

Math 100: Topics in Probability

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The class meets Mondays and Wednesdays at 4:00 in 201 Kemeny Hall. The subject for the quarter is ‘A Brownian approach to spectral geometry’. We will use the theory of Brownian motion (a polite term for ‘drunkard’s walk’) as a source of tools and insight into problems in spectral geometry. A list of some possible topics is given below.

The geometry of diffusion

- Drunkard’s walk
- Brownian motion on the line
- Brownian motion in higher dimensions
- Transience and recurrence
- Laplace’s equation
- The heat equation
- Brownian motion on manifolds
- Conformal mapping
- Brownian motion on Riemann surfaces
- The type problem
- Shorting and cutting

- Ergodicity of geodesic flow
- Eigenvalues
- The wave equation
- The free Gaussian field

Spectral geometry

- Spectral invariants
- Isospectrality
- Unions of flat tori
- Drums
- Drumsticks
- Dirichlet versus Neumann
- Homogeneous geometries
- The method of images
- The Selberg trace formula
- The counting trace
- Accidental isospectrality
- Scenarios
- Tetra and Didi, the cosmic spectral twins
- Hyperbolic 2-manifolds
- Hearing cone-points
- Disconnected flat 2-orbifolds
- The Prime Geodesic Theorem
- Hyperbolic 2-orbifolds
- Lens manifolds and lens orbifolds

Diffraction

- The wave equation
- Reflection and diffraction
- Diffraction by an infinite half-plane
- Harmonic sources
- Repeated diffraction
- Waves in a polyhedron