# ... for $L_2(11)$.

This page contains all webs with an 11-face.
It is closed under "decimation".

| | | $\tau^2/\delta$ |
|---|---|---|
| $\gamma_0$ | $\infty 0 \cdot 1X \cdot 25 \cdot 37 \cdot 68 \cdot 69$ | 0 |
| $\gamma_1$ | $\infty 10 \cdot 26X \cdot 385 \cdot 497$ | 1 |
| $\gamma_2$ | $\infty 2754X39860 \cdot 1$ | 4 |
| $\gamma_3$ | $\infty 3X40 \cdot 12876 \cdot 59$ | 9 |
| $\gamma_4$ | $\infty 4130 \cdot 29X56 \cdot 78$ | 5 |
| $\gamma_5$ | $\infty 57890 \cdot 142X63$ | 3 |
| $\gamma_6$ | $\infty 64320 \cdot 158X79$ | 3 |
| $\gamma_7$ | $\infty 7X80 \cdot 16592 \cdot 3 \cdot 4$ | 5 |
| $\gamma_8$ | $\infty 8170 \cdot 2 \cdot 345X9 \cdot 6$ | 9 |
| $\gamma_9$ | $\infty 9467182350 \cdot X$ | 4 |
| $\gamma_X$ | $\infty X0 \cdot 195 \cdot 247 \cdot 368$ | 1 |

So $\tau^2/\delta$ = 0 1 4 9 5 3

$\Rightarrow$ orders = 2 3 11 5A 5B 6

So we distinguish between 5s by:-

5A: $\delta=1$, $\tau = \pm 3$ $\quad \tau^2/\delta = 9$

5B: $\delta=1$, $\tau = \pm 4$ $\quad \tau^2/\delta = 5$

A $\quad 2^1 3^2 5^1 \bar{5}^2 6\ 11$

D $\quad 2^1 3^2 5^2 \bar{5}^1 6\ 11$ ] 5-way?

B $\quad (2)^1 2^1\ 3^2 5^1 \bar{5}^1 6^1\ 11$

E $\quad (2)^1 2^1\ 2^2 5^1 \bar{5}^1 6^1\ 11$ ] 5-way?

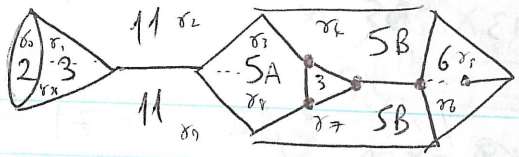
C $\quad 2^3 3^3 5^3 \bar{5}^3 6^3\ 11^3$ $\supset$ 5-way?

(40)$_{10}$

(36)$_{12}$

# ODD CASES

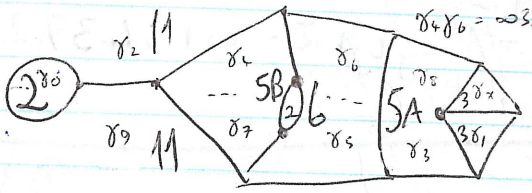
$\gamma_3\gamma_4 = \infty 04 \cdots$ $\quad \theta = \infty 0X5369721 \cdot 4 \cdot 8$




A (40)$_{10}$

$\theta = \infty 05463 1X7982$

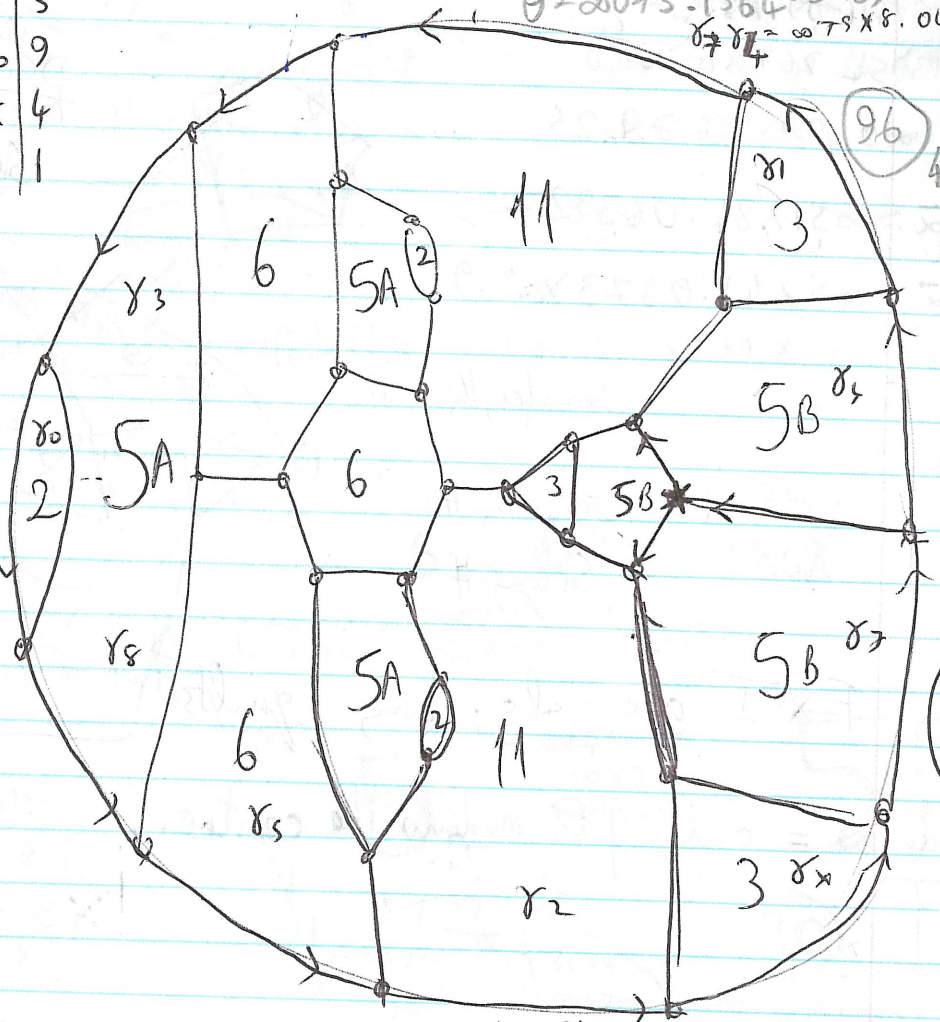
$\gamma_4\gamma_6 = \infty 3 \cdot 06 \cdots$




B (36)$_{12}$

$\theta = \infty 073 \cdot 1564 \cdot 2X89$

$\gamma_7\gamma_4 = \infty 79X8 \cdot 04123$

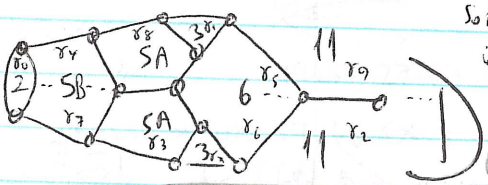



(96)$_4$

C

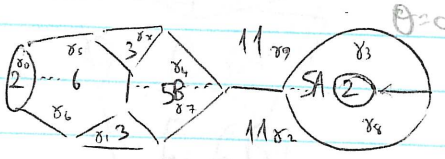
$\infty 3681X497025$, i.e. $t \to \frac{3t+1}{t-5}$, takes $\gamma_7 \to \gamma_7\gamma_4 \to \gamma_4$

So the 3-fold sym about * is achieved by a conjugacy in $PGL_2(11)$




$\theta = \infty 083X2574 \cdot 6 \cdot 9$

$\theta = \infty 02761 8 4X935$




E

(36)$_{12}$

$\gamma_8\gamma_3 = \infty 7 \cdot 03 \cdots$

# 11-3 friday

$\infty 587X.0134\overline{2.6.9}\ \beta$

$\infty 49.087.13X.\overline{285}\ \alpha$

$L = \infty 2.45.\overline{69.0X.38.17}$ $\quad L\beta\bar{\alpha} = \infty 7138.09264.5.X$

$\infty 75X8.0417\overline{3.6.9} = \beta'$ $\quad L\beta'\bar{\alpha} = \infty 16439285X.0$

$a^3 = (ac)^3 = (\bar{a}^1 c)^5 = c^5 = 1$

$\bar{a}b = c$

base

$\alpha = \infty 27\overline{415.09X83}$

$\beta = 1395L.267X8.\infty.0$

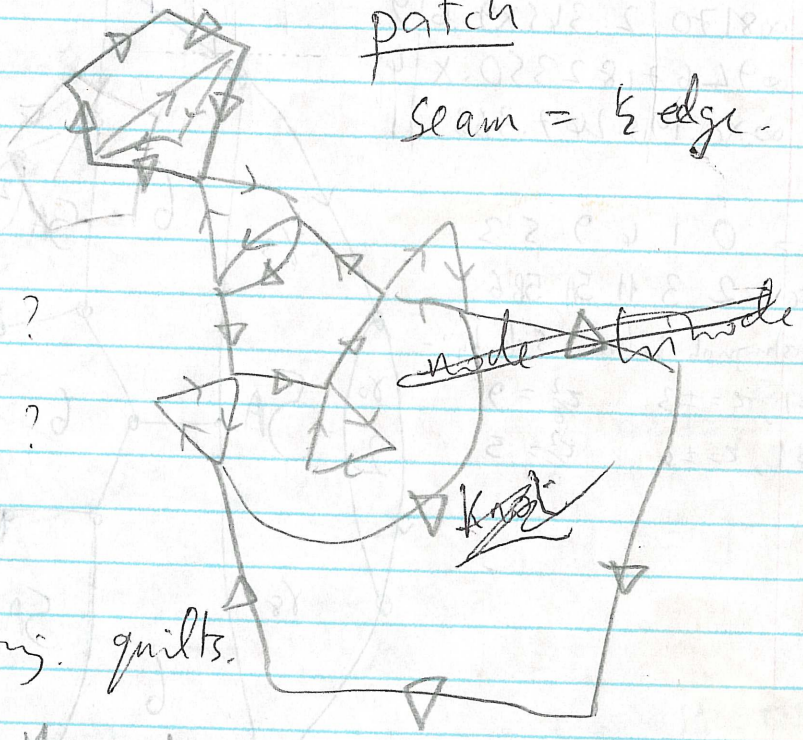
$\bar{c} = \infty 0.69.1X.68.73.25$

$L\beta\bar{\alpha} = \infty 51X84.063927$

$L\beta^2\bar{\alpha} = \infty 5241.0673X.8.9$

patch

seam = $\frac{1}{2}$ edge.



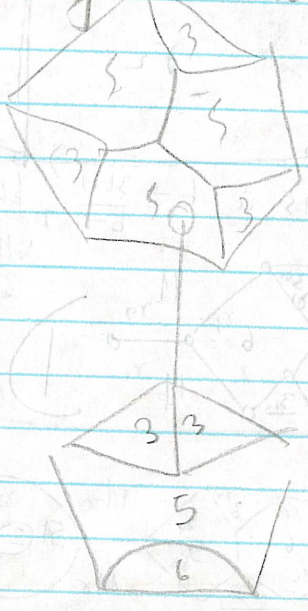

seam length ?

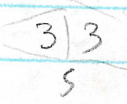
sewing # ?

seam #

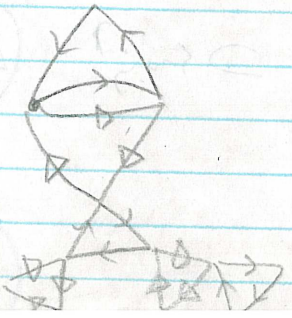
F & I are alg. conj. quilts.

modulus = order of $\theta$ modulo the centre.



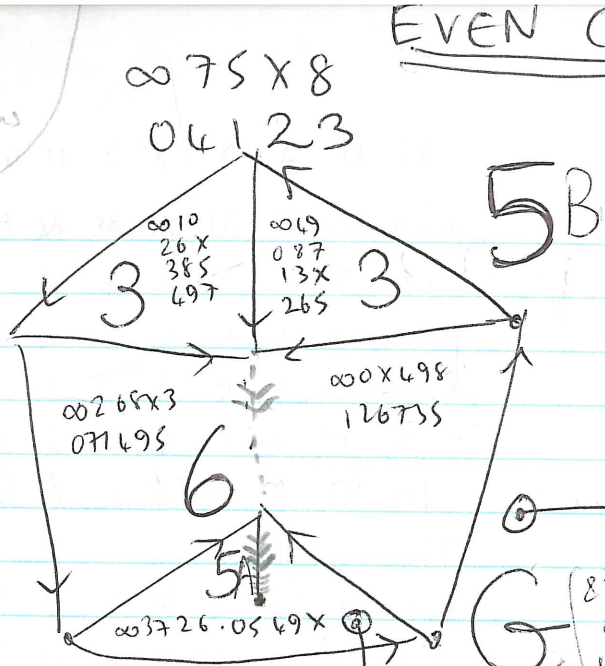



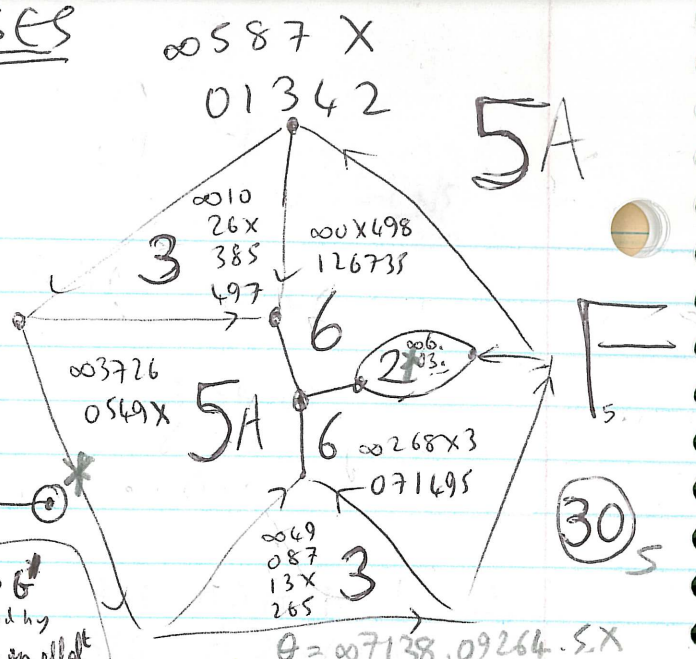

$\langle a, b \mid a^3 = b^3 = (ab)^5 = (a^{-1}b)^5 = 1 \rangle$



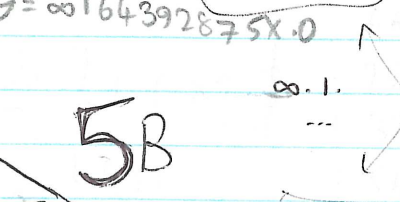
seam no. / modulus

# EVEN CASES



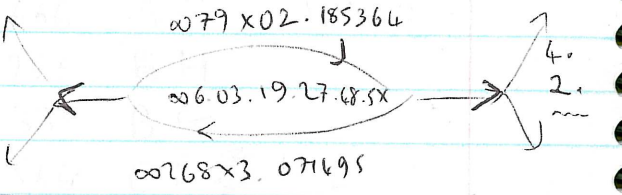

$\theta = \infty 7138.09264.5X$




$\theta = \infty 1643928 75X.0$




$\theta = \infty 51X86.063927$




$2\left(\frac{t-3}{t-2}\right)$ N.B. $\infty 2.03.14.5.67.89.X$ rotates this.
So 2-fold sym. is a conj" in $PGL_2(11)$

~~$\infty 6.05.17.28.34.9$~~

~~$\infty 6.05.17.2X.34.89$~~

The sym here is a conj" in $PGL_2(7)$:—

$\alpha = \infty 75X8.04123.69$

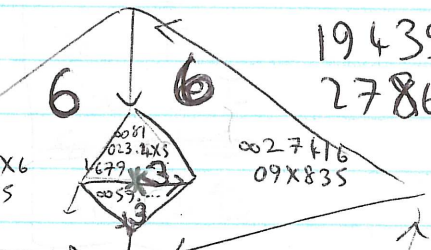
$\beta = \infty 7832.04X59.1.8$

$\alpha^\beta = \infty 6958.4X1\infty 2$

$\kappa = \alpha^{\beta\bar{\alpha}} = \infty 18\%.032.45X.769$

~~$\beta_1 = \kappa\bar{\alpha} = \infty 81.039.$~~

$\beta_1 = \kappa\bar{\rho} = \infty 8126.03975.4.X$




$\theta = \infty 5241.0673X.8.9$

$\infty X796.01875.2.3$

$\infty 6...$

$19435.2786X.\infty.0$

we want $\beta \to \alpha \to \beta_1$ as a conj" by $\lambda$

so $\lambda: \frac{1}{8} \to \frac{8}{9} \to \frac{4}{X}$

$\lambda = X389k2\infty 75016$ works

$\left(\frac{7t-2}{t-2}\right)_4$

$3\left(\frac{t-2}{t-3}\right)$ $\infty 3.02.17.46.5X.8.9$ rotates this.
So 2-fold sym is conj" in $PGL_2(11)$

memo

$\tau^2/f = 9$ for 5A

$5$ for 5B

660 12 6 5 5 6 11 11

| | 1A | 2A | 3A | 5A | 5B | 6A | 11A | 11B | hs |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | : |
| 5 | 1 | −1 | 0 | 0 | 1 | b11 | ∞ | | |
| 5 | 1 | −1 | 0 | 0 | 1 | ∞ | b11 | | |
| 10 | 2 | 1 | 0 | 0 | −1 | −1 | −1 | | : |
| 10 | −2 | 1 | 0 | 0 | 1 | −1 | −1 | | : |
| 11 | −1 | −1 | 1 | 1 | −1 | 0 | 0 | | : |
| 12 | 0 | 0 | b5 | ∞ | 0 | 1 | 1 | | : |
| 12 | 0 | 0 | ∞ | b5 | 0 | 1 | 1 | | : |

$b_5 = \frac{-1+\sqrt{5}}{2}$

(3, 5A, 6)

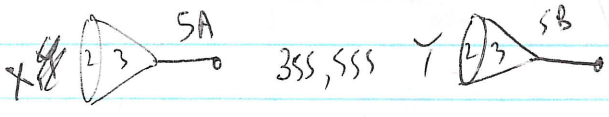
(3, 5B, 6)

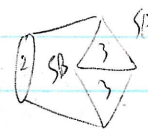
255 AAAF
556 ADGGGHHI
666 AC
256 BH
335 BG
566 CFHI
555 CDDDGHHH
356 DEFG
255 EI
266 F
336 GI
~~566 HI~~
366 I

255 1 $-1/11 = \frac{10}{11}\cdot\frac{660}{5\cdot 6\cdot 11} = 2$ } EI x2
* 255' → 2 } x1
256 $1 + 1/11 = \frac{12}{11}\cdot\frac{660}{5\cdot 6\cdot 11} = 2$ } BH x1
266 $1 + 1/5 + 1/5 + 2/10 - 2/10 - 1/11 \to 2$ } F x2
* 335 $1 + 1/11 = \frac{12}{11}\cdot\frac{660}{6\cdot 6\cdot 11} = 4$ } BG x2
336 $1 + \frac{1}{5} + \frac{1}{5} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11} = \frac{72}{55}\cdot\frac{660}{666} = 4$ } GI x2
* 355 $1 - 1/11 = \frac{10}{11}\cdot\frac{660}{6\cdot 5\cdot 11} \Rightarrow 4 = 4$ } AAAF x2
* 355' → 4 } x1
356 $1 + \frac{1}{11} = \frac{12}{11}\cdot\frac{660}{6\cdot 5\cdot 6} \to 4$ } DEFG x1
§ 366 $1 + \frac{1}{5} + \frac{1}{10} + \frac{1}{10} - \frac{1}{11} = \frac{39}{55}\cdot\frac{660}{666} \to 2\frac{1}{6}$ } I x2
* 555 $1 + \frac{1}{11} + \frac{b5^3 2_L}{12} = \frac{25}{55}\cdot\frac{660}{555} = 4$ } CDDDGHHHM x2 ?
† 555' $1 + \frac{1}{11} \pm \frac{b5^3 b5^* 2_L}{12} = \frac{155}{155}\cdot\frac{660}{555} = 6\frac{1}{5}$ }
556 $1 - \frac{1}{11} = \frac{10}{11}\cdot\frac{660}{5\cdot 5\cdot 6} = 4$ } ADGGGHHI x2
55'6 → 4 } x1
566 $1 + \frac{1}{11} = \frac{12}{11}\cdot\frac{660}{5\cdot 6\cdot 6} \Rightarrow 4$ } CFHI X2
666 $1 + \frac{1}{5} + \frac{1}{5} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11} = \frac{72}{55}\cdot\frac{660}{666} \to 4$ } AC x2

$b5^3 = \frac{-1+3\sqrt{5}-3\cdot 5+5\sqrt{5}}{8} = -2+\sqrt{5}$

$\therefore \frac{b5^3 2_L}{12} = \frac{-4}{12} = -\frac{1}{3}$




$b5\,b5^* = \frac{1-5}{4} = -1$

$\therefore b5^2\, b5^*\, 2_L = -b5\, 2_L = 1$

*: these happen in A5
†: in 11:5 & C5
§: in C66

a c b^c

a b^c

# P Buser

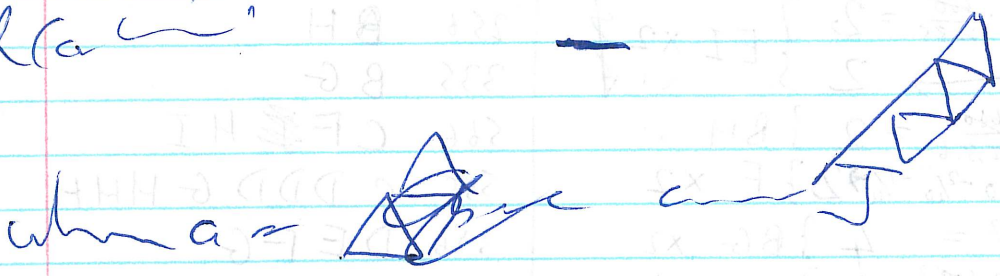
212 988 9500 <9.00pm

Wife arrives Feb 6 ish (thru holidays)

(a, b, c)

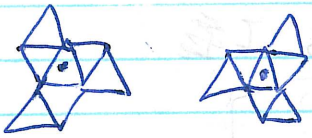
L(a, b, c)

R(a ~~~ '



L(a, b, c) = (b^a, a, c)

R(a, b, c) ———



a, b, c = ———

a = ( , ) ... b = c =

L(a, b, c)

R( ———

LR( ———)

A

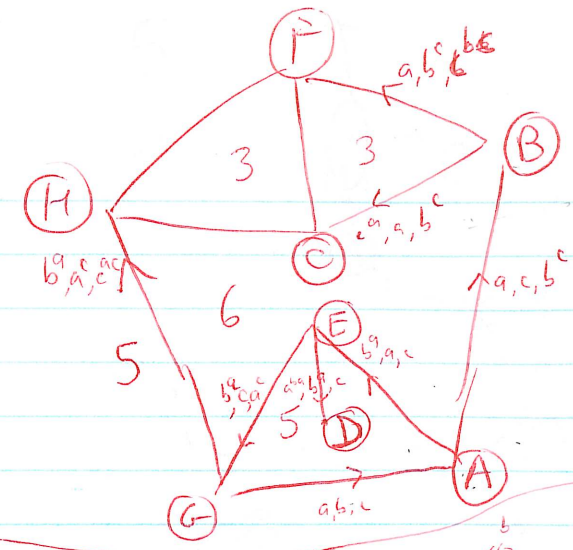
B

C

———

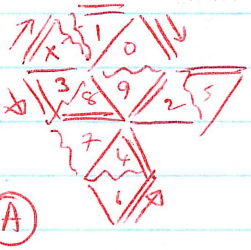
**Symmetry Note:** - the mirror image of G is G or G'. Which?
comparing (A) & (G), we see it is G'.

| | |
|---|---|
| a | 01.3X.47.89 / 07.12.36.9X |
| b | 0X.29.36.78 / 01.25.6X.89 |
| c | 09.25.38.46 / 08.24.5X.79 |
| $b^c$ | 9X.05.68.37 / 18.4X.65.07 |
| $c^a$ | 18.25.9X.67 / 78.14.59.0X |
| $b^a$ | 13.28.X6.49 / 27.15.39.8X |
| $a^{ba}$ | 03.16.97.24 / 02.57.96.38 |
| $c^{bc}$ | 5X.20.74.86 / 17.2X.46.09 |
| $a^c$ | 91.8X.67.30 / 89.14.36.57 |
| $c^{ac}$ | 31.25.0X.47 / 09.21.7X.58 |

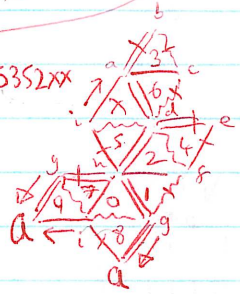



(A)

a,b,c *5532xx



*5352xx

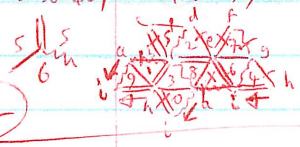


abcdefghi
*153152111 xx

(H)

abcdefghi
*115315211 xx *5352xx

91.8X.67.30 / 89.14.3657
13.28.X6.49 / 27.15.39.8X
09.25.38.46 / 08.24.5X.79



(G)

3*53 each time but cones flip

a 01.3X.47.89 / 07.12.369X
$b^c$ 9X.05.68.37 / 18.4X.65.07
$c^{bc}$ 5X.20.74.86 / 17.2X.46.09

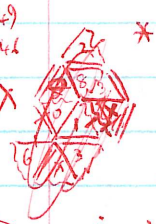
(F)

AB abcdefgh
31*11151311

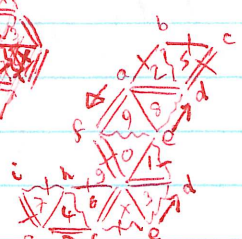
b,a,c

01.3X.47.89
13.28.X6.49
09.25.38.46

*5352xx

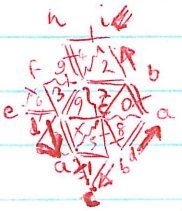


abcdefghi
*135111125 xx



(E)

07.12.36.9X
27.15.39.8X
08.24.5X.79



abcdefghi
*111153152 xx

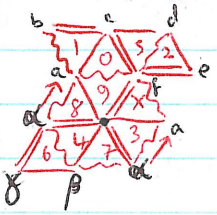
*5352xx each time

(D)

A abcdef αβγ
3*1153111*211

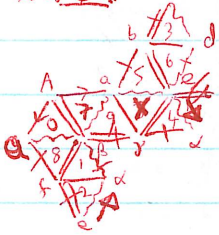
(B)

a,c,$b^c$ *533*32



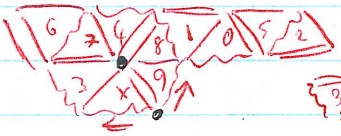
3*53*2

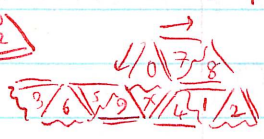
07.12.36.9X
18.4X.65.07
08.24.5X.79



3*3332 but cones flip!

a,$b^c$,c

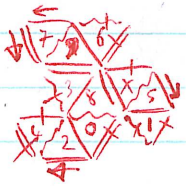




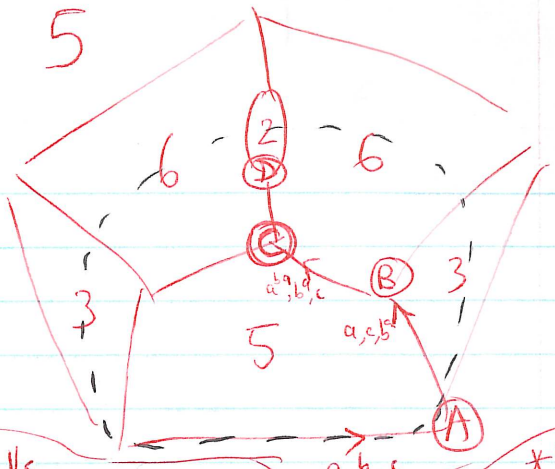
(C)

a,$b^a$ b,c *5*5*5 each time

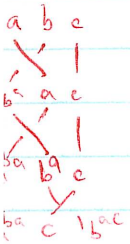
02.57.96.38
27.15.39.8X
08.24.5X.79



# F




a 01.3X.47.89/07.12.36.9X
b 0X.29.36.78/01.25.6X.89
c 05.37.48.9X/07.18.4X.56
$b^a$ 13.28.6X.49/27.15.39.8X
$a^{ba}$ 03.16.97.42/02.57.69.38
$b^{ac}$ 17.24.69.8X/20.68.39.14




## On the symmetry of this example

If $\alpha, \beta$ is on a quilt $Q$
then $\bar{\beta}, \bar{\alpha}$ is on a quilt $\overline{Q^*}$ called the reflection of $Q$

If a,b,c are words with $ab=\alpha$, $bc=\beta$
Then c,b,a have $cb=\bar{\beta}$, $ba=\bar{\alpha}$

So as far as we are concerned,
$\overline{Q^*}$ & $Q$ give the same things.

In this case, $\overline{Q^*}=Q$, so mirror image vertices give the same examples as ~~to~~ rotates [same the rotation is achieved by duality]

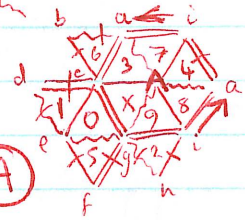
So (A), (B), (C), (D)
give all cases.

Which is the mirror?

Swapping 1,4 fixes (A).
Swapping 11,1 doesn't fix (C). (look at b ©
in fact it takes © to ©'.

So (A) is on mirror, (C) not.

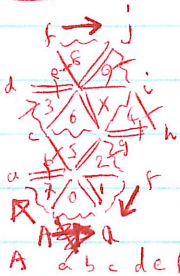
5 ‖ 5 / 3



(A)

A abcdefghi
1*131515131X

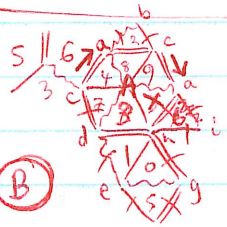
*5533X
3*55X



A abcdefghi
3*111511151X

01.3X.47.89/07.12.36.9X
13.28.6X.49/27.15.39.8X
05.37.48.9X/07.18.4X.56

5 ‖ 6 / 3



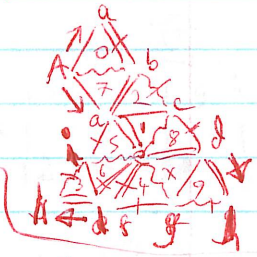
(B)

*53332X
3*532X

A abcdefghi
1*131215313X
B*121115131X

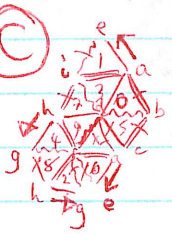
03.16.97.42/02.57.69.38
13.28.6X.49/27.15.39.8X
05.37.48.9X/07.18.4X.56

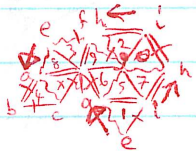
5 ‖ 6 / 6



abcd efghi
*1351*31212X
*1531*23121X

(C)





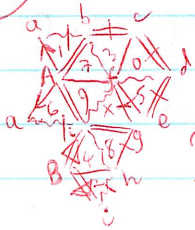
*53*322X

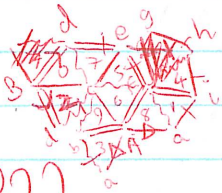
03.16.97.42/02.57.69.38
17.24.69.8X/20.68.39.14
05.37.48.9X/07.18.4X.51

2 ‖ 6 / 6

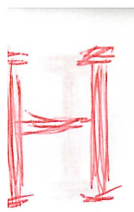
AB abcdefghi
12*32132112 1
12*23121123 1

(D)





2*32322



a 2X.34.59.67 / 19.26.45.78

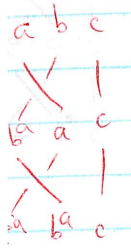
b 0X.29.36.78 / 01.25.6X.89

c 03.16.24.79 / 02.57.38.69

$b^a$ 02.X5.47.68 / 09.64.2X.71

$a^{ba}$ 05.37.X9.48 / 07.4X.65.18

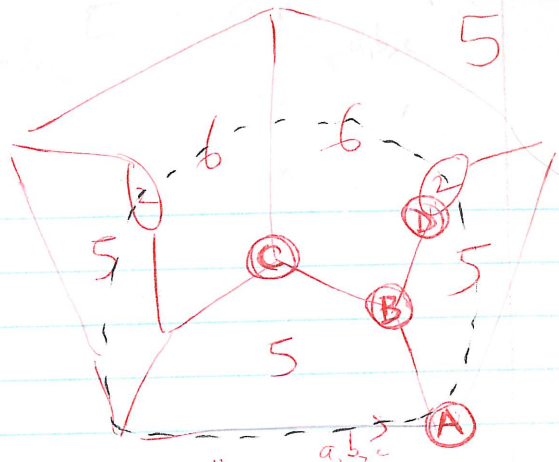
$a^c$ 4X.02.57.19 / 16.09.47.35

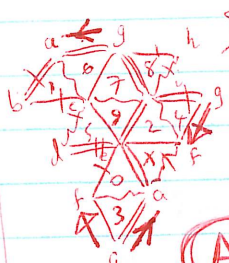



## On Symmetry

(A) is not dual-symmetric ∴ must be on the mirror. So (C) must be dual-symmetric – this checks

Aha! we could have deduced this from the existence of the 2-sided faces!



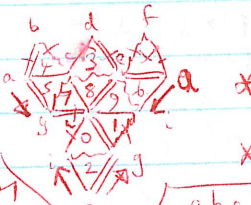



a b c d e f g h i

*1515111*151 X

*1515115*111 X

(A)



*555*X

*55*5X

2X.34.67.59 / 19.26.45.78

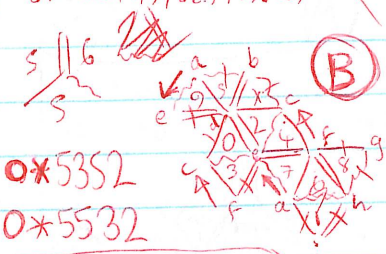
02.X5.47.68 / 09.64.2X.71

03.16.24.79 / 02.57.38.69

a b c d e f g h i

0*13 1111525

0*15121135 1



(B)

0*5352

0*5532

05.37.X9.48 / 07.4X.65.18

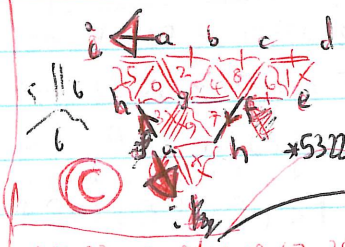
02.X5.47.68 / 09.64.2X.71

03.16.24.79 / 02.57.38.69

a b c d e f g h i

*1225311*13X

*1225131*31X



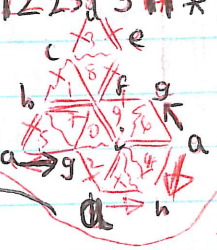
*5322*3X

(C)

4X.02.57.19 / 16.09.47.35

02.X5.47.68 / 09.64.2X.71

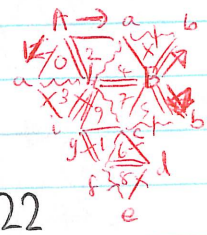
03.16.24.79 / 02.57.38.69



A B a b c d e f g h i

21*13 1152112

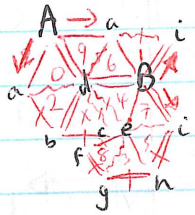
21*1151123 12



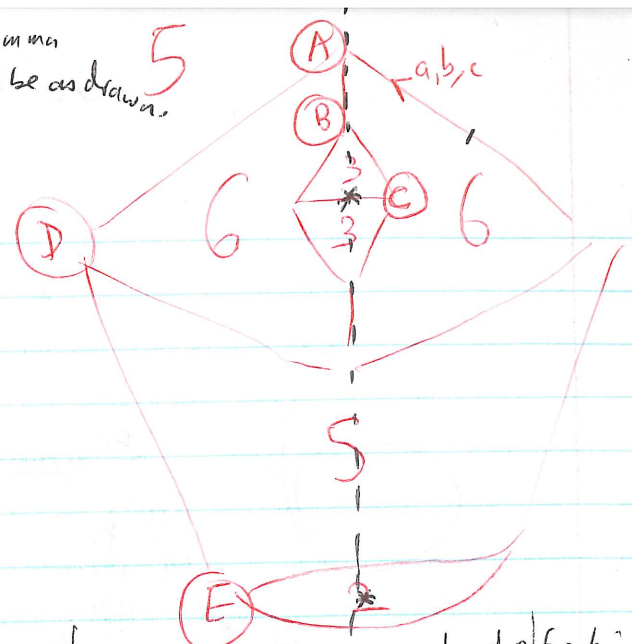
(D)

2*5322

2*5232



Symmetry note: the 2-sided face lemma shows the mirror to be as drawn.




a = 07.12.38.6X / 04.3X.27.89

b = 2X.34.67.59 / 19.26.45.78

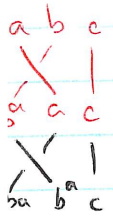
c = 19.68.7X.45 / 14.2X.35.67

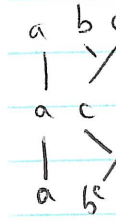
$b^a$ = 16.84.X0.59 / 18.76.05.29

$a^{ba}$ = X7.62.34.10 / 54.3X.96.12

$b^c$ = 27.35.8X.41 / 49.X7.13.68

$c^{bc}$ = 49.6X.28.13 / 39.27.15.8X



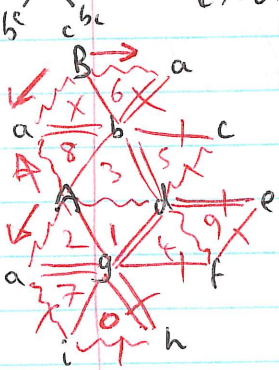



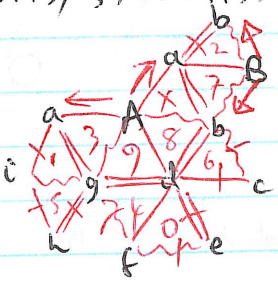

07.12.38.6X / 04.3X.27.89

27.35.8X.41 / 49.X7.13.68

69.6X.28.13 / 39.27.15.8X

5 || 5
2





2*552

E

A B a b c d e f g h i

1 2*1 1 2 1 5 1 1 5 1

1 2*1 1 2 1 5 1 1 5 1

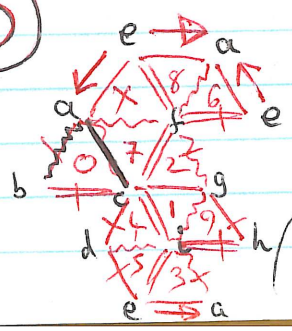
07.12.38.6X / 04.3X.27.89

27.35.8X.41 / 49.X7.13.68

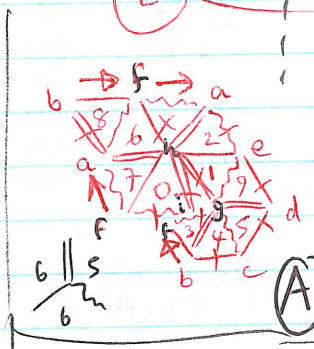
19.68.7X.45 / 14.2X.35.67

5 || 5
6

D



*5532XX




a b c d e f g h i

*12332*1115X

*11252*3131X

*3322*5X

*522*33X

07.12.38.6X / 04.3X.27.89

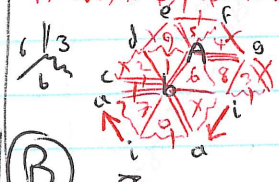
16.84.X0.59 / 18.67.05.29

19.68.7X.45 / 14.2X.35.67

A a b c d e f g h i

1*2 1 3 2 3 3 2 3 1 X

3*1 1 2 1 3 1 3 1 2 X




*3323232X

3*3322X

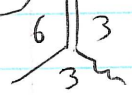
7X.62.34.10 / 54.3X.96.12

16.84.X0.59 / 18.76.05.29

19.68.7X.45 / 14.2X.35.67

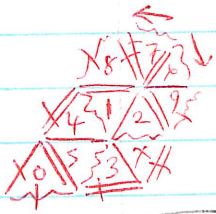
A B a b c d e f g h i

1 3*1 1 3 1 2 3 1 3 1




3*3332

C



a b c d e f g h i

*1513112 51 XX

*1151311 25 XX

TOWEX

RNHFGI

MAIL