Exploring the Padovan Sequence

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INTRODUCTION

Fibonnaci sequence: F(n)=F(n-1)+F(n-2)

1 1 2 3 5 8 13 21…

Limiting ratio between consecutive #s= 1.61803398875… (ϕ- the golden ratio)

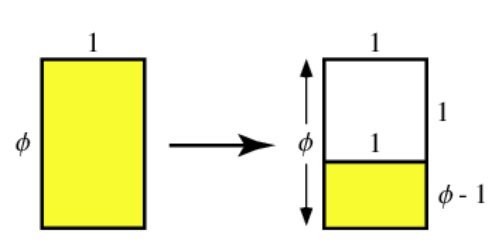
Padovan sequence: P(n)=P(n-2)+P(n-3)

1 1 1 2 2 3 4 5 7 9….

Limiting ratio between consecutive #s= 1.32471795724… (ϱ-the plastic number)

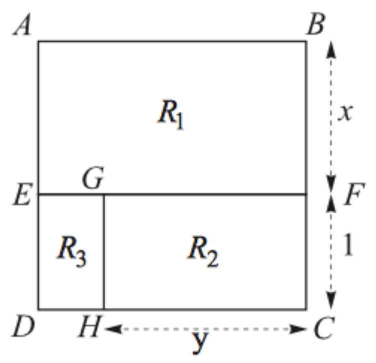
Solutions to equations of the form x^n-x-1=0:
n=2,3 are the only equations of the form that have real solutions:

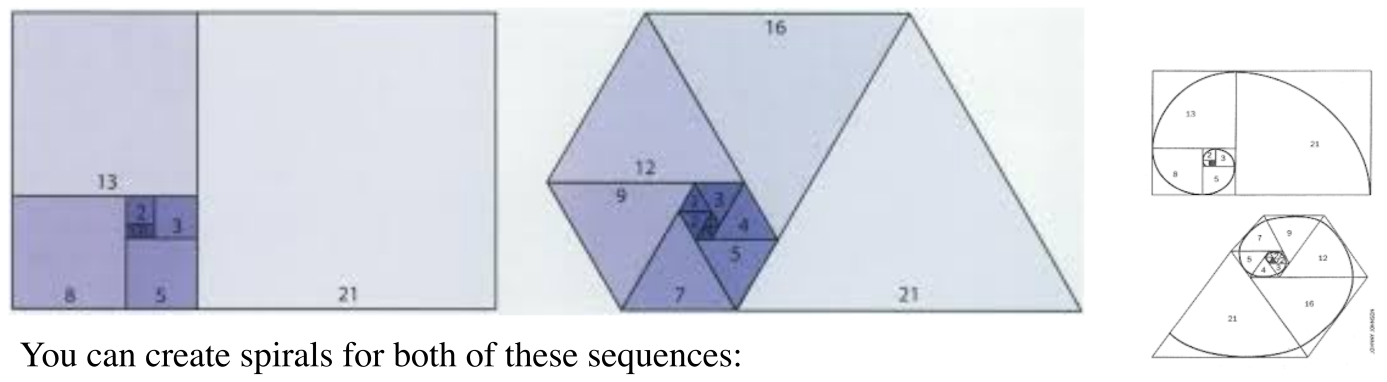
- ϕ is the only real solution to x^2-x-1=0
- ϱ is the only real solution to x^3-x-1=0



If a rectangle has the same ratio as the leftover rectangle after a square is removed, the ratio of their sides is ϕ

If a square is split into 3 rectangles of different sizes that have the same ratio of sides, that ratio is ϱ





You can create spirals for both of these sequences:
A Fibonacci spiral of Squares and a Padovan Spiral of Equilateral Triangles

BUILDING A PADOVAN CUBOID SPIRAL

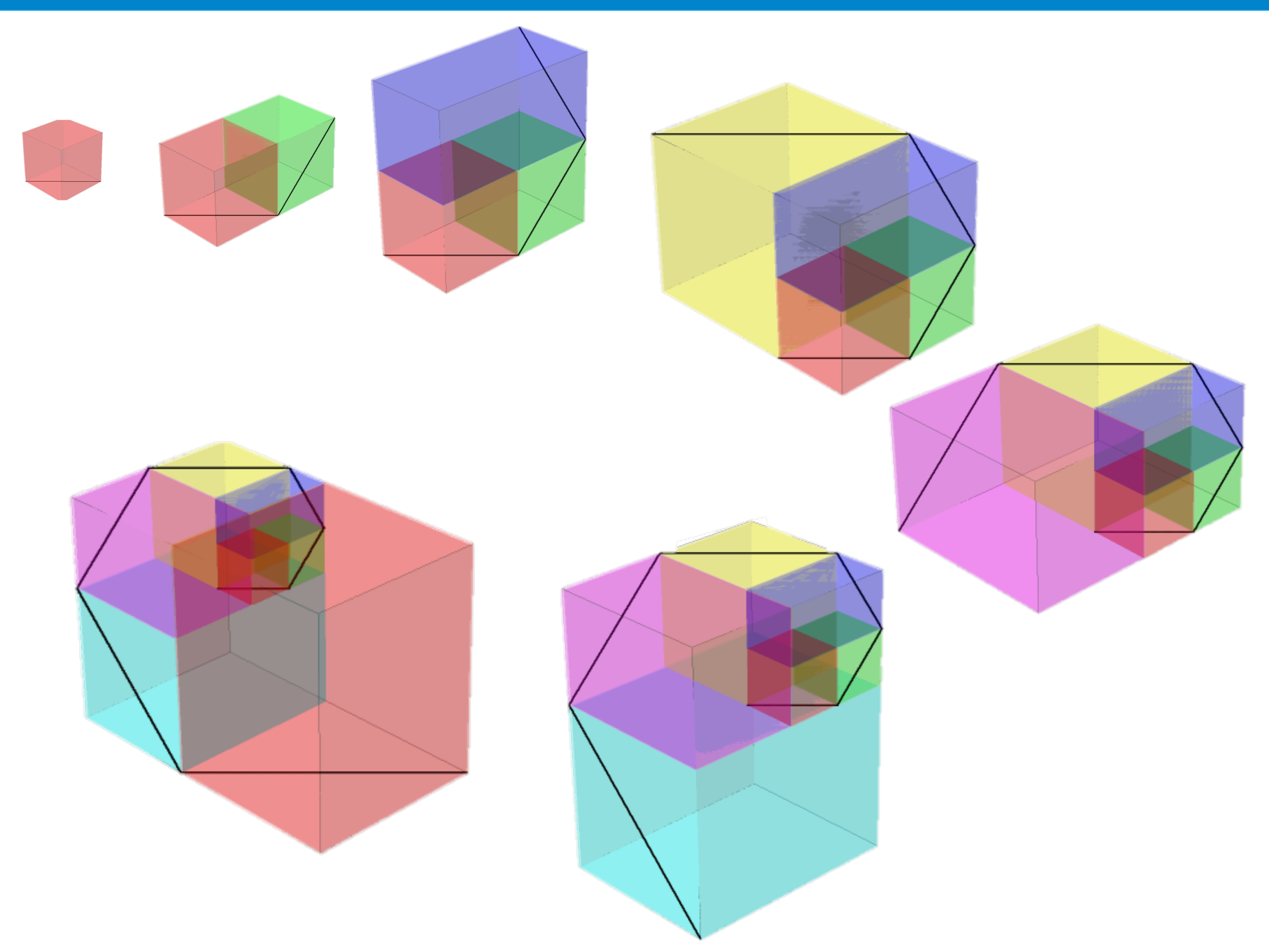
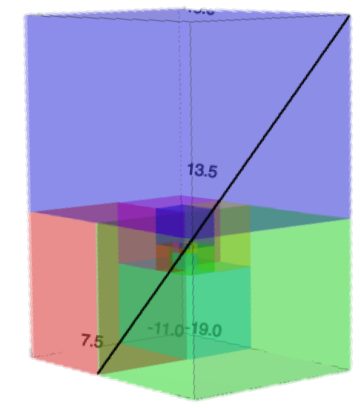
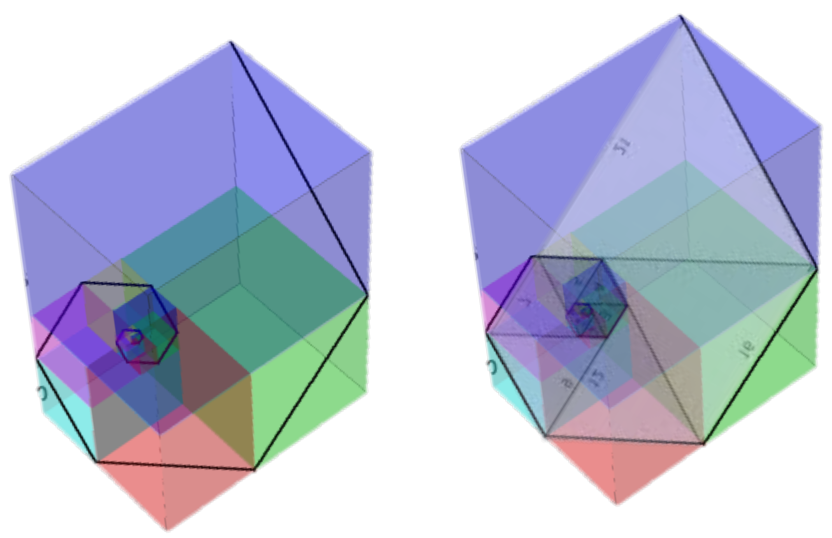


DIAGRAM OR EXAMPLE OF STIMULI

Why does the Spiral lie in a plane?

Equation of the spiral plane:
x – y + z = 0
Each diagonal is such that either:
x=y, z=0 OR x=0,y=z OR x=-z, y=0





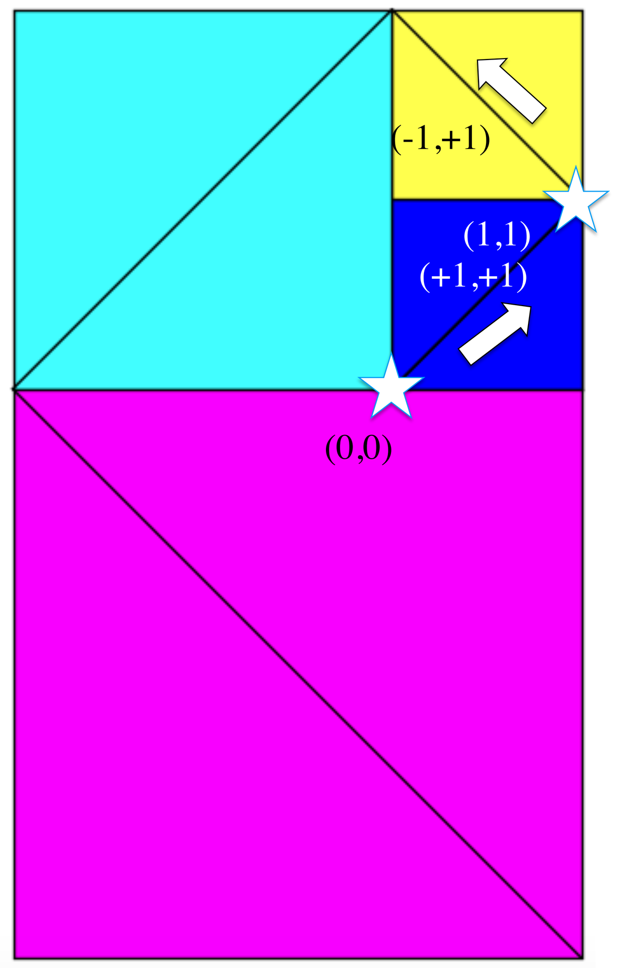
Notice that the Padovan Triangular 2D spiral is the same as the Cuboid Spiral!

HOW TO PROGRAM?

Starting Simple with the Fibonnaci Spiral:

Keeping track of ***starting corner*** (star) and ***direction*** of diagonal movement (arrow)

- The magnitude of the direction is the Fibbonaci number
- The signs of the direction are determined by a modulus 4 pattern
- The starting corner is the sum of the last starting corner and direction



For the Cuboid Spiral:

Needed to figure out patterns for how to determine the next ***starting point***, ***direction of motion***, and ***dimension of the diagonal*** from that information from previous squares

- The pattern repeats based on modulus 6

For example:

```
if i%6==0:
    x=L[last][0]+L[last][3]
    y=L[last][1]
    z=L[last][2]+L[last][5]
    a=L[last][4]
    b=L[last][4]
    c=-L[last][5]-L[last-1][5]
    s=(1,0,0)
    dim="z"
```

For this method I stored previous values of ***starting point***, ***direction of motion***, and ***dimension of the diagonal*** in lists so there was no need for a Padovan function because you are storing the values instead of having to recalculate them recursively each time. This is good because it keeps the program from going too slow.

PATTERNS OF COORDINATES

The Padovan Sequence:
1•1•1•2•2•3•4•5•7•9•12•16•21•28•37•49•65•86•114•151•200•265•351•465

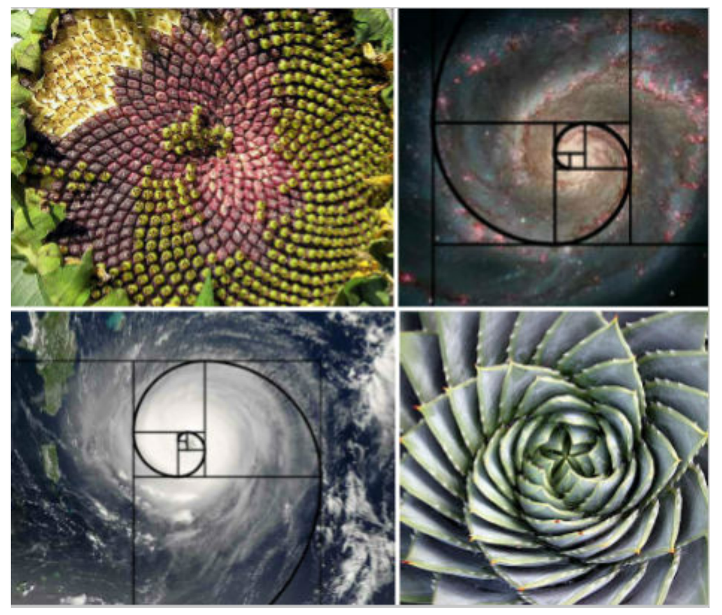
This table shows how the bottom –back-left corner of each cuboid follows a pattern related the Padovan sequence. The other corners follow similar patterns.

	0 (mod 6)	***1 (mod 6)***	***2 (mod 6)***	***3 (mod 6)***	***4 (mod 6)***	***5 (mod 6)***
1st covering	(0,0,0)	(0,1,0)	(0,0,1)	(-2,0,0)	(-2,-2,0)	(-2,-2,-3)
2nd covering	(1,-2,-3)	(-2,2,-3)	(-2,-2,2)	(-11,-2,-3)	(-11,-14,-3)	(-11,-14,-19)
3rd covering	(5,-14,-19)	(-11,7,-19)	(-11,-14,9)	(-60,-14,-19)	(-60,-79,-19)	(-60,-79,-105)
4th covering	(26,-79,-105)	(-60,35,-105)	(-60,-79,46)	(-325,-79,-105)	(-325,-430,-105)	(-325,-430,-570)

X differences	**1,4,21**	**-2,-9,-49**	**-2,-9,-49**	**-9,-49,-265**	**-9,-49,-265**	**-9.-49,-265**
	0 (mod 6)	3 (mod 6)	3 (mod 6)	3 (mod 6) *	3 (mod 6) *	3 (mod 6) *
Y differences	**-2,-12,-65**	**1,5,28**	**-2,-12,-65**	**-2,-12,-65**	**-12,-65,-351**	**-12,-65,-351**
	4 (mod 6)	**1 (mod 6)**	4 (mod 6)	4 (mod 6)	4 (mod 6) *	4 (mod 6) *
Z differences	**-3,-16, -86**	**-3,-16,-86**	**1,7,37**	**-3,-16,-86**	**-3,-16,-86**	**16,-86, -465**
	5 (mod 6)	5 (mod 6)	**2 (mod 6)**	5 (mod 6)	5 (mod 6)	5 (mod 6) *

FUTURE DIRECTIONS…

The Fibonacci spiral and golden ratio are given a lot of hype for appearing in plant growth, sea shells, human faces, and even in pictures of the galaxy. Does the Padovan Spiral or the plastic number occur in nature like the Fibonacci Spiral and golden ratio do? Could it be that some of the things we associate with the Fibonanci spiral are better fit with the Padovan Spiral?



What other geometric patterns related to the Padovan spiral can we find? What can we say about the irregular pentagon that forms with each equilateral triangle added to make the Padovan spiral? How can we precisely draw in the curve for this spiral? Can it be done with parts of circles, like that of the Fibonacci Spiral.

ACKNOWLEDGEMENTS

Thank you to Professor Doyle for helping me with this project.
I also looked at a few websites:
http://www.heldermann-verlag.de/jgg/jgg13/j13h2spin.pdf
http://www.ingelec.uns.edu.ar/pds2803/Materiales/Articulos/SA1996-06.pdf
http://mathworld.wolfram.com/PadovanSequence.html