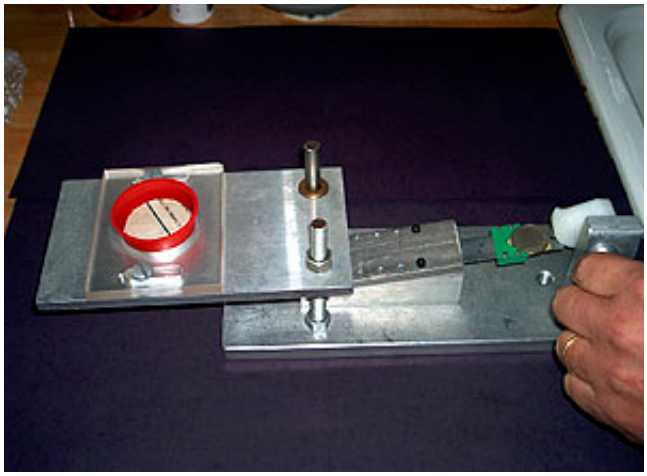
Type	Number of trials	Number of heads	Proportion of heads	Standard deviations from mean for fair coin	95% Confidence interval for proportion of heads
Flips	29,015	14,709	.507	2.37	(.501, .513)
Spins	20,422	9,197	.450	-14.19	(.443, .457)

Tips	14,611	10,087	.690	46.02	(.683, .698)

This suggests that flipping a coin is a reasonable way to make a random choice between two outcomes but spinning or tipping are not.

Of course when we spin a coin with our thumb we usually don't pay attention to which side is up before we spin it. Now Perci Diaconis, Richard Montgomery, and Susan Holmesand have studied what happens if you toss a coin in the usual way but always have heads up when you spin it.

In his NPR interview Diaconis starts by discussing his experiments with a mechanical coin flipper built for him when he was at Harvard.



Persi Diaconis' mechanical coin flipper, designed by Harvard University engineers. Diaconis says that if a coin is flipped exactly the same way, the coin lands the same way. Credit: Susan Holmes

Diaconis describes how the machine works and reports that if the machine projects a coin with heads up it will land in the cup with heads up every time. This requires that the machine is exactly as designed including a piece of tape on one side. This result led Diaconis and his colleagues to see what happens when humans toss a coin starting with heads up each time. Diaconis reports that theory and experiments obvserved with a high speed cameras, led to the conclusion that a coin tossed, starting with heads up before spinning, will be biased in favor of heads. The bias is small but he predicts that heads will come up at least 51% of the time with 10,000 such tosses of a coin.

Keller [1] and others have studied the trajectories of a coin toss as a deterministic processes subject to Newton's laws. Endre Csáki's in his review of Keller's paper for Math Reviews:

> The probability of heads is calculated by analyzing the mechanics of a tossed coin. It is assumed that a circular coin of negligible thickness performs two kinds of motions: a vertical one due to gravity and rotation about a horizontal axis. Solving certain differential equations with random initial conditions and assuming that the coin starts out with heads up, the author determines the probability of heads and shows that, under certain circumstances, this probability approaches 1/2 .

Here "under certain circumstances" means choosing the initial conditions, upward velocity and angular velocity, randomly using an appropriate distribution. Since this theoretical experiment always starts with heads up, this would seem to contradict the conjecture of Diaconis and his colleagues.

But Diaconis observes that when we toss a coin it does not simply rotate around a horizontal axis as in Keller's model. He mentions two differences. First the coin, in addition to rotating around the horizontal axis, also rotates around the vertical axis like pizza dough does when it is tossed into the air. Secondly, it does not turn over as often as you think it does. He describes a rather homey experiment in which he fastened a light ribbon, about three feet long, to a coin and tossed the coin expecting to find the ribbon rolled up as the coin spun. But he reported that in about 4 times in 100 it did not roll up at all-- the coin had not flipped at all even though it's other gyrations gave the impression of spinning-- perhaps a clue as to how, as a magician, Diaconis can control the outcome of a toss of a coin.

Diaconis remarks that he would like to check their claims by a massive experiment by statistics classes accross the country.

REFERENCES:

[1] Keller, J.B. 1986. The probability of heads. *American Mathematical Monthly* 93 (March) :191-197.

DISCUSSION QUESTIONS:

(1) What odds should you give that at least 51% heads turn up in 10,000 tosses of a coin with each flip starting with heads up, if each toss has an equal probability ending heads and tails? What odds would you give for such an experiment?

(2) Note that all the historical exeriments tossing a coin without regard to which coin was up when the coin is tossed as well as those of Roblin Lock's students resulted in more heads than tails. How might this affect the design of the experiment proposed by Diaconis.

Love lasts when the maths is right.
***ABC Science online*, 13 February, 2004**
Mark Horstman

The news reporters at the meeting of the AAAS are always looking for talks that might interest their readers. This year they concentrated on a session called "The Science of Marriage" based on work of a mathematician James Murray and psychologist John Gottman. Murray has in recent years worked in the area of mathematical models in biology. Gottman has done extensive research on family relations and directs the Gottman Institute dedicated to research and restoring relationships. Together they developed a mathematical model to predict divorce which was described in their book *The Mathematics of Marriage: Dynamic Nonlinear Models* [1].

We read in this article:

> Newly married couples, or people about to get married, came to Dr Gottman's lab and researchers videotaped them for 15 minutes while they discussed topics such as sex, in-laws, housing or money.
>
> The researchers scored their conversations between - 4 and + 4, according to an accepted psychological system. Angry or contemptuous actions (such as eye-rolling) lost points, while happiness and humor won points.
>
> The sum of the positive numbers minus the negative numbers was plotted against

time as a cumulative graph, a kind of Dow Jones index of marital conversation.

Murray told ABC Science Online that the ratio of positive to negative interactions was crucial. "If the marriage is in good shape and stable, the ratio of positive to negative interactions is five-to-one or stronger. That means, for every negative thing, there were five times as many positive ones. Less than that, the marriage is shaky. By the time they get down to one-to-one, then their marriage has real problems."

Another way to use the data is to plot the husband's scores against the wife's, to express their influence on each other.

"It's not complicated mathematics at all, actually," Murray said. "With half an hour I could teach it to 15 year olds."

Well, you might doubt that since since their book has 128 pages of discussion of calculus, nonlinear dynamics, phase space etc. before it even discusses their marriage model. However, there is a germ of truth to this claim. Tom Peterkin, in an article Algebra shows how two can live as one, posted on The Telegraph Website August 8, 2003, bravely includes the formulas used:

Marriage mathematics

Wife's equation:

$$w(t+1)=a+r1*w(t)+ihw[h(t)]$$

w= wife, **h**= husband, **t**= time
a= a constant representing the wife's state of mind when she is not with her husband. **r1*w(t)**= represents how easy it is to change her state of mind when she is in conversation with her husband. **ihw**= "influence function" – a measure of the influence that a husband's remarks have on his wife. **h(t)**= the husband's "score" during their 15-minute conversation. **w(t+1)**= how the wife has reacted to her husband's conversation – the higher the number then the greater the likelihood of divorce

Husband's equation:

$$H(t+1) =b+r2*h(t)+iwh[w(t)]$$

b= a constant representing the husband's state of mind when he is not with his wife. **r2*w(t)**= represents how easy it is for him to change his state of mind when he is in conversation with his wife. **iwh**= "influence function" – a measure of the influence that a wife's remarks have on her husband. **w(t)**= the wife's score during their 15-minute conversation. **H(t+1)**= how the husband has reacted to his wife's conversation – the higher the number then the greater the likelihood of divorce

As these equations suggest, the author's model is similar to the classical predatory-prey model but uses difference equations rather than differential equations. Since this avoids calculus, it might be true that it could be explained to a bright 15 year old.

In their book, the authors show how they estimate the parameters from data and from this the dynamic behavior of a conversation. Again from data they estimate which kinds of behavior lead to stable marriages and which lead to divorce.

The book is very well written and would be useful for showing students how dynamic systems have been successfully used in biological sciences and how they can also be used in the social sciences.

The ABC Science article explains the success of the model as follows:

> A person's ability to affect their partners' mood involves 'repair' or 'damping' functions. These represent conscious attempts to direct conversations in positive or negative directions. Murray said that the model could mimic the interaction with only three elements in the equation.
>
> "I was just astonished," he said. "When we compared the statistics, we found it predicts which couples will divorce with more than 90% accuracy."
>
> The researchers tracked more than 700 participating couples with regular questionnaires to check their marriages against the model's predictions. The unions predicted to end mostly managed between six and 14 years before divorce.
>
> The model can also suggest specific therapies for couples who seek marriage counseling.
>
> "You play games that explore different scenarios," Murray said. "You can show the husband what would happen if you ran the conversation again, without him being quite so rigid about some things."
>
> Gottman was impressed by the power of mathematical modeling."Maths revealed something that we didn't know before: that people are mismatched because of differences in how they influence each other."

REFERENCES:

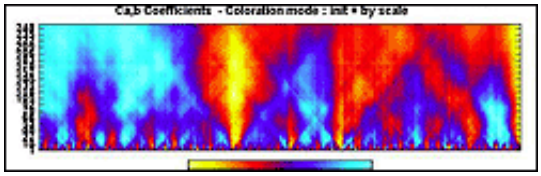
[1] Gottman, Murray, Swanson, Tyson, and Swanson, *The Mathematics of Marriage: dynamic nonlinear models*, Cambridge Mass.: MIT Press, c2002.

DISCUSSION QUESTION:

(1) In their book the authors remark that in the United States the chances of first marriages ending in divorce range between 50% and 67%. Taking this into account is the 90% accuracy still impressive?

Forecasting future wars.

The National Science Foundation's Digital Government Research Program
Karen Heyman



The Gulf War shows as a yellow plume of data in this chart from Rice University's Digital Government project.

See complete chart

Two Rice University researchers, one a computer scientist, the other an expert in international relations, are developing a computer system they hope will have the ability to predict regional conflicts.

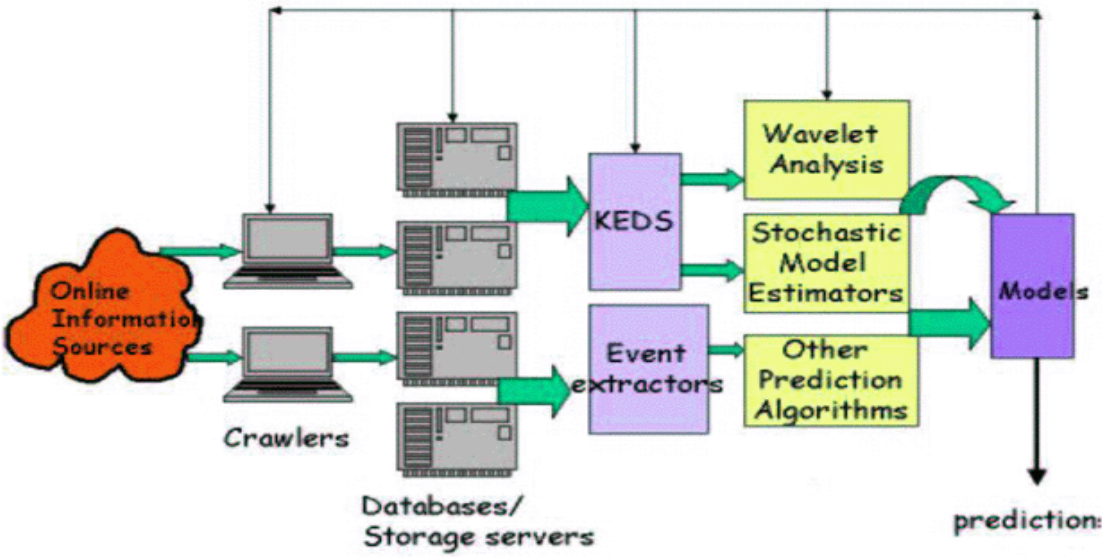
The project goal is to create a tool that supplements rather than replaces work done by traditional policy analysts.

Functionally, the tool is designed to do the following:

- Screen online news sources quickly.
- Extract "events" and "actors" from data sources.
- Make predictions of future conflicts by identifying patterns in time-series data with a technique called wavelet analysis.

See the researcher's position paper [1] and/or the Digital Government article [2] for more detail.

The graphic above is an example of the system's output. The yellow burst in the middle represents the Gulf War. The underlying data are news stories on Middle East nations. The first figure in their paper shows the steps leading to their predictions:



The project is funded to the tune of $400,000 into 2005. Interestingly, the project has the attention of the CIA, though not their active participation.

DISCUSSION QUESTION:

Assuming this system were to develop into a viable predictor of events as the designers envision, can you think of potential weaknesses such a tool might have?

REFERENCES:

[1] Subramanian, D. and Stoll, R., "Events, Patterns, and Analysis: Forecasting International Conflict in the Twenty-First Century," (version current as of 02/14/2004).

[2] Heyman, Karen, "Forecasting Future Wars, DG Research Project Scours News Coverage for Patterns of Conflict," (version current as of 02/14/2004).

Missing the moving target.
Economic Policy Institute
Jared Bernstein, Lee Price, and Isaac Shapiro

On Feb 9, 2004 the White House announced projected job gains of 2.6 million over the balance of 2004. Better still, by Tuesday the figure was revised upward to 3.8 million because of an oversight in the accounting methods used for the first projection.

This article suggests that previous experience does not lead one to great confidence about this prediction.

The Council of Economic Advisers (CEA) writes the Economic Report of the President, the administration's major annual assessment of economic trends. In each of the last three years, the report has included the administration's employment forecast for coming years

Bernstein and his colleagues show the comparison of these predictions and what actually happened by the following table:

TABLE 1
Actual and predicted year-over-year job growth

	Year-over-year job growth, prediction by CEA published early in the year	Year-over-year job growth, actual
2002	-100,000	-1,494,000
2003	+1,700,000	-406,000
2004	+2,600,000	?

DISCUSSION QUESTION:

Would you bet on it?

Editor's comment: After this was written President Bush backed off from this estimate.

Our next item was suggested by Mike Cox who remarked: they cast an interesting light (I think) on acceptable and unacceptable false positives.

The dangers of inaccurate pregnancy tests.
***The Times* (London), 18 February, 2004**
Thomas Stuttaford

Alarm over faulty pregnancy tests.
***The Times* (London), 18 February, 2004**
Oliver Wright and Laura Peek

It is reported that a batch of SAS One Step testing kits, used to determine pregnancy, may have a false negative rate as high as 30%. These kits are provided by a Texas company SA Scientific Inc. and are used in the U.K. by the National Health Service.

The faulty tests were picked up by the University Hospital of North Durham's routine checks. They found that the tests were not sensitive enough to detect early pregnancies. The Medicine and Healthcare Products Regulatory Authority (MHR) confirmed a problem with one batch of the kit and issued an alert advising that the affected batch not be used. In the first article we read:

> MHRA said it did not know how many faulty kits were in use in the NHS. Early estimates suggest there could be as many as 87,000, and 15,000-20,000 women could be affected.

A false negative result has serious medical implications for women needing an abortion. It also has medical implications for women planning to have the baby since the care of women in early pregnancy is important to the health of the baby. For example, women are typically advised to quit smoking when pregnant. And there are obvious emotional problems caused by a false positive test.

DISCUSSION QUESTIONS:

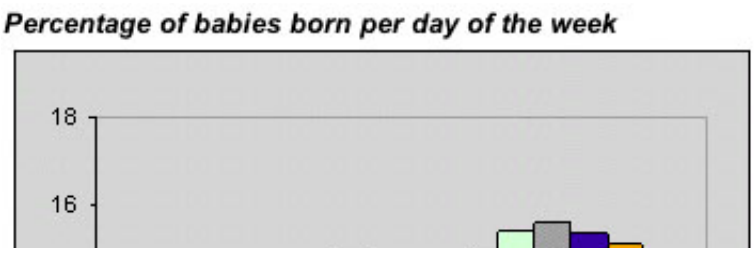
(1) The first article remarks that the false negative rate should be 1% and the second article says "A 99 per cent accuracy rate is expected." Are these saying the same thing as regards false negative tests?

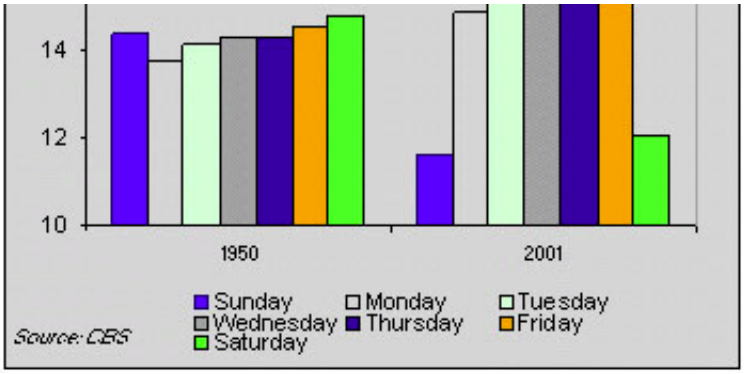
(2) As we know, the false positive and false negative rates are not sufficient to estimate the probability that a woman is pregnant given a false positive test. Do you think articles like these should mention this problem? If so how would they do it for this particular example?

In Chance News 13.01 we mentioned that the probability of being born on February 29 is sometimes given as 1/1461 and other times as 1/1506 and asked which is correct? The first probably comes from assuming that every year divisible by 4 is a leap year giving one leap year every four years and 4*365 + 1 = 1461 days in a four year period making the chance of being born on February 29 equal to 1/1461.

But that's not quite correct. According to the Gregorian calendar, which is the calendar in use today, years evenly divisible by 4 are leap years, with the exception of centurial years that are not evenly divisible by 400. Thus in a 400 year period there are only 97 leap years and so 400*365 + 97 = 146097 days making the probability of being born on February 29 is 97/146097 = 1/1506.15.

John Strohsacker who maintains a Leap Year website pointed out that this year February 29 is a Sunday and it is well known that fewer babies are born on weekends than on weekdays. He cited an article "Few Sunday's children", written by Andries de Jong for the WebMagazine of Statistics Netherlands which has the following graphics related to the births in the Netherlands in 1950 and 2001.





As this graphic shows, the probability of a leap year birthday is further complicated by the fact that there now appears to be significantly fewer birthdays when February 29 falls on Sunday. Note that this was not the case in 1950. This is often explaned by the fact that doctors like to play golf on the weekend. Note that signficantly fewer births are on Saturday also.

Jeff Dunham suggested we look at the following hockey statistics:

2003-2004 Division III Women Ratings Percentage Index

If you look here you will see that Chatham, which has an 0-14-0 record, is ranked ahead Hamline and Trinity, both of which have two wins to their credit. Why should this be? As explained on the web site

> The Ratings Percentage Index is one tool used to select teams for the national collegiate ice hockey tournament. Only results from games between two teams that each play 20 or more games against Division III opponents are used. Factors involved are 1) the team's winning percentage; 2) the average winning percentage of the team's opponents; and 3) the average winning percentage of the team's opponents' opponents. These factors are multiplied by 35%, 50%, and 15% respectively.

DISCUSSION QUESTION:

Suppose that in the first game of the season, Hamline plays Trinity and wins. How are these teams ranked at this point according to the formula (ignoring the 20 game requirement)? Do you agree with the result?

In the mathematics of matrimony, 32 + 27 = your best shot at love: If you play the field for too long, you'll end up 'on the shelf.'
***Ottowa Citizen*, 22 February 2004, A1.**
John Elliot

The article reports that an emeritus statistics professor of statistics at University College London, Dennis Lindley, has devised an formula to determine how long one should "play the field" before deciding on a spouse. As explained here:

> This is calculated by taking Y (the age at which you started searching) and adding it to one divided by E (where E is 2.718) multiplied by X (the age at which you would expect to stop looking) minus Y. Each person can choose different values for X and Y depending on when they start and when they expect to finish the hunt for a spouse.

For men he takes Y = 16 and X = 60, and finds that 32 is the optimal age to stop looking and prepare to settle down.

We recognize here a continuous adaptation a classic probability puzzle, which involves trying to find the best candidate on a list of n, given that the candidates arrive sequentially in random order and any candidate not accepted on the spot is lost forever. The solution is to reject the first (n/e) and accept the next one who beats all those seen so far. Asymptotically, this gives a (1/e) chance of ending up with the best candidate overall.

DISCUSSION QUESTION:

What assumptions are being made in the version described in the article?

Dreaming big: Why do people play the power ball?
Senior Honors Thesis, Harvard University, 14 March, 2002
Emily Oster

The question of why people play the lottery has been the subject of a great deal of research. But much of it is theoretical and, when not entirely theoretical, it is often based on a limited amount of data. In this senior thesis, Emily Oster studies three well known-theories as to why people play lotteries. She uses two substantial data sets relating to the Powerball lottery to assess which theory appears to best explain why people play the lottery.

The first data set was obtained from the Multi-State Lottery Corporation(MUSL) that oversees the Powerball lottery. It provides ticket sales by state and the size of the jackpot (advertised and actual) for each drawing from 1992 (when the Powerball was started)to the end of the year 2000. The second data set was obtained from the Connecticut Lottery office in July 2001. It provides daily Powerball sales for each retailer in the state of Connecticut from August 1999 to May 2001. The address including zip code of each retailer was also provided. In addition Oster used demographic data from the 2000 census.

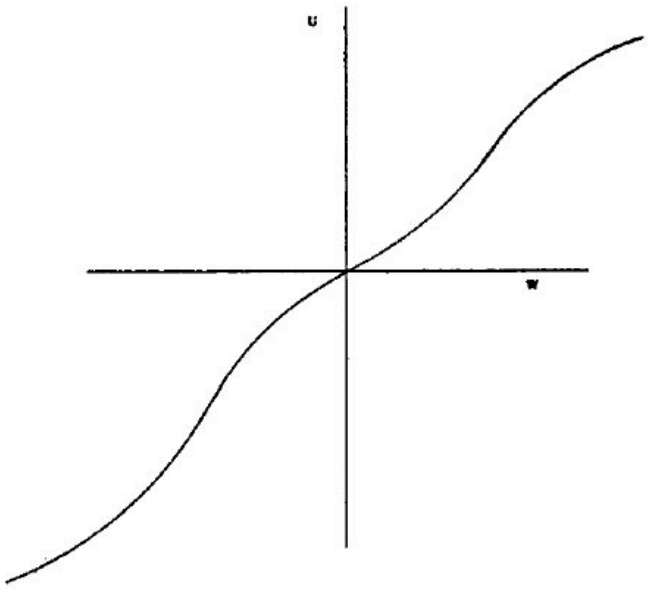
The way that the Powerball lottery is played, the prizes, and the odds are well described on the MUSL Homepage. For a more detailed discussion see Oster's thesis or our discussion of Powerball lottery.

The three theories tested by Oster are:

1. Prospect Theory: As applied here, this means that people who play the lottery estimate their chance of winning to be higher than the true probability theory.

Prospect Theory was developed by Kahneman and Tversky [1] and, as used here, allows Oster to determine the utility function for a lottery ticket in terms of the size of the jackpot, the probability of winning, a value function and a probability weighting function.

2. Convex Utility Functions: People who play the lottery have a utility function for increases or decreases in their normal wealth which is convex for large winnings and small losses illustrated by the following graph. The origin is a persons normal wealth:



This explanation for why people like lotteries was first proposed by Friedman and Savage[2] but in terms of the utility for the amount won or loss rather than the increase or decrease in wealth. Later Markowitz [3] showed that assuming the utility for the change in weath has this form led to a better description of the choices people actually make when presented with the choice of a lottery or a fixed amount of money.

Let's see how this applies to the current Powerball lottery where the probability of winning the jackpot is 1/120,526,770. Suppose that the jackpot is 50 million and we ignore the smaller prizes. Then if you buy a $1 ticket the change in your wealth is +$50,000,000 with probability 1/120,526,770 and -$1 with probability 120,526,769/120,526,770 which gives an expected change inwealth of -$.5851... . However, with the above utility function, the utility of a $50 million increase in wealth is more than $50 million and the utility of - $1 is less than -$1. So the expected utility of the change in wealth could well be positive, justifying playing the lottery.

Here is one person who clearly has such a utility function!

> Seen from a rut, the lottery is essential.
> *New York Times*, 16 July, 1996, A 16
> John P. Rach (letter to the editor)
>
> To the Editor:
> Your July 14 Week in Review article on lottery advertising repeats stereotypes about lottery players' being poor and uneducated and swept up into a gambling addiction. No doubt many are. But Gov. George E. Pataki's statement that "it has always bothered me to hold up the prospect of instant riches" could also be recast as, "I want to take away the only prospect poor people have of getting out of their rut."
>
> Before lotteries, other options existed for people to improve their lives, and the barriers were not so high. Graduate education today is expensive; many professions require training as well as licensing, and investing in stocks requires substantial equity. Most people are not so brilliant that they can start an Apple computer company in their garages. So, playing the lottery becomes a good investment when no alternative is better.
>
> Lottery players know that the odds are stacked against them, but they don't have to

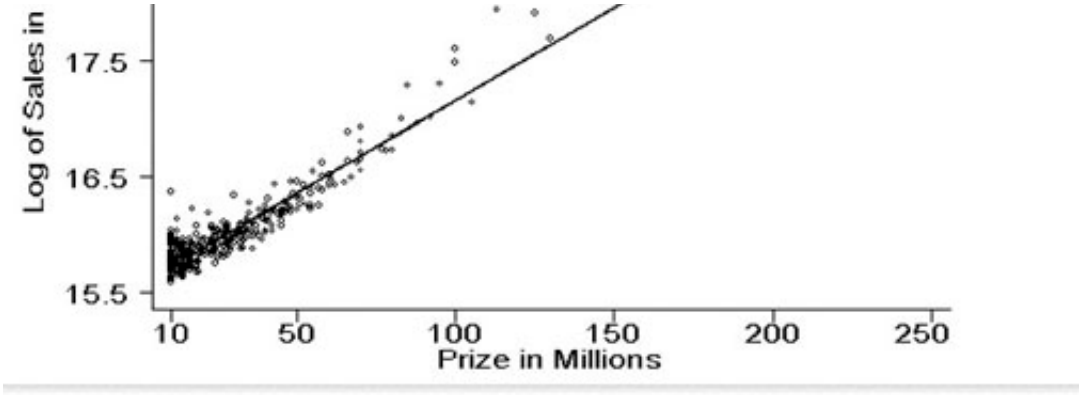
spend a lot of money. Quite a few win regularly. Many who play Lotto or Take 5 use wheeling systems, which, contrary to statements by lottery officials, do increase the chances of winning. I know; I play Lotto and I've won everything except the jackpot, and I win several times a year.
Yes, I have lost more than I've won. But in the tedious world I inhabit along with so many other New Yorkers, I've bought a fantasy. If I ever win the jackpot, I'll wave to you from Sutton Place.

3. Additive Utility of Gambling. This theory assumes that the utility function is concave but an additional amount is added to the utility for the fun of playing--choosing your favorite number, telling your friends when you win etc. Then a player's utility function for the lottery is determined in terms of the size of the jackpot, the number of tickets sold, the probability of winning, a normal concave utility function, and a free parameter capturing the person's feelings about the lottery.

After discussing these three theories in detail, Oster makes predictions as to how income, the odds of winning, and the buyers feelings would affect the sales of lottery tickets. Here are her predictions:

	Prospect Theory	Convex Utility	Additive Utility
Income	In the simplest version everyone purchases the same number of tickets but for other versions the results are ambiguous.	For the Savage version people purchase more tickets but in the Markowitz version all people purchase the same number of tickets.	Richer people start buying lottery tickets at higher jackpot levels. As a result in richer zip codes sales should change more when the jackpot increases.
Odds	A percentage decrease in odds leads to a smaller percentage decrease in sales which is independent of the size of the jackpot.	A percentage decrease in odds leads to the same percentage decrease in sales, independent of the size of the jackpot.	A decrease in odds causes everyone to change their jackpot entry. A decrease in odds should cause a decrease in sales at higher jackpots.
Feelings	No effect on ticket sales	No effect on ticket sales	May cause sales to change over time, for example at holidays.

Then Oster looks at the data to see which theory is most consistent with these predictions. She starts by looking at the effect of the size of the lottery on the ticket sale. She finds that the sales appear to increase according to a log-linear relation.

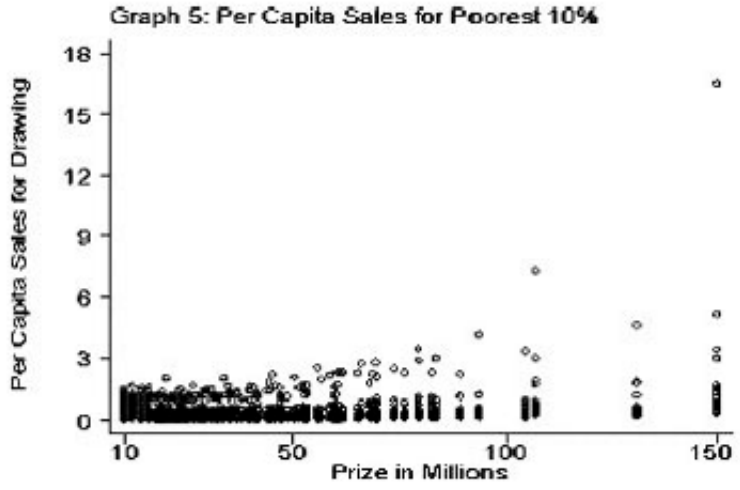


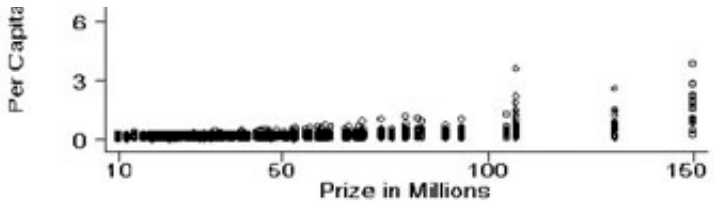
Oster writes:

> There are two possible explanations for the increase in sales when the jackpot increases. One possibility is that the pool of buyers stays the same size at all jackpots, but they increase their purchases more when the jackpot is higher. Alternatively, it may be the case that increased sales are due to new individuals entering the lottery, and more new individuals enter at higher jackpot levels. In reality it is likely that both are happening.

This uncertainty means that the mere fact that sales increase with increased jackpot is not terribly useful in testing which theory is best.

Next Oster ranks the zip codes according to increasing average income. She then compares the per-capita sales within the first 10% (the poor) with those in the top 10% (the rich) for different jackpot sizes. She obtains the following graphs:

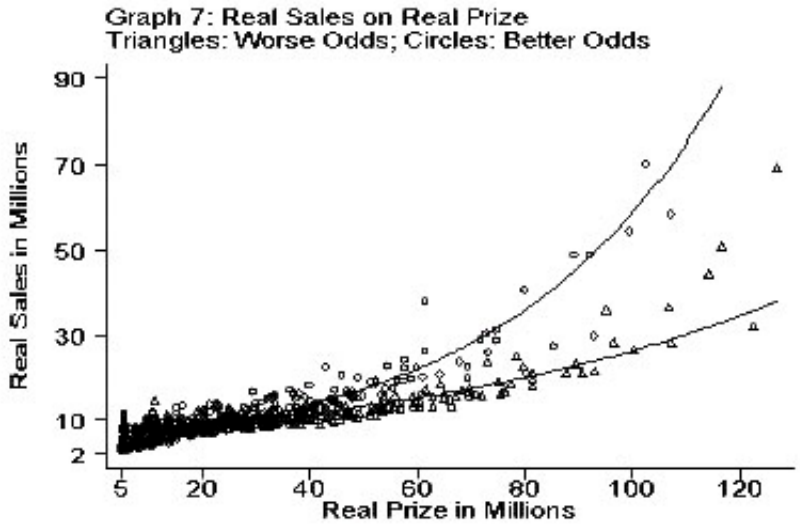




Oster comments:

> It is clear from these graphs that the poorest zip code areas purchase more tickets at the lower jackpot levels. However, at the highest jackpots the sales are about the same (slightly over $16 per capita in the poorest zip codes and around $17 per capita in the richest). This evidence is consistent with additive utility theory: the elasticity of sales with respect to prize seems to be larger in the richer zip codes. The graphs are generally not consistent with the predictions of convex utility theory.

Oster is able is able to consider the effect of changes in odds because, before 1997, a ticket required choosing 5 numbers from 45 numbers and then 1 additional number from 45 making the odds for winning a jackpot about 1 in 55 million. After 1997, to increase the size of the jackpots, a ticket required choosing 5 numbers from 49 and 1 additional number from 41 decreasing the odds for wining a jackpot to about 1 in 80 million. The effect of the change in odds is shown in the following graphic:



From this graph we see that, for a given jackpot size, the sales are better at the better odds. This also gives evidence that a decrease in odds will give a decrease in sales at a given jackpot size which is consistent only with the Additive Utility theory.

Both the prospect theory model and the convex utility model sales involve only the odds of winning and the size of the jackpot. The additive utility model has a parameter that allows one to add other things that might affect the sales. Oster gives a number such things. For example, the following table shows the effect of the day of the week on sales. Recall that the Powerball drawings are made on Wednesday and Saturday.

Sales by Days of Week

Day of Week	Mean
Sunday	756.8
Monday	1626.1
Tuesday	2651.7
Wednesday	6282.3
Thursday	1443.8
Friday	2767.5
Saturday	6267.0

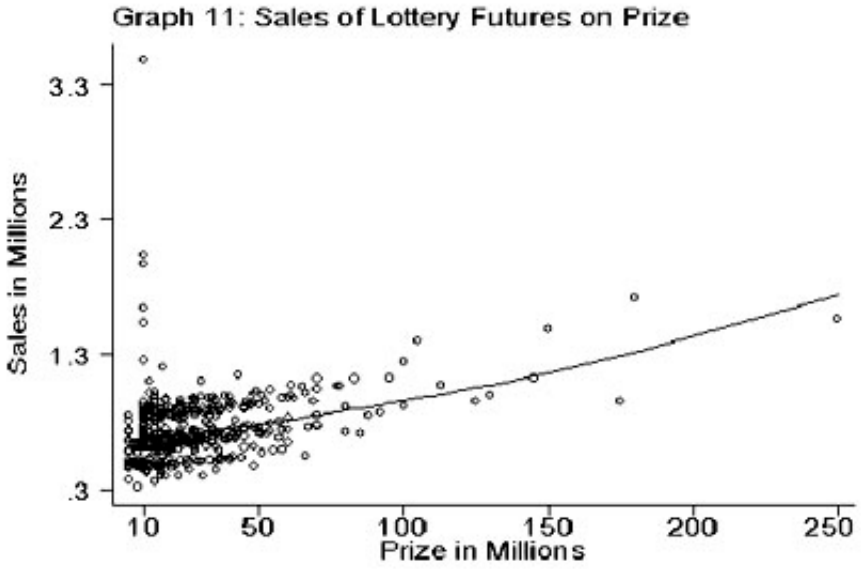
Data used: Overall Powerball; Daily November 2, 1997 through December 30, 2000

[a] All sales figures reported in thousands

Oster calls this the "deadline" effect. To check that it is not simply the normal pattern for gambling she sees if the same effect occurs for the instant tickets and it does not.

Oster also shows that sales for a given jackpot size are significantly higher than the average sales for the end of the month, before Christmas, and between Christmas and New Years.

Finally, Oster argues that the fact that people take advantage of lottery futures (the ability to buy tickets for future drawings) supports the additive utility theory. She regards this as evidence that people get more enjoyment out of being involved in several drawings than in just one while, at least when the lottery is high, it would make much more sense to buy 10 tickets for the present drawing. Her last graphic shows that the size of the jackpot has little effect on the sales of futures.



Well, that is the end of our story but you can read the full story in Emily Oster's thesis or an abbreviated version in her article "Are All Lotteries Regressive? Evidence from the Powerball" to appear in the National Tax Journal, June 2004. You can also obtain her data here and check your own theories.

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[1]Kahneman, D., Tversky, A., 1992. Advances in Prospect Theory: Cumulative Representation of Uncertainty. Journal of Risk and Uncertainty 5:297-323.

[2]Friedman, M., Savage, L.J., 1948. The Utility Analysis of Choices Involving Risk. Journal of Political Economy 56:279-304.

[3]Markowitz, H., 1952. The Utility of Wealth. Journal of Political Economy 60:151-158.

CHANCE News 13.02
Feb. 10, 2004 to March 9, 2004
