

DE PROPAGATIONE PVLSVVM
PER MEDIUM ELASTICVM

AVCTORE
L. EVLERO.

§. 1.

Medium elasticum in statu aequilibrii versari nequit, Tab. III. nisi omnes eius particulae aequalibus viribus elasticis in se mutuo agant. Quod si autem una particula adepta fuerit maiorem elasticitatem, quam reliquae, tum ab statu aequilibrii sublatum haec sese expandendo, ac reliquas magis comprimendo tamdiu agitatitur, donec perfectum aequilibrium inter omnes vires fuerit restitutum. Particularum enim elasticarum eiusmodi est indoles, ut quo magis expanduntur, eo minorem obtineant vim elasticam, contra vero, quo magis comprimuntur et in minus volumen rediguntur, earum vis elastica augeatur. Quanquam autem hoc incrementum ac decrementum elasticitatis pro ratione aucti et minuti voluminis diuersissimas proportiones sequi potest, tamen si mutatio voluminis fuerit quam minima, augmentum vel decrementum vis elasticae his ipsis mutationibus proportionale comprehenditur.

§. 2. Difficillima autem maximeque ardua videtur quaestio, qua commotio singularum particularum medii elasticci, cum aequilibrium semel fuerit sublatum, quaeritur, simul autem resolutio huius quaestionis in physica maximi est momenti, cum formatio et propagatio soni

in huiusmodi commotione particularum aeris consistat. Neque etiam amplius dubitare licet, quin ipsum lumen, radiorumque lucidorum propagatio a sublato aequilibrio inter particulas aetheris proficiscatur. Quando enim aeris quaepiam portio in maius minusve spatum compellitur, ob minutam vel auctam eius elasticitatem status aequilibrii cum vicinis aeris particulis turbatur, hincque in ipsis agitatio oritur, quae sese continuo ad particulas ulteriores extendit, donec ubique tranquillitas fuerit restituta. Hinc igitur sonus, et si in aethere similis agitatio eueniat, inde lumen originem suam trahit.

Fig. 1.

§. 3. Cum itaque haec quaestio sit maximi momenti, operam dabo, ut ad eam resoluendam ex primis principiis mechanicae viam sternam. Quo igitur a casu simplicissimo ordinar, primum unicam considerabo partículam A, quae quidem in se spectata nullius mutationis sit capax, sed quae filis elasticis inertiae expertibus AP et AQ intra parietes firmos P et Q detineatur. Sint autem haec fila seu elastræ AP et AQ ita comparata, ut quo fiant breviora eo maiori vi elastica polleant; dum autem elongantur, eorum elasticitas diminuantur. His positis manifestum est corpus A fore in aequilibrio, si utriusque elastri AP et AQ eadem fuerit vis: quod euenire ponamus; si utriusque elastri longitudine AP et AQ fuerit aequalis. Sit tamen $AP = AQ = \alpha$; et utriusque vis elastica $= g$; quoniam corpusculum A utrinque aequaliter urgetur, si semel quieuerit, perpetuo quiescere perseuerabit.

Fig. 2.

§. 4. Concipiamus nunc hoc corpusculum A ex situ aequilibrii semel fuisse dimotum, ita ut alterum elastrum

strum longius alterum vero breuius fit factum. Cum igitur hoc modo ex altera parte vis elastica sit minuta, ex altera vero aucta, necesse est ut corpusculum A motum conceperit, quem hic determinabo, in hypothesi quod elongatio et contractio amborum elastrorum sit minima, ita ut augmentum vel decrementum vis elasticae ipsi contractioni seu elongationi proportionale censeri possit. Elapsò ergo tempore t peruenient corpus A, cuius massa littera A exprimatur, in situm quem figura refert. Ponatur longitudo elasti AP = $a+x$; erit ob x prae a valde paruum, eius vis elastica = $\frac{ag}{a+x} = g(1 - \frac{x}{a})$: alterius elasti AQ longitudo consequenter erit = $a-x$, eiusque vis elastica = $\frac{ag}{a-x} = g(1 + \frac{x}{a})$: vnde corpus A secundum directionem AP urgetur vi = $\frac{gx}{a}$.

§. 5. Ponamus tempusculo dt corpus progredi per elementum spatii = dx , erit eius celeritas = $\frac{dx}{dt}$. Tempus autem t ita exprimatur, vt haec fractio $\frac{dx}{dt^2}$ exhibeat altitudinem celeritati, quam corpus in A habet debitam. Sumto ergo elemento dt constante, erit vis solicitans = $\frac{2Addx}{dt^2}$, cui aequalis poni debet vis qua corpus actu urgetur $\frac{2gx}{a}$ quae cum motui renitur, habebimus hanc aequationem:

$$\frac{2Addx}{dt^2} - \frac{2gx}{a} \text{ seu } Aaddx + gxdt^2 = 0.$$

Multiplicetur haec aequatio per dx , et integretur, erit $Aadx^2 + gxxdt^2 = gbbdt^2$; vnde fit $dt = \frac{dx\sqrt{Aa}}{\sqrt{g(bb-xx)}}$; et $t = \frac{\sqrt{Aa}}{\sqrt{g}} \text{ A sin. } \frac{x}{b} - C$ hincque $x = b \text{ sin. } (t+C)\sqrt{\frac{g}{Aa}}$.

§. 6. Vocetur breuitatis gratia $\sqrt{\frac{r}{n\omega}} = n$, et mutatis constantibus valor ipsius x ita exprimetur:

$$x = b \sin. nt + c \cos. nt$$

quae constantes ex primo aequilibrii turbati statu definiri debent. Posito scilicet $t=0$, habebitur $x=c$; Deinde cum corporis celeritas sit $= \frac{dx}{dt} = nb \cos. nt - nc \sin. nt$, initio, quo $t=0$, eius celeritas erat $=nb$. Quod si ergo corpus A ipso initio quiescens ponatur, atque intervallum AP tum fuerit $=a+\omega$: fiet $b=0$, et $c=\omega$; unde quouis tempore t elapsō erit situs corporis A

$$PA = a + x = a + \omega \cos. nt$$

$$\text{eiisque celeritas } = -n\omega \sin. nt$$

wbi signum $-$ indicat eius motum versus parietem P fore directum.

§. 7. Corpus ergo A celeritatem habebit maximam, si angulus nt fiat rectus, quo casu fit $PA=a$ ita ut perpetuo in ipso situ aequilibrii celerrime moueatur. Tum vero cum angulus nt ad duos rectos exsurgit, celeritas iterum fit $=0$, et intervallum $PA=a-\omega$, quod in altera elongatione maxima a puncto medio euenit. Vnde patet corpus alternis motibus circa punctum medium instar penduli motum iri; huncque motum perpetuo esse duraturum, nisi quatenus a resistentia diminuatur. Pendulum igitur simplex assignari poterit, cuius motus oscillatorius conueniat cum isto corporis A motu reciproco; reperietur autem longitudo huius penduli simplicis isochroni $= \frac{4\pi}{\omega} = \frac{1}{2\pi n}$. Quod si ergo fiat $nt=180^\circ$, vt sit $t = \frac{180}{\pi}$; tum tempus t aequabitur temporis unius oscillationis

lationis penduli, cuius longitudo $= \frac{1}{2}m$. Hinc generaliter, si angulus 180° exprimatur per π , reperiaturque tempus $t = \pi m$, tum hoc tempus cognoscetur in mensura consueta, quoniam aequabitur durationi unius oscillationis penduli cuius longitudo est $= \frac{1}{2}m$: quae mensura in sequentibus adhiberi poterit.

§. 8. Cuius hoc primo eoque facilissimo expedito Fig. 3. contemplemur duo corpuscula A et B, quae cum inter se tum inter parietes immobiles P et Q elastris PA, AB, BQ defineantur. Sint corpora ambo inter se aequalia, et in aequilibrio constituta, quando tria interualla AP, AB, BQ fuerint aequalia. Ponatur hoc casu uniuscuiusque elastri longitudo $= a$ et vis elastica $= g$: itemque vtriusque corporis massa $= A$. Quodsi iam corpus A ex statu aequilibrii deturbatur, dum proprius vel ad P vel ad B impellitur, corpus quoque B mox ad motum concitatatur, hocque vicissim in A aget; unde motus in utroque orientur, qui a casu praecedente maxime discrepabit, neque amplius motui oscillatorio similis erit, atque ob hoc ipsum multo difficilius definietur. Ad eum autem resoluendum ponamus elasto tempore $= t$, ambo corpora in punctis A et B versari, esseque.

$$PA = a + x; AB = a + y; BQ = a + z$$

ita ut sit $x + y + z = 0$.

§. 9. Erit ergo vis elastica elastri AP $= g(1 - \frac{x}{a})$ elastri AB $= g(1 - \frac{y}{a})$ et elastri BQ $= g(1 - \frac{z}{a})$ unde corpus A versus Q propelletur vi $= \frac{g(y-x)}{a}$, et corpus B vi $= \frac{g(z-y)}{a}$. Cum iam sit $PA = a + x$, erit corporis A

A celeritas $= \frac{dx}{dt}$, et vis ad eius motum requisita $= \frac{zA ddx}{dt^2}$, quae ipsi vi $\frac{g(y-x)}{a}$ aequalis esse debet. Deinde ob $PB = 2a + x + y$, erit corporis B celeritas $= \frac{dx+dy}{dt}$ et vis ad eius motum requisita $= \frac{zA(ddx+ddy)}{dt^2}$ ipsi $\frac{g(z-y)}{a}$ aequanda; vnde consequimur has binas aequationes: $\frac{zA ddx}{dt^2} = \frac{g(y-x)}{a}$; $\frac{zA(ddx+ddy)}{dt^2} = \frac{g(z-y)}{a}$ quarum illa ab hac subtracta relinquit: $\frac{zA ddy}{dt^2} = \frac{g(z-y+x)}{a}$ existente $x+y+z=a$.

§. 10. Haec posterior aequatio ob $x+z=-y$ abibit in hanc: $\frac{zA ddy}{dt^2} = -\frac{gy}{a}$, quae per dy multiplicata et integrata dabit $\frac{zA dy^2}{dt^2} = C - \frac{zgy^2}{a}$; vnde fit $dt = \frac{dy \sqrt{zAa}}{\sqrt{zg(bb-yy)}}$, fit vt supra $\sqrt{\frac{g}{Aa}} = n$ erit $ndt \sqrt{\frac{z}{2}} = \frac{dy}{\sqrt{bb-yy}}$; vnde integrando obtinebitur $y=b$ fin. $nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}}$. Aequatio vero prior hanc induet formam: $\frac{zddx}{dt^2} = nn(y-x)$, quae transit in $\frac{zddx}{nn dt^2} + x = b$ fin. $nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}}$. Ad quam integrandam ponatur $x=vu$, et aequatio $\frac{zvddu+4vdvdu+zuddu}{nn dt^2} + vu = b$ fin. $nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}}$ discerpatur in has duas: $\frac{zddu}{nn dt^2} + u = 0$ et $\frac{zuddv+4dudv}{nn dt^2} = b \sin. nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}}$, quarum prior integrata dabit $u = a \sin. nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}}$ vnde et valor ipsius v , hincque porro $x=vu$ inventi poterit.

§. 11. Ponatur breuitatis gratia b fin. $nt \sqrt{\frac{z}{2}} + c \cos. nt \sqrt{\frac{z}{2}} = T$, erit $zuddv+4dudv=nnTdt^2$; quae per u multiplicata et integrata dabit $zuddv=nn dt \int T u dt$ et

et $v = \frac{1}{2}nn\int \frac{dt}{uu} \int T u dt$. At valores u et T seu y in sequentes formas transmutari possunt: vt sit

$$T = y = b \sin. ntV^{\frac{1}{2}} + c \cos. ntV^{\frac{1}{2}} = E \cos. (ntV^{\frac{1}{2}} + \mu)$$

$$u = a \sin. ntV^{\frac{1}{2}} + \xi \cos. ntV^{\frac{1}{2}} = F \cos. (ntV^{\frac{1}{2}} + \nu)$$

$$\text{vnde fit } \int \frac{dt}{uu} = \frac{1}{2} \frac{\sin. (ntV^{\frac{1}{2}} + \nu)}{\cos. (ntV^{\frac{1}{2}} + \nu)}; \text{ atque}$$

$$Tu = EF \cos. (ntV^{\frac{1}{2}} + \mu) \cos. (ntV^{\frac{1}{2}} + \nu). \text{ Ponatur}$$

$$\int Tu dt = P \sin. (ntV^{\frac{1}{2}} + \mu) \cos. (ntV^{\frac{1}{2}} + \nu) + Q \cos. (ntV^{\frac{1}{2}} + \mu) \sin. (ntV^{\frac{1}{2}} + \nu) \text{ fietque differentiando}$$

$$Tu = nP V^{\frac{1}{2}} \cos. (ntV^{\frac{1}{2}} + \mu) \cos. (ntV^{\frac{1}{2}} + \nu) - nP V^{\frac{1}{2}} \cos. (ntV^{\frac{1}{2}} + \mu) \sin. (ntV^{\frac{1}{2}} + \nu) - nQ V^{\frac{1}{2}}$$

$$\sin. (ntV^{\frac{1}{2}} + \mu) \sin. (ntV^{\frac{1}{2}} + \nu) \text{ vnde est } P = -Q V^{\frac{1}{2}};$$

$$\text{et } \frac{nQ - nP}{\sqrt{2}} = EF. \text{ ergo } Q = -\frac{EV}{n\sqrt{2}}; \text{ et } P = \frac{EV\sqrt{2}}{n\sqrt{2}}.$$

$$\text{Ex his porro fiet: } v = \frac{En}{2F\sqrt{2}}$$

$$\int \frac{dt(V^{\frac{1}{2}} \sin. (ntV^{\frac{1}{2}} + \mu) \cos. (ntV^{\frac{1}{2}} + \nu) - \cos. (ntV^{\frac{1}{2}} + \mu) \sin. (ntV^{\frac{1}{2}} + \nu))}{\cos. (ntV^{\frac{1}{2}} + \nu)^2}$$

$$\text{seu } v = \frac{-E \cdot \cos. (ntV^{\frac{1}{2}} + \mu)}{2F \cos. (ntV^{\frac{1}{2}} + \nu)} + G \text{ ideoque } x = Gu$$

$$- \frac{1}{2}y; \text{ Valoribus ergo pro } u \text{ et } y \text{ restitutis erit}$$

$$x = a \sin. ntV^{\frac{1}{2}} + \xi \cos. ntV^{\frac{1}{2}} - \frac{1}{2}b \sin. ntV^{\frac{1}{2}} - \frac{1}{2}c \cos. ntV^{\frac{1}{2}}$$

$$y = b \sin. ntV^{\frac{1}{2}} + c \cos. ntV^{\frac{1}{2}}.$$

§. 12. Elapso ergo tempore t , erit corpus A in A ita vt sit

$$PA = a + a \sin. ntV^{\frac{1}{2}} + \xi \cos. ntV^{\frac{1}{2}} - \frac{1}{2}b \sin. ntV^{\frac{1}{2}} - \frac{1}{2}c \cos. ntV^{\frac{1}{2}} \text{ eiusque celeritas, qua a pariete P recedit erit: } = naV^{\frac{1}{2}} \cos. ntV^{\frac{1}{2}} - nE V^{\frac{1}{2}} \sin. ntV^{\frac{1}{2}} - \frac{1}{2}nbV^{\frac{1}{2}} \cos. ntV^{\frac{1}{2}} + \frac{1}{2}ncV^{\frac{1}{2}} \sin. ntV^{\frac{1}{2}}.$$

Eodemque momento alterum corpus erit in B vt sit

$$PB = 2a + a \sin. ntV^{\frac{1}{2}} + \xi \cos. ntV^{\frac{1}{2}} + \frac{1}{2}b \sin. ntV^{\frac{1}{2}} + \frac{1}{2}c \cos. ntV^{\frac{1}{2}}$$

Tom. I. K.

$\cos(ntV\frac{1}{2})$ eiusque celeritas qua pariter a pariete P remouetur, erit $= n\alpha V\frac{1}{2} \cos(ntV\frac{1}{2}) - n\beta V\frac{1}{2} \sin(ntV\frac{1}{2}) + \frac{1}{2}nbV\frac{1}{2} \cos(ntV\frac{1}{2}) - \frac{1}{2}ncV\frac{1}{2} \sin(ntV\frac{1}{2})$. Corporis ergo A celeritas erit maxima iis temporibus quae ex hac aequatione definientur: $\circ = -\alpha \sin(ntV\frac{1}{2}) - \beta \cos(ntV\frac{1}{2}) + \frac{1}{2}b \sin(ntV\frac{1}{2}) + \frac{1}{2}c \cos(ntV\frac{1}{2})$. Corporis vero B celeritas erit maxima, quando t habuerit valorem ex hac aequatione

$$\circ = -\alpha \sin(ntV\frac{1}{2}) - \beta \cos(ntV\frac{1}{2}) - \frac{1}{2}b \sin(ntV\frac{1}{2}) - \frac{1}{2}c \cos(ntV\frac{1}{2}).$$

§. 13. Ponamus ipso initio, quo erat $t = \circ$, ambo corpora quieuisse; alterum B quidem in situ suo naturali alterum vero A versus P retractum fuisse e situ suo aequilibrii, ita ut eius distantia AP fuerit $= a - \omega$. Prior conditio praebet hos valores $a = 0$, et $b = 0$; Deinde ob $AP = a - \omega$ fit $\beta = -\frac{1}{2}\omega = -\omega$, et ob $B P = 2a$ erit $\beta + \frac{1}{2}c = 0$: ideoque $\beta = -\frac{1}{2}\omega$, et $c = \omega$. Hoc ergo casu postquam elapsum fuerit tempus t , erit $PA = a - \frac{1}{2}\omega \cos(ntV\frac{1}{2}) - \frac{1}{2}\omega \cos(ntV\frac{1}{2})$

$$PB = 2a - \frac{1}{2}\omega \cos(ntV\frac{1}{2}) + \frac{1}{2}\omega \cos(ntV\frac{1}{2})$$

Celeritas ipsius A $= \frac{n}{2}\omega V\frac{1}{2} \sin(ntV\frac{1}{2}) + \frac{n}{2}\omega V\frac{1}{2} \sin(ntV\frac{1}{2})$

Celeritas ipsius B $= \frac{n}{2}\omega V\frac{1}{2} \sin(ntV\frac{1}{2}) - \frac{n}{2}\omega V\frac{1}{2} \sin(ntV\frac{1}{2})$

Quare corpus A maximam acquirit celeritatem cum fuerit $\cos(ntV\frac{1}{2}) + 3 \cos(ntV\frac{1}{2}) = 0$ corporis vero B celeritas erit maxima, quando fiet $\cos(ntV\frac{1}{2}) - 3 \cos(ntV\frac{1}{2})$.

Corporis vero A celeritas evanescet, quoties fit $\sin(ntV\frac{1}{2}) + \sqrt{3} \cdot \sin(ntV\frac{1}{2}) = 0$ et corporis B celeritas ad nihilum redigitur, quando est: $\sin(ntV\frac{1}{2}) = \sqrt{3} \cdot \sin(ntV\frac{1}{2})$

§. 14. Circa motus ergo initium, quando angulus nt est adhuc valde parvus, celeritas corporis ita se habebit, ut sit

cele-

celeritas corporis A $= nn\omega t - \frac{1}{2}n^2\omega t^2$ et
 celeritas corporis B $= -\frac{1}{2}nn\omega t + \frac{1}{6}n^2\omega t^2$

Initio ergo corpus A a pariete P recedit, corpus vero B ad eundem accedit, donec ad quietem redigatur; atque interea habuerit necesse est maximam celeritatem. Postquam autem versus P accedere desierit, tum demum versus Q promouebitur, et cum acquisierit maximum celeritatis gradum, pulsus acceptum maxima vi in parietem Q exerere erit censendum. Cum igitur corpus B, postquam corpus A iam habuit maximam celeritatem, motu versus Q directo maximum velocitatis gradum adiiscatur, hinc iam euidens est tempore opus esse, antequam pulsus ex corpore A in corpus B transferatur; siquidem in quavis medii elastici particula pulsus tum inesse assumamus, cum maximo velocitatis gradu versus parietem Q mouetur. Si enim in Q organum sensus concipiatur, id hoc momento maximam patietur impressionem.

S. 15. Patet ergo haec duo corpora A et B diversissimos motus recipere posse, prout initio tam eorum situs quam motus fuerit diuersimode comparatus. Quo autem in eam agitationem accuratius inquiramus, cuiusmodi in productione soni et luminis oriri solet, ponamus initio corpus A ex situ suo quietis per interuum valde paruum $= \omega$ versus P diductum, ibique detentum suisce, quoad alterum corpus B cedendo quietuerit, tum vero corpus A subito dimitti. Status ergo iste initialis ita erit comparatus, vt posito $t = 0$, utriusque corporis celeritas sit nulla: unde fit $a = 0$ et $b = 0$: Deinde quia

K 2

corpus

corpus B ipso initio nulla vi afficitur, erit quoque eius acceleratio nulla, hincque differentiale ipsius celeritatis $\equiv 0$; ex quo erit $\ell + \frac{3}{2}c \equiv 0$. Denique cum isto motus initio sit $PA \equiv a - \omega$ erit $\ell - \frac{3}{2}c \equiv -\omega$; ideoque $c \equiv \frac{1}{2}\omega$ et $\ell \equiv -\frac{3}{4}\omega$. Quibus valoribus substitutis, postquam elapsum fuerit tempus t , erit

$$PA \equiv a - \frac{3}{4}\omega \cos. ntV^{\frac{1}{2}} - \frac{1}{4}\omega \cos. ntV^{\frac{3}{2}}$$

$$PB \equiv 2a - \frac{3}{4}\omega \cos. ntV^{\frac{1}{2}} + \frac{1}{4}\omega \cos. ntV^{\frac{3}{2}}$$

$$\text{Celeritas ipsius } A \equiv \frac{3}{4\sqrt{2}}n\omega \sin. ntV^{\frac{1}{2}} + \frac{\sqrt{3}}{4\sqrt{2}}n\omega \sin. ntV^{\frac{3}{2}}$$

$$\text{Celeritas ipsius } B \equiv \frac{3}{4\sqrt{2}}n\omega \sin. ntV^{\frac{1}{2}} - \frac{\sqrt{3}}{4\sqrt{2}}n\omega \sin. ntV^{\frac{3}{2}}.$$

§. 16. Si tempus elapsum t adhuc fuerit tam parvum vt anguli $ntV^{\frac{1}{2}}$ et $ntV^{\frac{3}{2}}$ sit minimi; quia tum erit proxima:

$$\sin. ntV^{\frac{1}{2}} \equiv ntV^{\frac{1}{2}} - \frac{1}{16}n^5t^5V^{\frac{1}{2}}$$

$$\sin. ntV^{\frac{3}{2}} \equiv ntV^{\frac{3}{2}} - \frac{3}{16}n^5t^5V^{\frac{3}{2}} \text{ erit}$$

$$\text{Celeritas ipsius } A \equiv \frac{3}{4}nnnt\omega - \frac{1}{8}n^4t^4\omega$$

$$\text{Celeritas ipsius } B \equiv \frac{1}{16}n^4t^4\omega$$

Statim ergo ab initio corpus B tardissime moueri incipit cum eius celeritas se habeat ad celeritatem corporis A vt $nntt$ ad 12: nt autem sit fractio minima. Tempore ergo quopiam opus est, antequam corpus B sensibiliter moueri incipiat. Inuestigemus ergo momenta, quibus vtrumque corpus maximam celeritatem attingit. Ac primo quidem corpus A celerrime mouebitur, cum fuerit: $\cos. ntV^{\frac{1}{2}} \equiv \cos. ntV^{\frac{3}{2}} \equiv 0$.

Corpus vero B celeritatem habebit maximam, quando fit $\cos. ntV^{\frac{1}{2}} \equiv \cos. ntV^{\frac{3}{2}}$.

§. 17. Definiamus primum momenta, quibus corpus A maximo celeritatis gradu concitatur, et quia hoc fit, quando summa cosinum angulorum $ntV\frac{1}{2}$ et $ntV\frac{2}{2}$ euaneat: primus casus habebitur, si

$$ntV\frac{1}{2} = \frac{1}{2}\pi - s \text{ et } ntV\frac{2}{2} = \frac{1}{2}\pi + s$$

vnde fit $\frac{n(1+\sqrt{3})}{\sqrt{2}} = \pi$ et $nt = \frac{\pi\sqrt{2}}{1+\sqrt{3}}$ seu $t = \frac{\pi\sqrt{2}}{(1+\sqrt{3})n}$ quod tempus definitur una oscillatione penduli, cuius longitudo est $= \frac{1}{(1+\sqrt{3})^2 nn} = \frac{\Delta a}{(1+\sqrt{3})^2 g}$. erit autem hoc casu $s = \frac{1}{2}\pi - \frac{\pi}{1+\sqrt{3}} = \frac{(\sqrt{3}-1)\pi}{2(1+\sqrt{3})} = \frac{\pi}{(1+\sqrt{3})^2}$: et celeritas ipsius A $= \frac{3+\sqrt{3}}{4\sqrt{2}} n \omega \cos \frac{\pi}{(1+\sqrt{3})^2}$. Dehinc vero iterum maximum celeritatis gradum acquirit, si fit $ntV\frac{1}{2} = \pi + s$ et $ntV\frac{2}{2} = 2\pi - s$ seu $ntV\frac{1}{2} = \frac{s\pi}{1+\sqrt{3}}$ et $s = \frac{(2-\sqrt{3})\pi}{1+\sqrt{3}}$. Tertio quoque maxima celeritas dabitur in corpore A cum fuerit $ntV\frac{1}{2} = \pi + s$ et $ntV\frac{2}{2} = 2\pi + s$

vnde fit $ntV\frac{1}{2} = \frac{\pi}{\sqrt{3}-1}$, et $s = \frac{(2+\sqrt{3})\pi}{\sqrt{3}-1}$. Generaliter vero corpus A toties habebit maximum celeritatis gradum, quoties fuerit $ntV\frac{1}{2} = \frac{(2i+1)\pi}{\sqrt{3}-1}$ denotante i numerum integrum quemcunque.

§. 18. Corpus autem alterum B maximam celeritatem consequitur, quando fit:

$$\cos ntV\frac{1}{2} = \cos ntV\frac{2}{2}$$

primum ergo hoc euenit quando $ntV\frac{1}{2} = \pi - s$ et $ntV\frac{2}{2} = \pi + s$, seu $ntV\frac{1}{2} = \frac{2\pi}{1+\sqrt{3}}$, ideoque $t = \frac{2\pi\sqrt{2}}{(1+\sqrt{3})n}$. Cum igitur corpus A primum maxima celeritatis gradum nanciscatur elapso tempore $t = \frac{\pi\sqrt{2}}{(1+\sqrt{3})n}$, patet tempus quo corpus B maximam celeritatem acquirit duplo maius esse tempore, quo corpori A maximus celeritatis gradus pri-

mum inducitur. Si ergo pulsus tum effectum exercere censatur, cum quaeque particula citissime mouetur, pulsus a particula A in particulam B hoc est per interuum α transfertur tempore $t = \frac{\pi\sqrt{2}}{(1+\sqrt{3})g} = \frac{\pi\sqrt{2}\alpha}{(1+\sqrt{3})\sqrt{g}}$.

§. 19. Quodsi ergo ponamus pulsum eadem celeritate per reliquas ultra Q sequentes medii elastici partes propagari, et si multitudo particularum aliam formulam sit suppeditatura, hinc tempus, quo pulsus ad quanuis distantiam transfertur definiri poterit. Sit enim α distan-
tia proposita; eritque multitudo particularum seu massa A ipsi longitudini α proportionalis. Atque si vis elasti-
ca medii per pondus columnae eiusdem medii exprimatur,
ita ut g sit longitudo columnae; cuius pondus aequetur
vi elasticae, pro A ipsa longitudo poni poterit, atque ideo
pulsus per spatum α propagabitur tempore $t = \frac{\pi\alpha\sqrt{2}}{(1+\sqrt{3})\sqrt{g}}$:
quae formula si diuidatur per 250, et longitudines α et
 g in particulis millesimis pedis Rhenani exprimantur,
exhibebit tempus in minutis secundis.

§. 20. Si in hac hypothesi pro medio elastico, per quod pulsus propagatur, aerem substituamus, erit eius elasticitas $g = 27980$ ped. Rhen. Vnde tempus quo pulsus in aere seu sonus per interuum $= \alpha$ propagatur erit $= \frac{\pi\alpha\sqrt{2}}{250(1+\sqrt{3})\sqrt{27980000}}$ minutorum secundorum. Hinc ergo primum patet tempora spatiis esse proportionalia, pulsusque motu uniformi propagari. Si ergo ponatur $\frac{\pi\alpha\sqrt{2}}{250(1+\sqrt{3})\sqrt{27980000}} = 1$ prodibit spatum α per quod sonus uno minuto secundo propagatur, quod erit in partibus millesimis pedis rhenani: $\alpha = \frac{250(1+\sqrt{3})\sqrt{15990000}}{\pi}$ ideoque in pedi-

pedibus rheanis $a = \frac{(1+\sqrt{3})\sqrt{13990000}}{4\pi} = \frac{(1+\sqrt{3})\sqrt{874775}}{\pi}$ quae formula evoluta dat $a = 813$ ped. Constat autem solum minuto secundo peragrade spatium circiter 1000 pedum; quod accrementum a multitudine particularum oritur.

§. 21. Antequam autem plures particulas contempnemur, operae pretium erit annotasse ambobus corporibus A et B initio eiusmodi simum tribui posse, ut motu regulari ad similitudinem penduli oscillantis moueantur. Hoc autem dupli modo euenire potest, si quidem vtrumque corpus ab initio quiescere ponamus, ita vt sit $a = 0$, et $b = 0$. Primum scilicet huiusmodi motus orietur si fuerit $c = 0$, et $\xi = -\omega$, quo casu fit: $PA = a - \omega \cos. ntV^{\frac{1}{2}}$; $PB = 2a - \omega \cos. ntV^{\frac{1}{2}}$; motusque conformis erit motui penduli, cuius longitudo est $= \frac{1}{nn} = \frac{Aa}{g}$. Deinde quoque motus oscillatorius simplex orietur si sit $\xi = c$, et $c = 2\omega$, vt sit: $PA = a - \omega \cos. ntV^{\frac{1}{2}}$ et $PB = 2a + \omega \cos. ntV^{\frac{1}{2}}$ hocque casu longitudo penduli simplicis isochroni erit $= \frac{1}{3nn} = \frac{A^2}{3g}$. Ad oscillationes scilicet prioris generis producendas, initio ambo corpora per aequalia interualla ex locis suis naturalibus in eandem plagam deduci debent; pro posteriori vero genere in plagas oppositas. Posteriori autem casu oscillationes celeriores erunt, quam priori.

§. 22. Sint nunc tria corpuscula A, B, C aequalia ^{ig. 4.}
filis elasticis inuicem connexa, quae in aequilibrio versentur cum aequalibus interuallis tum inter se, tum a partibus P et Q distent. Ponatur vt ante uniuscuiusque massa $= A$, distantia binorum contiguorum naturalis $= a$,

et

et vis elastica in hoc statu $= g$. Agitata autem sint haec corpuscula vtcunque, ac post tempus t peruenient in simum figura exhibatum, in quo sit:

$$PA = a + x; AB = a + y; BC = a + z; \text{ et } CQ = a + v \\ \text{erit } x + y + z + v = 0.$$

Erit ergo vis elastica filii $PA = g(1 - \frac{x}{a})$; filii $AB = g(1 - \frac{y}{a})$ filii $BC = g(1 - \frac{z}{a})$ et filii $CQ = g(1 - \frac{v}{a})$. Vires autem ad singulorum corporum motus conferuandos requisitae sunt:

$$\begin{aligned} \text{pro corpore A} &= \frac{2\Lambda ddx}{dt^2} \\ \text{pro corpore B} &= \frac{2\Lambda(ddx + ddy)}{dt^2} \\ \text{pro corpore C} &= \frac{2\Lambda(ddx + ddy + ddz)}{dt^2}. \end{aligned}$$

§. 23. Ob tensionem vero singulorum elastrorum corpus A reuera secundum directionem PQ vrgetur vi $= \frac{g(y-x)}{a}$; Corpus B vi $= \frac{g(z-y)}{a}$; Corpus C vi $= \frac{g(v-z)}{a}$. Posito ergo breuitatis gratia $\sqrt{\frac{g}{2\Lambda a}} = n$ seu $\frac{g}{2\Lambda a} = n^2$, habebuntur sequentes aequationes:

$$\begin{aligned} \frac{ddx}{dt^2} &= n^2(y - x) \\ \frac{ddx + ddy}{dt^2} &= n^2(z - y) \\ \frac{ddx + ddy + ddz}{dt^2} &= n^2(v - z) \end{aligned}$$

ex quibus cum hac $x + y + z + v = 0$ coniunctis motus ad quoduis tempus determinabitur.

§. 24. Quoniam ex praecedentibus forma valorum x, y, z , et v iam colligi potest, ponamus:

$$\begin{aligned} x &= \alpha \cos. npt + \mathfrak{A} \sin. npt \\ y &= \beta \cos. npt + \mathfrak{B} \sin. npt \\ z &= \gamma \cos. npt + \mathfrak{C} \sin. npt \\ v &= \delta \cos. npt + \mathfrak{D} \sin. npt \end{aligned}$$

erit

erit primo : $\alpha + \beta + \gamma + \delta = 0$ et $A + B + C + D = 0$.

Deinde erit positio dt constante :

$$\frac{ddx}{dt^2} = -\alpha nnpp \cos. npt - A nnpp \sin. npt$$

$$\frac{ddy}{dt^2} = -\beta nnpp \cos. npt - B nnpp \sin. npt$$

$$\frac{ddz}{dt^2} = -\gamma nnpp \cos. npt - C nnpp \sin. npt$$

vnde sequentes orientur aequationes :

$$-\alpha pp = \beta - \alpha \quad | -A pp = B - A$$

$$-(\alpha + \beta)pp = \gamma - \beta \quad | -(A + B)pp = C - B$$

$$-(\alpha + \beta + \gamma)pp = \delta - \gamma \quad | -(A + B + C)pp = D - C$$

§. 25. Manifestum ergo est ex similitudine harum aequationum coefficientes A, B, C, D simili modo determinari, quo coefficientes $\alpha, \beta, \gamma, \delta$. Hos autem investigantes inueniemus :

$$\beta = \alpha - \alpha pp; \quad \alpha + \beta = 2\alpha - \alpha pp$$

$$\gamma = \beta - (\alpha + \beta)pp; \quad = \alpha - 3\alpha pp + \alpha p^4$$

$$\alpha + \beta + \gamma = 3\alpha - 4\alpha pp + \alpha p^4$$

$$\delta = \gamma - (\alpha + \beta + \gamma)pp = \alpha - 6\alpha pp + 5\alpha p^4 - \alpha p^6.$$

Quare cum sit $\alpha + \beta + \gamma + \delta = 0$ habebitur

$$0 = 4 - 10\alpha pp + 6\alpha p^4 - \alpha p^6$$

cuius aequationis factores sunt :

$$0 = (2 - \alpha pp)(2 - 4\alpha pp + \alpha p^4)$$

vnde pro αpp sequentes tres valores reperiuntur :

$$\alpha pp = 2; \quad \alpha pp = 2 + \sqrt{2}; \quad \alpha pp = 2 - \sqrt{2}.$$

§. 26. Triplices hi valores pro pp inuenti sequentes praebent coefficientes:

$pp = 2$	$pp = 2 + \sqrt{2}$	$pp = 2 - \sqrt{2}$
$\alpha = \alpha$	$\alpha = +\alpha$	$\alpha = +\alpha$
$\beta = -\alpha$	$\beta = -(1+\sqrt{2})\alpha$	$\beta = -(1-\sqrt{2})\alpha$
$\gamma = -\alpha$	$\gamma = +(1+\sqrt{2})\alpha$	$\gamma = +(1-\sqrt{2})\alpha$
$\delta = +\alpha$	$\delta = -\alpha$	$\delta = -\alpha$
$\mathfrak{A} = \mathfrak{A}$	$\mathfrak{A} = \mathfrak{A}$	$\mathfrak{A} = \mathfrak{A}$
$\mathfrak{B} = -\mathfrak{A}$	$\mathfrak{B} = -(1+\sqrt{2})\mathfrak{A}$	$\mathfrak{B} = -(1-\sqrt{2})\mathfrak{A}$
$\mathfrak{C} = -\mathfrak{A}$	$\mathfrak{C} = +(1+\sqrt{2})\mathfrak{A}$	$\mathfrak{C} = +(1-\sqrt{2})\mathfrak{A}$
$\mathfrak{D} = +\mathfrak{A}$	$\mathfrak{D} = -\mathfrak{A}$	$\mathfrak{D} = -\mathfrak{A}$

Cum igitur pro pp triplicem valorem inuenimus, in expressionibus intergralibus assumtis termini sunt triplicandi; eritque:

$$\begin{aligned} x &= -\alpha \cos nt\sqrt{2} + \alpha' \cos nt\sqrt{(2+\sqrt{2})} + \alpha'' \cos nt\sqrt{(2-\sqrt{2})} \\ &= -\mathfrak{A} \sin nt\sqrt{2} + \mathfrak{A}' \sin nt\sqrt{(2+\sqrt{2})} + \mathfrak{A}'' \sin nt\sqrt{(2-\sqrt{2})} \\ y &= -\alpha \cos nt\sqrt{2} - (1+\sqrt{2})\alpha' \cos nt\sqrt{(2+\sqrt{2})} - (1-\sqrt{2})\alpha'' \cos nt\sqrt{(2-\sqrt{2})} \\ &= -\mathfrak{A} \sin nt\sqrt{2} - (1+\sqrt{2})\mathfrak{A}' \sin nt\sqrt{(2+\sqrt{2})} - (1-\sqrt{2})\mathfrak{A}'' \sin nt\sqrt{(2-\sqrt{2})} \\ z &= -\alpha \cos nt\sqrt{2} + (1+\sqrt{2})\alpha' \cos nt\sqrt{(2+\sqrt{2})} + (1-\sqrt{2})\alpha'' \cos nt\sqrt{(2-\sqrt{2})} \\ v &= -\alpha \cos nt\sqrt{2} - \alpha' \cos nt\sqrt{(2+\sqrt{2})} - \alpha'' \cos nt\sqrt{(2-\sqrt{2})} \\ &= -\mathfrak{A} \sin nt\sqrt{2} - \mathfrak{A}' \sin nt\sqrt{(2+\sqrt{2})} - \mathfrak{A}'' \sin nt\sqrt{(2-\sqrt{2})} \end{aligned}$$

§. 27. Si assumamus motus initio, quo erat $t=0$, singula corpora quiesce, coefficientes $\mathfrak{A}', \mathfrak{A}'', \mathfrak{A}'''$ nulli sunt statuendi, sicque post elapsum tempus t situs corporum sequenti modo determinabitur:

$$\begin{aligned} PA &= a + \alpha \cos nt\sqrt{2} + \alpha' \cos nt\sqrt{(2+\sqrt{2})} + \alpha'' \cos nt\sqrt{(2-\sqrt{2})} \\ PB &= 2a + * -\alpha' \sqrt{2} \cos nt\sqrt{(2+\sqrt{2})} + \alpha'' \sqrt{2} \cos nt\sqrt{(2-\sqrt{2})} \\ PC &= 3a - \alpha \cos nt\sqrt{2} + \alpha' \cos nt\sqrt{(2+\sqrt{2})} + \alpha'' \cos nt\sqrt{(2-\sqrt{2})}. \end{aligned}$$

Hinc

Hinc porro cognoscentur singulorum corporum celeritates, erit enim celeritas secundum directionem PQ corporis

$$A = n\alpha\sqrt{2} \cdot \sin. nt \sqrt{2} - n\alpha' \sqrt{(2+\sqrt{2})} \cdot \sin. nt \sqrt{(2+\sqrt{2})} - n\alpha'' \sqrt{(2-\sqrt{2})} \cdot \sin. nt \sqrt{(2-\sqrt{2})}$$

$$B = +n\alpha' \sqrt{(4+2\sqrt{2})} \cdot \sin. nt \sqrt{(2+\sqrt{2})} - n\alpha'' \sqrt{4-2\sqrt{2}} \cdot \sin. nt \sqrt{(2-\sqrt{2})}$$

$$C = +n\alpha\sqrt{2} \cdot \sin. nt \sqrt{2} - n\alpha' \sqrt{(2+\sqrt{2})} \cdot \sin. nt \sqrt{(2+\sqrt{2})} - n\alpha'' \sqrt{(2-\sqrt{2})} \cdot \sin. nt \sqrt{(2-\sqrt{2})}$$

Quae expressiones si denuo differentientur, prodibunt accelerationes singulorum corporum secundum plagam PQ.

$$A = 2nn\alpha \cos. nt \sqrt{2} - (2+\sqrt{2})nn\alpha' \cos. nt \sqrt{(2+\sqrt{2})} - (2-\sqrt{2})nn\alpha'' \cos. nt \sqrt{(2-\sqrt{2})}$$

$$B = + (2+\sqrt{2})nn\alpha' \cos. nt \sqrt{(2+\sqrt{2})} - (2-\sqrt{2})nn\alpha'' \sqrt{2} \cdot \cos. nt \sqrt{(2-\sqrt{2})}$$

$$C = + (2+\sqrt{2})nn\alpha' \cos. nt \sqrt{(2+\sqrt{2})} - (2-\sqrt{2})nn\alpha'' \cos. nt \sqrt{(2-\sqrt{2})}$$

§. 28. Ponamus nunc corpus A initio de situ suo quietis deductum fuisse per spatiolum ω versus P, ibique tamdiu fuisse detentum, donec reliqua corpora se ad statum aequilibrii composuerint; tum vero corpus A subito dimitti, sicque motum paulatim in corpora B et C transferri. Quo igitur formulas inuentas ad hunc casum accommodemus, primo erit:

$$\alpha + \alpha' + \alpha'' = -\omega$$

Deinde quia reliqua corpora B et C ipso motus initio nullam accelerationem patiuntur, erit:

$$(2+\sqrt{2})\alpha' \sqrt{2} = (2-\sqrt{2})\alpha'' \sqrt{2}$$

$$\text{et } 2\alpha = (2+\sqrt{2})\alpha' + (2-\sqrt{2})\alpha'' = 2(2-\sqrt{2})\alpha''$$

$$\text{ergo } \alpha'' = \frac{\alpha}{2-\sqrt{2}}; \text{ et } \alpha' = \frac{\alpha}{2+\sqrt{2}}$$

ideoque $\alpha' + \alpha'' = 2\alpha$; et $3\alpha = -\omega$. Quamobrem habebimus:

$$\alpha = -\frac{1}{3}\omega; \alpha' = -\frac{1}{3}\omega(2-\sqrt{2}); \alpha'' = -\frac{1}{3}\omega(2+\sqrt{2}).$$

§. 29. His igitur valoribus pro α , α' , α'' inuentis, momenta assignare licet, quibus singula corpora maximum celeritatis gradum adipiscuntur. Ac primo quidem corpus A celerrime mouebitur, si fuerit:

L 2

o =

$$o = -2 \cos(nt\sqrt{2}) + \cos(nt\sqrt{2+\sqrt{2}}) + \cos(nt\sqrt{2-\sqrt{2}})$$

Corpus vero B maximum celeritatis gradum habebit si sit:

$$o = \cos(nt\sqrt{2+\sqrt{2}}) - \cos(nt\sqrt{2-\sqrt{2}}) \quad \text{At corpus C maximam acquireret celeritatem, quando sit } o = -2 \cos(nt\sqrt{2}) \cos(nt\sqrt{2+\sqrt{2}}) + \cos(nt\sqrt{2-\sqrt{2}}).$$

Hinc facillime momenta assignantur, quibus corpus B celerime concitatatur: primum scilicet hoc fiet, quando erit

$$nt\sqrt{2-\sqrt{2}} = \pi - s \quad \text{et} \quad nt\sqrt{2+\sqrt{2}} = \pi + s$$

vnde fit $nt\sqrt{4+2\sqrt{2}} = 2\pi$ et $t = \frac{\pi\sqrt{2}}{n\sqrt{2+\sqrt{2}}} = \frac{\pi\sqrt{(2-\sqrt{2})}}{n}$; seu $t = \frac{\pi\sqrt{2}(2-\sqrt{2})}{\sqrt{g}}$. Tanto ergo tempore pulsus in secundum corpus B transfertur: neque vero hoc tempus duplo maius est eo, quo corpus A primum celerime mouetur, neque pari interuallo pulsus in corpus C progreditur. Haec autem experientiae non aduersantur, qua constat pulsus motu aequabili propagari; numerus enim particularum hic consideratarum nimis est parvus, quam ut inde conclusio ad numerum quasi infinitum inferri queat.

§. 30. Si has formulas attentius consideremus, iam ordinem in angulis, quorum sinus et cosinus hic occurront, obseruare licebit. Hoc enim casu, quo tria corpora A, B, C sumus contemplati, anguli $nt\sqrt{2}$, $nt\sqrt{2+\sqrt{2}}$ et $nt\sqrt{2-\sqrt{2}}$ ita se habent, ut posito e angulo recto sit:

$$nt\sqrt{2} = 2nt \cos \frac{1}{2}\pi; \quad nt\sqrt{2+\sqrt{2}} = 2nt \cos \frac{1}{4}\pi; \\ \text{et} \quad nt\sqrt{2-\sqrt{2}} = 2nt \cos \frac{3}{4}\pi.$$

isti ergo anguli ex quadrisectione anguli recti determinantur. Erat vero hic $n = \sqrt{\frac{g}{2A\alpha}}$. Si pro casu duorum

rum corporum posuissimus pariter $n = V \frac{g}{2Aa}$; tum producissent hi anguli nt , et $ntV3$; qui ita exhibebuntur per trisectionem anguli recti:

$$nt = 2nt \cos \frac{\pi}{3}\varrho; ntV3 = 2nt \cos \frac{1}{3}\varrho.$$

simili modo in casu vniuersi corporis, posito $n = V \frac{g}{2Aa}$ occurrebat angulus $ntV2 = 2nt \cos \frac{1}{2}\varrho$: ideoque ex bisectione anguli recti ϱ definitur. Ex his iam colligere possumus, si numerus corporum sit $= m - 1$ fore angulos solutionem ingredientes:

$$2nt \cos \frac{1}{m}\varrho; 2nt \cos \frac{2}{m}\varrho; 2nt \cos \frac{3}{m}\varrho \dots 2nt \cos \frac{m-1}{m}\varrho.$$

§. 31. Ponamus nunc intra parietes P et Q corpora quotcunque aequalia A, B, C, D, E, etc. in linea Fig. 55 recta esse constituta, quae interpositis elastris aequalibus in se inuicem nitantur. Sit massa cuiusque corporis $= A$, longitude singulorum elastrorum, cum se mutuo in aequilibrio seruant $= a$, et vis elastica eiusque elastri in hoc statu aequilibrii sit $= g$. Postquam autem ab actione quacunque status aequilibrii fuerit perturbatus, elapsso tempore t singula corpora eum situm teneant, qui in figura representatur, sitque numerus corporum $= \lambda - 1$ erit elastorum PA, AB, BC, etc. numerus vnitate maior $= \lambda$.

Vocetur nunc:

$$PA = a + x$$

$$PB = 2a + x^I$$

$$PC = 3a + x^{II}$$

$$PD = 4a + x^{III}$$

$$PE = 5a + x^{IV}$$

:

L 3

; PG

$$PG = (\lambda - 1)a + x^{(\lambda-2)}$$

$$PQ = \lambda a + x^{(\lambda-2)}$$

eritque $x^{(\lambda-1)} = 0$, $x^{\lambda} = 0$, $x^{(\lambda+1)} = 0$ etc.

§. 32. Hinc longitudines singulorum elastrorum cum suis viribus elasticis ita se habebunt

Longitudo	vis elastica
$PA = a + x^I - x$	$g \left(1 - \frac{x}{a} \right)$
$AB = a + x^I - x$	$g \left(1 - \frac{x^I}{a} + \frac{x}{a} \right)$
$BC = a + x^{II} - x^I$	$g \left(1 - \frac{x^{II}}{a} + \frac{x^I}{a} \right)$
$CD = a + x^{III} - x^{II}$	$g \left(1 - \frac{x^{III}}{a} + \frac{x^{II}}{a} \right)$
	etc.

Vires ergo quibus singula copora secundum directionem PQ sollicitantur erunt:

pro corpore vis sollicitans

$$A \cdots \cdots \cdots \frac{g}{a} (x^I - 2x)$$

$$B \cdots \cdots \cdots \frac{g}{a} (x^{II} - 2x^I + x)$$

$$C \cdots \cdots \cdots \frac{g}{a} (x^{III} - 2x^{II} + x^I)$$

$$D \cdots \cdots \cdots \frac{g}{a} (x^{IV} - 2x^{III} + x^{II})$$

etc.

$$G \cdots \cdots \cdots \frac{g}{a} (x^{(\lambda-1)} - 2x^{(\lambda-2)} + x^{(\lambda-3)})$$

§. 33. Celeritates porro singulorum corporum sequenti modo exprimentur, secundum directionem PQ :

$$\text{Celeritas corporis } A = \frac{dx}{dt}$$

$$B = \frac{dx^I}{dt}$$

Celeri-

PER MEDIUM ELASTICUM

87

$$\text{Celeritas corporis } C = \frac{dx^{II}}{dt}$$

$$- - - - - D = \frac{dx^{III}}{dt}$$

:

:

$$(n-2)^2$$

$$\text{Celeritas ultimi } G = \frac{dx}{dt}$$

Atque vires, quae ad accelerationem singulorum secundum eandem directionem PQ requiruntur, erunt

Corpus sollicitabitur vi

$$A \quad \frac{Addx}{dt^2} = \frac{g}{a} (x^I - 2x)$$

$$B \quad \frac{Addx^I}{dt^2} = \frac{g}{a} (x^{II} - 2x^I + x)$$

$$C \quad \frac{Addx^{II}}{dt^2} = \frac{g}{a} (x^{III} - 2x^{II} + x^I)$$

$$D \quad \frac{Addx^{III}}{dt^2} = \frac{g}{a} (x^{IV} - 2x^{III} + x^II)$$

$$E \quad \vdots$$

$$F \quad \vdots$$

$$G \quad \frac{Addx}{dt^2} = \frac{g}{a} (x^{n-1} - 2x^{n-2} + x^{n-3})$$

S. 34. Ponamus vt ante $\frac{s}{A_a} = nn$, et habebimus
has aequationes:

$$\frac{ddx}{nn dt^2} = x^I - 2x$$

$$\frac{ddx^I}{nn dt^2} = x^{II} - 2x^I + x$$

$$\frac{ddx^{II}}{nn dt^2} = x^{III} - 2x^{II} + x^I$$

$$\frac{ddx^{III}}{nn dt^2} = x^{IV} - 2x^{III} + x^II$$

ad

$$\frac{d\alpha}{ndt^2} = x^{(\lambda-1)} - 2x^{(\lambda-2)} + x^{(\lambda-3)}$$

Ad quas aquationes resoluendas ponamus:

$$x = \alpha \cos. nt p$$

$$x^I = \alpha^I \cos. 2nt p$$

$$x^{II} = \alpha^{II} \cos. 2nt p$$

$$x^{III} = \alpha^{III} \cos. 2nt p$$

$$x^{(\lambda-2)} = \alpha^{(\lambda-2)} \cos. 2nt p$$

eritque $\alpha^{(\lambda-1)} = 0$, ob $x^{(\lambda-1)} = 0$. Potuissemus hic quoque sinus eiusdem anguli $2nt p$ adiicere, sed cum eorum coefficientes eandem legem teneant, inuentis coefficientibus $\alpha, \alpha^I, \alpha^{II}, \alpha^{III}$ etc. cum valoribus constantis quantitatis p , hi termini nullo negotio adiiciuntur.

§. 35. Cum igitur posito dt constante sit:

$$-\frac{ddx}{ndt^2} = 4\alpha pp \cos. 2nt p$$

$$-\frac{ddx^I}{ndt^2} = 4\alpha^I pp \cos. 2nt p$$

$$-\frac{ddx^{II}}{ndt^2} = 4\alpha^{II} pp \cos. 2nt p$$

$$-\frac{ddx^{III}}{ndt^2} = 4\alpha^{III} pp \cos. 2nt p$$

$$-\frac{ddx^{(\lambda-2)}}{ndt^2} = 4\alpha^{(\lambda-2)} pp \cos. 2nt p$$

sequentes adipiscemur aequationes :

$$\begin{array}{l} -4\alpha pp = \alpha^1 - 2\alpha \\ -4\alpha^1 pp = \alpha^{II} - 2\alpha^I + \alpha \\ -4\alpha^{II} pp = \alpha^{III} - 2\alpha^{II} + \alpha^I \\ -4\alpha^{III} pp = \alpha^{IV} - 2\alpha^{III} + \alpha^{II} \\ \vdots \\ -4\alpha^{(\lambda-2)} pp = \alpha^{(\lambda-1)} - 2\alpha^{(\lambda-2)} + \alpha^{(\lambda-3)} \end{array} \quad \left| \begin{array}{l} \alpha^I = 2(1-2pp)\alpha \\ \alpha^{II} = 2(1-2pp)\alpha^I - \alpha \\ \alpha^{III} = 2(1-2pp)\alpha^{II} - \alpha^I \\ \alpha^{IV} = 2(1-2pp)\alpha^{III} - \alpha^{II} \\ \vdots \\ \alpha^{(\lambda-1)} = 2(1-2pp)\alpha^{(\lambda-2)} - \alpha^{(\lambda-3)} \end{array} \right.$$

§. 36. Ponamus nunc esse $p = \sin. \Phi$, erit
 $1-2pp = \cos. 2\Phi$, hincque fiet

$$\begin{aligned} \alpha^I &= 2\alpha \cos. 2\Phi \\ \alpha^{II} &= 4\alpha \cos. 2\Phi \cos. 2\Phi - \alpha = \alpha(1 + \cos. 4\Phi) \\ \alpha^{III} &= \alpha(2\cos. 2\Phi + 4\cos. 2\Phi \cos. 4\Phi - 2\cos. 2\Phi) = \alpha(2\cos. 2\Phi \\ &\quad + 2\cos. 6\Phi) \\ &\quad \text{etc.} \end{aligned}$$

quo autem lex harum formularum clarius perspiciatur, ponamus $\alpha = \mathfrak{A} \sin. 2\Phi$ eritque

$$\begin{aligned} \alpha &= \mathfrak{A} \sin. 2\Phi \\ \alpha^I &= \mathfrak{A} \sin. 4\Phi \\ \alpha^{II} &= \mathfrak{A} \sin. 6\Phi \\ \alpha^{III} &= \mathfrak{A} \sin. 8\Phi \\ \alpha^{(\lambda-1)} &= \mathfrak{A} \sin. 2\lambda\Phi = 0. \end{aligned}$$

Quia ergo $\sin. 2\lambda\Phi = 0$, sumto ϱ pro angulo recto angulum $2\lambda\Phi$ esse oportet aequalem termino cuiquam huius seriei $0, 2\varrho, 4\varrho, 6\varrho, 8\varrho$, etc. Generaliter ergo erit $2\lambda\Phi = 2m\varrho$ denotante m numerum quemcunque integrum; unde fit $\Phi = \frac{m}{\lambda}\varrho$; et $p = \sin. \frac{m}{\lambda}\varrho$.

§. 37. Pro p igitur tot inuenimus diuersos valoreſ quot vnitates continentur in numero $\lambda - 1$, ſen quo fuerint corpora in ſerie PQ: totidemque terminis conſtabunt valořeſ x , x^I , x^{II} , etc. Sumto ergo pro m numero quoctunque minori quam λ , erit

$$\textcircled{c} \quad p = \sin. \frac{m}{\lambda} \varrho$$

$$\alpha = \mathfrak{A} \sin. \frac{2m}{\lambda} \varrho$$

$$x^I = \mathfrak{A} \sin. \frac{4m}{\lambda} \varrho$$

$$x^{II} = \mathfrak{A} \sin. \frac{6m}{\lambda} \varrho$$

$$x^{III} = \mathfrak{A} \sin. \frac{8m}{\lambda} \varrho$$

:

$$x^{(\lambda-1)} = \mathfrak{A} \sin. 2m \varrho = \textcircled{c}$$

vnde ſequentes obtinebuntur valořeſ :

$$x = \mathfrak{A} \sin. \frac{2}{\lambda} p. \cos. 2nt \sin. \frac{p}{\lambda} + \mathfrak{B} \sin. \frac{4}{\lambda} \varrho. \cos 2nt \sin^2 \frac{p}{\lambda} + \\ (\mathfrak{C} \sin. \frac{6}{\lambda} \varrho. \cos. 2nt \sin. \frac{3p}{\lambda} + \dots + \mathfrak{D} \sin. \frac{4(\lambda-1)p}{\lambda}. \cos 2nt \sin. \frac{2(\lambda-1)p}{\lambda})$$

$$x^I = \mathfrak{A} \sin. \frac{4}{\lambda} \varrho. \cos. 2nt \sin. \frac{p}{\lambda} + \mathfrak{B} \sin. \frac{8}{\lambda} \varrho. \cos 2nt \sin^2 \frac{p}{\lambda} + \\ (\mathfrak{C} \sin. \frac{12}{\lambda} p. \cos. 2nt \sin. \frac{3p}{\lambda} + \dots + \mathfrak{D} \sin. \frac{4(\lambda-1)p}{\lambda}. \cos 2nt \sin. \frac{(\lambda-1)p}{\lambda})$$

$$x^{II} = \mathfrak{A} \sin. \frac{6}{\lambda} \varrho. \cos. 2nt \sin. \frac{p}{\lambda} + \mathfrak{B} \sin. \frac{12}{\lambda} p. \cos 2nt \sin^2 \frac{p}{\lambda} + \\ (\mathfrak{C} \sin. \frac{18}{\lambda} p. \cos. 2nt \sin. \frac{3p}{\lambda} + \dots + \mathfrak{D} \sin. \frac{6(\lambda-1)p}{\lambda}. \cos 2nt \sin. \frac{(\lambda-1)p}{\lambda})$$

$$x^{(I-2)} = \mathfrak{A} \sin. \frac{2(\lambda-1)}{\lambda} p. \cos 2nt \sin. \frac{p}{\lambda} + \mathfrak{B} \sin. \frac{4(\lambda-1)}{\lambda} p. \cos 2nt \sin^2 \frac{p}{\lambda} + \\ (\mathfrak{C} \sin. \frac{6(\lambda-1)}{\lambda} p. \cos 2nt \sin. \frac{3p}{\lambda} + \dots + \mathfrak{D} \sin. \frac{2(\lambda-1)p}{\lambda}. \cos 2nt \sin. \frac{(\lambda-1)p}{\lambda}).$$

§. 38. Aequationes iſtae iam ita ſunt comparatae, wt ipſo motuſ initio, quo erat $t = 0$, ſinguloruſ cor-
poruſ

porum celeritates evanescant; in quem finem sinus angularum ω nt data opera omisimus. Pro vario ergo situ cuiusque corporis initiali, respectu situs aequilibrii, vnde valores litterarum A , B , C , D , etc. pendent, innumerabiles diuersarum agitationum modi resultant, quos quidem si valores litterarum A , B , C , D , etc. fuerint cogniti, facile determinare licet, cum ex aequationibus inuentis ad quoduis temporis momentum singulorum corporum tam situs quam motus assignari queat. Longe autem difficilis est pro quoquis statu initiali proposito, idoneos litterarum A , B , C , D , etc. valores inuestigare, cum tot prodeant aequationes, quot adesse ponuntur corpora: vnde si horum corporum numerus fuerit indefinitus, via vix patet, quae ad cognitionem istorum valorum perducat.

§. 39. Si ponamus initio omnia corpora praeter primum in situ suo naturali fuisse constituta, primum autem internallo $= \omega$ de loco suo naturali fuisse dimotum, necesse est ut posito $t = 0$ fiat $x = \omega$, et $x^I = 0$, $x^{II} = 0$, $x^{III} = 0$, etc. Hinc ergo sequentes aequationes resultabunt.

$$\begin{aligned} A \sin \frac{1}{\lambda} \xi + B \sin \frac{2}{\lambda} \xi + C \sin \frac{3}{\lambda} \xi + \dots + D \sin \frac{s(\lambda-1)}{\lambda} \xi &= \omega \\ A \sin \frac{1}{\lambda} \xi + B \sin \frac{3}{\lambda} \xi + C \sin \frac{12}{\lambda} \xi + \dots + D \sin \frac{s(\lambda-1)}{\lambda} \xi &= 0 \\ A \sin \frac{6}{\lambda} \xi + B \sin \frac{12}{\lambda} \xi + C \sin \frac{18}{\lambda} \xi + \dots + D \sin \frac{s(\lambda-1)}{\lambda} \xi &= 0 \\ A \sin \frac{1}{\lambda} \xi + B \sin \frac{16}{\lambda} \xi + C \sin \frac{24}{\lambda} \xi + \dots + D \sin \frac{s(\lambda-1)}{\lambda} \xi &= 0 \end{aligned}$$

M 2

A sin.

$\mathfrak{A} \sin. \frac{z(\lambda-1)}{\lambda} \xi + \mathfrak{B} \sin. \frac{z(\lambda-1)}{\lambda} \xi + \mathfrak{C} \sin. \frac{z(\lambda-1)}{\lambda} \xi + \dots + \mathfrak{D} \sin. \frac{z(\lambda-1)}{\lambda} \xi = 0$
 quarum aequationum numerus est $= \lambda - 1$, ideoque corporum A, B, C, etc. numero aequatur, et vnaquaeque aequatio totidem continet terminos.

§. 40. Videamus ergo, an inductio a casibus facilitioribus quicquam ad generalem litterarum A, B, C, etc. determinationem conferat. Sit igitur primo vnicum corpus A, et habebitur vnlca aequatio, ob $\lambda - 1 = 1$.

$$\mathfrak{A} = -\omega$$

Sit $\lambda - 1 = 2$ seu $\lambda = 3$, habebimus duas aequationes.

I. $\mathfrak{A} \sin. \frac{z}{3} \xi + \mathfrak{B} \sin. \frac{2}{3} \xi = -\omega; \mathfrak{A} \sin. \frac{z}{3} \xi = -\frac{1}{2} \omega$

II. $\mathfrak{A} \sin. \frac{z}{3} \xi - \mathfrak{B} \sin. \frac{2}{3} \xi = 0; \mathfrak{B} \sin. \frac{2}{3} \xi = -\frac{1}{2} \omega$

Hinc $\mathfrak{A} \sin. \frac{z}{3} \xi = -\frac{1}{2} \omega; \mathfrak{B} \sin. \frac{2}{3} \xi = -\frac{1}{2} \omega; \mathfrak{A} \sin. \frac{z}{3} \xi = -\frac{1}{2} \omega$
 et $\mathfrak{B} \sin. \frac{2}{3} \xi = +\frac{1}{2} \omega$.

Sit $\lambda - 1 = 3$ seu $\lambda = 4$, tres habebuntur aequationes.

I. $\mathfrak{A} \sin. \frac{z}{4} \xi + \mathfrak{B} \sin. \frac{3}{4} \xi + \mathfrak{C} \sin. \frac{6}{4} \xi = -\omega$

II. $\mathfrak{A} \sin. \frac{z}{4} \xi + \mathfrak{B} \sin. \frac{3}{4} \xi + \mathfrak{C} \sin. \frac{12}{4} \xi = 0$

III. $\mathfrak{A} \sin. \frac{z}{4} \xi + \mathfrak{B} \sin. \frac{12}{4} \xi + \mathfrak{C} \sin. \frac{18}{4} \xi = 0$

sive

ergo

I. $\mathfrak{A} \sin. \frac{z}{4} \xi + \mathfrak{B} \sin. \xi + \mathfrak{C} \sin. \frac{1}{2} \xi = -\omega \quad \mathfrak{C} = \mathfrak{A}$

II. $\mathfrak{A} \sin. \xi + * - \mathfrak{C} \sin. \xi = 0 \quad (\mathfrak{A} + \mathfrak{C}) \sin. \frac{z}{4} \xi = -\frac{1}{2} \omega$
 $\mathfrak{A} = -\frac{1}{4} \omega; \sin. \frac{z}{4} \xi$

III. $\mathfrak{A} \sin. \frac{z}{4} \xi - \mathfrak{B} \sin. \xi + \mathfrak{C} \sin. \frac{1}{2} \xi = 0 \quad \mathfrak{C} = -\frac{1}{4} \omega; \sin. \frac{z}{4} \xi$
 $\mathfrak{B} = -\frac{1}{2} \omega; \sin. \xi$

Erit ergo

$\mathfrak{A} \sin. \frac{z}{4} \xi = -\frac{1}{4} \omega | \mathfrak{A} \sin. \xi = -\frac{1}{2} \omega \cos. \frac{z}{4} \xi | \mathfrak{A} \sin. \frac{z}{4} \xi = -\frac{1}{4} \omega$

$\mathfrak{B} \sin. \xi$

$$\begin{array}{l|l|l} \mathfrak{B} \sin. \frac{4}{7}\varphi = -\frac{1}{2}\omega & \mathfrak{B} \sin. \frac{8}{7}\varphi = 0 & \mathfrak{B} \sin. \frac{12}{7}\varphi = +\frac{1}{2}\omega \\ \mathfrak{C} \sin. \frac{6}{7}\varphi = -\frac{1}{4}\omega & \mathfrak{C} \sin. \frac{12}{7}\varphi = +\frac{1}{2}\omega \cos. \frac{1}{2}\varphi & \mathfrak{C} \sin. \frac{18}{7}\varphi = -\frac{1}{4}\omega \end{array}$$

§. 41. Hos valores iam supra eruimus ; nunc igitur ulterius progrediamur ac ponamus $\lambda - i = 4$, seu $\lambda = 5$.

$$\mathfrak{A} \sin. \frac{2}{5}\varphi + \mathfrak{B} \sin. \frac{6}{5}\varphi + \mathfrak{C} \sin. \frac{6}{5}\varphi + \mathfrak{D} \sin. \frac{8}{5}\varphi = -\omega$$

$$\mathfrak{A} \sin. \frac{4}{5}\varphi + \mathfrak{B} \sin. \frac{8}{5}\varphi + \mathfrak{C} \sin. \frac{12}{5}\varphi + \mathfrak{D} \sin. \frac{16}{5}\varphi = 0$$

$$\mathfrak{A} \sin. \frac{6}{5}\varphi + \mathfrak{B} \sin. \frac{12}{5}\varphi + \mathfrak{C} \sin. \frac{18}{5}\varphi + \mathfrak{D} \sin. \frac{24}{5}\varphi = 0$$

$$\mathfrak{A} \sin. \frac{8}{5}\varphi + \mathfrak{B} \sin. \frac{16}{5}\varphi + \mathfrak{C} \sin. \frac{24}{5}\varphi + \mathfrak{D} \sin. \frac{32}{5}\varphi = 0$$

fit breuitatis gratia :

$$x = \sin. \frac{2}{5}\varphi = \sin. \frac{8}{5}\varphi = -\sin. \frac{12}{5}\varphi = -\sin. \frac{18}{5}\varphi = -\sin. \frac{32}{5}\varphi$$

$$y = \sin. \frac{4}{5}\varphi = \sin. \frac{6}{5}\varphi = -\sin. \frac{16}{5}\varphi = \sin. \frac{24}{5}\varphi \text{ erit}$$

$$\mathfrak{A}\alpha + \mathfrak{B}c + \mathfrak{C}e + \mathfrak{D}\epsilon = -\omega \quad | \quad \mathfrak{A}\alpha + \mathfrak{C}b = -\frac{1}{2}\omega$$

$$\mathfrak{A}c + \mathfrak{B}\alpha - \mathfrak{C}\epsilon - \mathfrak{D}e = 0 \quad | \quad \mathfrak{B}c + \mathfrak{D}\alpha = -\frac{1}{2}\omega$$

$$\mathfrak{A}e - \mathfrak{B}\epsilon - \mathfrak{C}\alpha + \mathfrak{D}c = 0 \quad | \quad \mathfrak{A}e - \mathfrak{C}\alpha = 0$$

$$\mathfrak{A}\epsilon - \mathfrak{B}c + \mathfrak{C}\alpha - \mathfrak{D}\epsilon = 0 \quad | \quad \mathfrak{B}c - \mathfrak{D}\epsilon = 0$$

$$\text{vnde fit } \mathfrak{A}(\alpha^2 + c^2) = -\frac{1}{2}\alpha\omega; \mathfrak{C}(\alpha\alpha + ee) = -\frac{1}{2}e\omega$$

$$\mathfrak{B}(\alpha\alpha + ee) = -\frac{1}{2}e\omega; \mathfrak{D}(\alpha\alpha + ee) = -\frac{1}{2}\alpha\omega \text{ ideoque}$$

$$\mathfrak{A} = \mathfrak{D} = \frac{-\alpha\omega}{2(\alpha^2 + e^2)}; \mathfrak{B} = \mathfrak{C} = \frac{-e\omega}{2(\alpha^2 + e^2)}$$

$$\text{et } \mathfrak{A}: \mathfrak{B} = \alpha: e.$$

§. 41. Ponamus iam esse $\lambda - i = 5$ seu $\lambda = 6$; fitque

$$\alpha = \sin. \frac{2}{5}\varphi = \sin. \frac{10}{5}\varphi = \sin. \frac{50}{5}\varphi$$

$$e = \sin. \frac{4}{5}\varphi = \sin. \frac{6}{5}\varphi = -\sin. \frac{16}{5}\varphi = -\sin. \frac{20}{5}\varphi = \sin. \frac{52}{5}\varphi = -\sin. \frac{40}{5}\varphi$$

$$c = \sin. \frac{6}{5}\varphi = -\sin. \frac{12}{5}\varphi = \sin. \frac{56}{5}\varphi$$

$$\omega = \sin. \frac{8}{5}\varphi = \sin. \frac{24}{5}\varphi$$

atque habebimus has aequationes :

$$\mathfrak{A}\alpha + \mathfrak{B}c + \mathfrak{C}y + \mathfrak{D}e + \mathfrak{E}\alpha = -\omega$$

M 3

$\mathfrak{A}c$

$$\mathfrak{A}\epsilon + \mathfrak{B}\epsilon + \mathfrak{C}\circ - \mathfrak{D}\epsilon - \mathfrak{E}\epsilon = 0$$

$$\mathfrak{A}\gamma + \mathfrak{B}\circ - \mathfrak{C}\gamma + \mathfrak{D}\circ + \mathfrak{E}\gamma = 0$$

$$\mathfrak{A}\epsilon - \mathfrak{B}\epsilon + \mathfrak{C}\circ + \mathfrak{D}\epsilon - \mathfrak{E}\epsilon = 0$$

$$\mathfrak{A}\alpha - \mathfrak{B}\epsilon + \mathfrak{C}\gamma - \mathfrak{D}\epsilon + \mathfrak{E}\alpha = 0$$

Harum media dat $\mathfrak{C} = \mathfrak{A} + \mathfrak{E}$, secunda et quarta vero $\mathfrak{A} - \mathfrak{C} = 0$; et $\mathfrak{B} - \mathfrak{D} = 0$; ergo erit $\mathfrak{E} = \mathfrak{A}$; $\mathfrak{D} = \mathfrak{B}$; $\mathfrak{C} = 2\mathfrak{A}$. Deinde prima et quinta dat:

$$\mathfrak{A}\alpha + \mathfrak{C}\gamma + \mathfrak{E}\alpha = -\frac{1}{2}\omega; \mathfrak{B}\epsilon + \mathfrak{D}\epsilon = -\frac{1}{2}\omega$$

$$\text{Ergo } \mathfrak{A} = \mathfrak{E} = \frac{-\omega}{4(\alpha+\gamma)}; \mathfrak{B} = \mathfrak{D} = \frac{-\omega}{4\epsilon}; \mathfrak{C} = \frac{-\omega}{2(\alpha+\gamma)}$$

Est vero hic $\alpha = \frac{1}{2}$; $\mathfrak{C} = \frac{\sqrt{3}}{2}$; et $\gamma = 1$: unde erit $\mathfrak{A} : \mathfrak{B} = \mathfrak{C} : \alpha + \gamma = \sqrt{3} : 3 = \alpha$; et ob $\gamma = 2\alpha$ fiet $\mathfrak{A} : \mathfrak{B} : \mathfrak{C} = \alpha : \mathfrak{C} : \gamma$.

§. 42. Hinc iam satis tuto per inductionem conclusio colligi posset pro generali coefficientium determinatione; sed quo magis confirmemur, ponamus adhuc $\lambda - 1 = 6$ seu $\lambda = 7$; sitque

$$\alpha = \sin. \frac{2}{7}\varrho = \sin. \frac{12}{7}\varrho = -\sin. \frac{16}{7}\varrho = \sin. \frac{30}{7}\varrho \sin. \frac{40}{7}\varrho = -\sin. \frac{72}{7}\varrho$$

$$\mathfrak{C} = \sin. \frac{4}{7}\varrho = \sin. \frac{10}{7}\varrho = -\sin. \frac{24}{7}\varrho = -\sin. \frac{18}{7}\varrho = \sin. \frac{32}{7}\varrho = \sin. \frac{60}{7}\varrho$$

$$\gamma = \sin. \frac{6}{7}\varrho = \sin. \frac{8}{7}\varrho = -\sin. \frac{20}{7}\varrho = \sin. \frac{36}{7}\varrho = -\sin. \frac{48}{7}\varrho = \sin. \frac{50}{7}\varrho,$$

atque sequentes obtinebuntur aequationes:

$$\mathfrak{A}\alpha + \mathfrak{B}\epsilon + \mathfrak{C}\gamma + \mathfrak{D}\alpha - \mathfrak{E}\gamma + \mathfrak{F}\alpha = -\omega$$

$$\mathfrak{A}\epsilon + \mathfrak{B}\gamma + \mathfrak{C}\alpha - \mathfrak{D}\alpha - \mathfrak{E}\gamma - \mathfrak{F}\epsilon = 0$$

$$\mathfrak{A}\gamma + \mathfrak{B}\alpha - \mathfrak{C}\epsilon - \mathfrak{D}\epsilon + \mathfrak{E}\alpha + \mathfrak{F}\gamma = 0$$

$$\mathfrak{A}\gamma - \mathfrak{B}\alpha - \mathfrak{C}\epsilon + \mathfrak{D}\epsilon + \mathfrak{E}\alpha - \mathfrak{F}\gamma = 0$$

$$\mathfrak{A}\epsilon - \mathfrak{B}\gamma + \mathfrak{C}\alpha + \mathfrak{D}\alpha - \mathfrak{E}\gamma + \mathfrak{F}\epsilon = 0$$

$$\mathfrak{A}\alpha - \mathfrak{B}\epsilon + \mathfrak{C}\gamma - \mathfrak{D}\gamma + \mathfrak{E}\epsilon - \mathfrak{F}\alpha = 0$$

Harum

Harum aequationum secundae, quartae, et sextae satis fit ponendo $\mathfrak{A} = \mathfrak{A}$; $\mathfrak{B} = \mathfrak{B}$ et $\mathfrak{D} = \mathfrak{C}$; ex quo tres reliquae abeunt in :

$$\mathfrak{A}\alpha + \mathfrak{B}\varepsilon + \mathfrak{C}\gamma = -\frac{1}{2}\omega$$

$$\mathfrak{A}\gamma + \mathfrak{B}\alpha - \mathfrak{C}\varepsilon = 0$$

$$\mathfrak{A}\varepsilon - \mathfrak{B}\gamma + \mathfrak{C}\alpha = 0$$

Duabus posterioribus autem satisfit ponendo :

$$\mathfrak{A} = ak; \mathfrak{B} = bk, \text{ et } \mathfrak{C} = \gamma k$$

est enim $a\gamma + a\varepsilon - b\gamma = 0$. Namque cum sit generatiter $\sin. p \sin. q = \frac{1}{2} \cos. (p-q) - \frac{1}{2} \cos. (p+q)$ erit

$$a\gamma = \frac{1}{2} \cos. \frac{2}{7}\varrho - \frac{1}{2} \cos. \frac{6}{7}\varrho = \frac{1}{2} \sin. \frac{5}{7}\varrho + \frac{1}{2} \sin. \frac{1}{7}\varrho$$

$$a\varepsilon = \frac{1}{2} \cos. \frac{2}{7}\varrho - \frac{1}{2} \cos. \frac{6}{7}\varrho = \frac{1}{2} \sin. \frac{5}{7}\varrho - \frac{1}{2} \sin. \frac{1}{7}\varrho$$

$$\mathfrak{C}\gamma = \frac{1}{2} \cos. \frac{2}{7}\varrho - \frac{1}{2} \cos. \frac{10}{7}\varrho = \frac{1}{2} \sin. \frac{5}{7}\varrho + \frac{1}{2} \sin. \frac{3}{7}\varrho$$

ideoque $a\gamma + a\varepsilon - b\gamma = 0$. Tum vero erit $k = \frac{\omega}{2(\alpha\alpha + \beta\beta + \gamma\gamma)}$

§. 43. Si igitur in genere pro casu quoevercunque corporum initio omnia corpora quiescant, ac primum quidem A in distantia ω a situ naturali, reliqua vero cuncta in ipso situ naturali; aequationibus in §. 39. repertis satisfit ponendo; si $\lambda - 1$ indicet numerum corporum:

$$\mathfrak{A} = k \sin. \frac{2}{\lambda}\varrho; \mathfrak{B} = k \sin. \frac{4}{\lambda}\varrho; \mathfrak{C} = k \sin. \frac{6}{\lambda}\varrho;$$

$$\mathfrak{D} = k \sin. \frac{8}{\lambda}\varrho; \dots \dots \dots \mathfrak{D} = k \sin. \frac{2(\lambda-1)}{\lambda}\varrho$$

Sic enim fieri, ut hic inuenimus $\mathfrak{A} = \mathfrak{D}$; $\mathfrak{B} = \mathfrak{B}$; $\mathfrak{C} = \mathfrak{C}$ etc. Tum vero littera k ita definitur ut sit :

$$k = \frac{\omega}{2(\sin. \frac{2}{\lambda}\varrho^2 + \sin. \frac{4}{\lambda}\varrho^2 + \sin. \frac{6}{\lambda}\varrho^2 + \dots + \sin. \frac{2(\lambda-1)}{\lambda}\varrho^2)}$$

Cum

Cum autem sit $2 \sin p^* = 1 - \cos 2p$ erit;
 $k = -\omega; (\lambda - 1 - \cos \frac{4}{\lambda} \xi - \cos \frac{8}{\lambda} \xi - \cos \frac{12}{\lambda} \xi - \dots - \cos \frac{4(\lambda-1)}{\lambda} \xi)$

Ponamus :

$$\begin{aligned} s &= 1 + \cos \frac{4}{\lambda} \xi + \cos \frac{8}{\lambda} \xi + \cos \frac{12}{\lambda} \xi + \dots + \cos \frac{4(\lambda-1)}{\lambda} \xi \\ &\text{erit ob } \sin p \cos q = \frac{1}{2} \sin(p+q) - \frac{1}{2} \sin(q-p) \\ s \sin \frac{2}{\lambda} \xi &= \sin \frac{2}{\lambda} \xi + \frac{1}{2} \sin \frac{6}{\lambda} \xi + \dots + \frac{1}{2} \sin \frac{2(2\lambda-3)}{\lambda} \xi + \frac{1}{2} \sin \frac{2(2\lambda-1)}{\lambda} \xi \\ &\quad - \frac{1}{2} \sin \frac{2}{\lambda} \xi - \frac{1}{2} \sin \frac{6}{\lambda} \xi - \frac{1}{2} \sin \frac{2(2\lambda-3)}{\lambda} \xi \\ \text{ideoque } s \sin \frac{2}{\lambda} \xi &= \frac{1}{2} \sin \frac{2}{\lambda} \xi + \frac{1}{2} \sin \frac{2(2\lambda-1)}{\lambda} \xi = 0 \\ \text{quia est } \sin \frac{2(2\lambda-1)}{\lambda} \xi &= \sin(4\xi - \frac{2}{\lambda} \xi) = -\sin \frac{2}{\lambda} \xi \end{aligned}$$

Hancobrem erit $k = \frac{-\omega}{\lambda}$.

§. 44. Quodsi iam hi valores substituantur, habebitur

$$\begin{aligned} \frac{-\lambda x}{\omega} &= \sin \frac{2}{\lambda} \rho \sin \frac{2}{\lambda} \rho \cos 2nt \sin \frac{\rho}{\lambda} + \sin \frac{4}{\lambda} \rho \sin \frac{4}{\lambda} \rho \cos 2nt \sin \frac{2\rho}{\lambda} + \text{etc.} \\ \frac{-\lambda x^3}{\omega} &= \sin \frac{2}{\lambda} \rho \sin \frac{4}{\lambda} \rho \cos 2nt \sin \frac{\rho}{\lambda} + \sin \frac{4}{\lambda} \rho \sin \frac{6}{\lambda} \rho \cos 2nt \sin \frac{2\rho}{\lambda} + \text{etc.} \\ \frac{-\lambda x^{11}}{\omega} &= \sin \frac{2}{\lambda} \rho \sin \frac{6}{\lambda} \rho \cos 2nt \sin \frac{\rho}{\lambda} + \sin \frac{4}{\lambda} \rho \sin \frac{12}{\lambda} \rho \cos 2nt \sin \frac{2\rho}{\lambda} + \text{etc.} \\ &\vdots \\ &\vdots \\ \frac{-\lambda x^{(-1)^v}}{\omega} &= \sin \frac{2}{\lambda} \rho \sin \frac{2^v}{\lambda} \rho \cos 2nt \sin \frac{\rho}{\lambda} + \sin \frac{4}{\lambda} \rho \sin \frac{2^v}{\lambda} \rho \cos 2nt \sin \frac{2\rho}{\lambda} + \text{etc.} \end{aligned}$$

quae series eo vsque continuari debent, quoad numerus terminorum in vnaquaque fiat $= \lambda - 1$. Hinc ergo uniuscuiusque corporis, cuius index a primo computando sit $= y$ ad quoduis tempus assignari poterit tam situs, quam celeritas.

§. 45. Casus autem ad propagationem pulsuum magis erit accommodatus, si ponamus initio, quo omnia cor-

corpora erant in quiete, vires acceleratrices singulorum practer primum fuisse nullas. Ut igitur superiori modo coefficientes \mathfrak{A} , \mathfrak{B} , \mathfrak{C} , etc. indagemus, ponamus primo esse $\lambda = 2$; et $\sin. \frac{1}{2} \rho = \alpha = \cos. \frac{1}{2} \rho$, erit $\sin. \rho = 2 \alpha \alpha$, sicutque ex acceleratione primi et solius corporis: $2\mathfrak{A}\alpha^2 = \omega$. Ponamus nunc $\lambda = 3$; sitque.

$$\sin. \frac{1}{3} \rho = \cos. \frac{2}{3} \rho = \alpha$$

$$\sin. \frac{2}{3} \rho = \cos. \frac{1}{3} \rho = \beta;$$

$$\text{erit } \mathfrak{A}\alpha^2 \cdot \beta + \mathfrak{B}\beta^2 \cdot \beta = \omega$$

$$\text{et } \mathfrak{A}\alpha^2 \cdot \beta - \mathfrak{B}\beta^2 \cdot \beta = 0$$

sicque patet easdem aequationes ut supra resultare, dummodo ibi pro \mathfrak{A} ponatur $\mathfrak{A} \sin. \frac{\rho^2}{\lambda}$; $\mathfrak{B} \sin. \frac{2\rho^2}{\lambda}$ pro \mathfrak{B} et ita porro. Sic igitur his constantibus mutatis, erit acceleratio singulorum corporum iisdem expressionibus, quas supra pro x , x' , x'' , x''' etc. invenimus proportionalis.

§. 46. Hinc ergo pro $\mathfrak{A} \sin. \frac{\rho^2}{\lambda}$, $\mathfrak{B} \sin. \frac{2\rho^2}{\lambda}$, etc. iidem prodibunt valores, quos supra pro litteris \mathfrak{A} , \mathfrak{B} , \mathfrak{C} etc. inuenimus. Quare elapsi tempore t erit corporis, cuius index in ordine a primo computato, est $= v$, vis acceleratrix huic expressioni proportionalis:

$$\sin. \frac{2}{\lambda} \rho \cdot \sin. \frac{2v}{\lambda} \rho \cos. 2nt \sin. \frac{\rho}{\lambda} + \sin. \frac{4}{\lambda} \rho \cdot \sin. \frac{4v}{\lambda} \rho \cos. 2nt \sin. \frac{2\rho}{\lambda} + \text{etc.}$$

Quodsi ergo acceleratio corporis ultimi quaeratur, facendum est $v = \lambda - 1$; eritque $\sin. \frac{2v}{\lambda} \rho = \sin. \frac{2}{\lambda} \rho$; $\sin. \frac{4v}{\lambda} \rho = - \sin. \frac{4}{\lambda} \rho$; $\sin. \frac{6v}{\lambda} \rho = \sin. \frac{6}{\lambda} \rho$, etc. Vnde acceleratio ultimi corporis erit isti expressioni proportionalis; $\sin. \frac{2}{\lambda} \rho^2 \cos. 2nt \sin. \frac{\rho}{\lambda} - \sin. \frac{4}{\lambda} \rho^2 \cos. 2nt \sin. \frac{2\rho}{\lambda} + \sin. \frac{6}{\lambda} \rho^2 \cos. 2nt \sin. \frac{3\rho}{\lambda} - \text{etc.}$

Tom. I

N

etc.
quae

quae expressio posita $\equiv 0$ ea indicabit temporis momenta, quibus vltimi corporis celeritas est maxima seu quibus pulsus ipsi inesse consendus erit.

§. 47. Si igitur quaeratur, quantum tempus a motus initio fit elapsum, antequam pulsus per totum interuum PQ propagetur, tempus hoc t definiri debet ex hac aequatione :

$$0 = \sin^2 \frac{\rho}{\lambda} \cdot \cos 2nt \sin \frac{\rho}{\lambda} - \sin^4 \frac{\rho}{\lambda} \cdot \cos 2nt \sin^2 \frac{\rho}{\lambda} + \sin^6 \frac{\rho}{\lambda} \cdot \cos 2nt \sin^3 \frac{\rho}{\lambda}$$

$$- \sin^8 \frac{\rho}{\lambda} \cdot \cos 2nt \sin^4 \frac{\rho}{\lambda} + \dots \dots + \sin^{\frac{2(\lambda-1)}{\lambda}} \rho^2 \cdot \cos 2nt \sin^{\frac{(\lambda-1)\rho}{\lambda}}$$

Sit tota longitudo PQ $= f$; et g longitudo columnae, cuius pondus ipsi vi elasticæ huius fluidi aequetur, erit $f = \lambda \cdot a$; $A = a$; ideoque $n = \sqrt{\frac{g}{2ad}} = \frac{1}{a} \sqrt{\frac{1}{2} g} = \frac{\lambda}{f} \sqrt{\frac{1}{2} g}$. Fingatur nunc tempus quae situm $t = m f$; $\sqrt{\frac{1}{2} g} t$: ita vt, si f et g in particulis millesimis pedis rhenani exprimantur, futurum fit tempus $t = \frac{1}{250} m f$; $\sqrt{\frac{1}{2} g} t$ minutis secundis. Totum ergo negotium redit ad determinationem numeri absoluti m , quam ex hac aequatione erui oportet :

$$0 = \sin^2 \frac{\rho}{\lambda} \cdot \cos 2\lambda m \sin \frac{\rho}{\lambda} - \sin^4 \frac{\rho}{\lambda} \cdot \cos 2\lambda m \sin^2 \frac{\rho}{\lambda} + \sin^6 \frac{\rho}{\lambda} \cdot \cos 2\lambda m \sin^3 \frac{\rho}{\lambda}$$

$$- \sin^8 \frac{\rho}{\lambda} \cdot \cos 2\lambda m \sin^4 \frac{\rho}{\lambda} + \dots \dots + \sin^{\frac{2(\lambda-1)}{\lambda}} \rho^2 \cdot \cos 2\lambda m \sin^{\frac{(\lambda-1)\rho}{\lambda}}$$

§. 48. Pendet ergo determinatio numeri m a numero λ seu a numero particularum A, B, C, D, etc. quae in interuum PQ $= f$ continentur; qui numerus cum in fluidis elasticis, cuiusmodi sunt aer et aether censi queat infinite magnus, erit $\lambda = \infty$, et valor numeri m ex aequatione infinita definiri debet. Cum autem arcus, quorum cosinus hic occurunt, sint incomparabiles inter se, patet hanc inuestigationem numeri

m esse difficultiam, neque sine insigni artificio institui posse.

§. 49. Quoniam in aequatione inuenta terminus ultimus sequens $\sin \frac{\lambda}{\lambda} p^2 \cos 2\lambda m \sin \frac{\lambda p}{\lambda}$ per se evanescit, cum adhuc in aequatione adicere poterimus. Quo igitur resolutionem aequationis propositae tentemus, singulos cosinus methodo consueta in series infinitas convertamus, denotetque signum summatorium \int summam huiusmodi seriei ad λ terminos continuatae, ita ut sit $\int \cos v = \cos v - \cos 2v + \cos 3v - 4v + \dots + \cos \lambda v$. Signum scilicet \int primo termino huiusmodi seriei praefixum indicet integrum eiusdem seriei valorem. Facta ergo ante memorata cofinuum resolutione fiet $\circ = f$

$$\sin \frac{2}{\lambda} p^2 - \frac{4\lambda^2 m^2}{1 \cdot 2} \int \sin \frac{2}{\lambda} p^2 \sin \frac{p^2}{\lambda} + \frac{16\lambda^4 m^4}{1 \cdot 2 \cdot 3 \cdot 4} \int \sin \frac{2}{\lambda} p^2 \sin \frac{p^4}{\lambda} - \frac{64\lambda^6 m^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \int \sin \frac{2}{\lambda} p^2 \sin \frac{p^6}{\lambda} + \text{etc.}$$

§. 50. Ut autem has summas definire queamus, ponamus esse λ numerum parem, reperiaturque $\int \cos v = \cos v - \cos 2v + \cos 3v - \cos 4v + \dots - \cos \lambda v = \cos \frac{1}{2} v - \cos (\lambda + \frac{1}{2}) v$, vbi imprimis notari conuenit, esse

casus, quibus haec expressio non veram progressionis summam indicet, qui casus evenerint; quando est $\frac{1}{2} v$, vel p , vel $3p$, vel $5p$, etc. his enim fractionis tam numerator quam denominator evanescit. His igitur casibus vera serie summa reperiatur

$$\frac{\sin \frac{1}{2} v - (2\lambda + 1) \sin (\lambda + \frac{1}{2}) v}{2 \sin \frac{1}{2} v}$$

quae ob $\frac{1}{2} v = p$ et λ numerum parem, dat $\sin (\lambda + \frac{1}{2}) v = \sin \frac{1}{2} v = v$, transit in λ , quod idem contingit si fuerit $\frac{1}{2} v = 3p$, vel $\frac{1}{2} v = 5p$, etc.

§. 51. Ponamus nunc pro v successiue angulos :
 $\frac{2}{\lambda}\rho; \frac{4}{\lambda}\rho; \frac{6}{\lambda}\rho; \frac{8}{\lambda}\rho; \text{ et generaliter } \frac{2\mu}{\lambda}\rho;$ erit facto $v = \frac{2\mu}{\lambda}\rho; \cos(\lambda + \frac{\mu}{2})v = \cos(2\mu\rho + \frac{\mu}{\lambda}\rho) = \pm \cos \frac{\mu}{\lambda}\rho,$
 vbi signorum ambiguorum superius valet, si fit μ numerus par, inferius vero si μ numerus impar: erit ergo
 $\int \cos \frac{2}{\lambda}\rho = \frac{1}{2}(-1),$ vnde sequentes orientur summationes :
 $\int \cos \frac{8}{\lambda}\rho = \int 1 = 0.$

$\int \cos \frac{2}{\lambda}\rho = 1$	excipiuntur casus
$\int \cos \frac{4}{\lambda}\rho = 0$	$\int \cos \frac{2\lambda}{\lambda}\rho = -\lambda$
$\int \cos \frac{6}{\lambda}\rho = 1$	$\int \cos \frac{6\lambda}{\lambda}\rho = -\lambda$
$\int \cos \frac{8}{\lambda}\rho = 0$	$\int \cos \frac{10\lambda}{\lambda}\rho = -\lambda$
$\int \cos \frac{10}{\lambda}\rho = 1$	$\int \cos \frac{14\lambda}{\lambda}\rho = -\lambda$
$\int \cos \frac{12}{\lambda}\rho = 0$	etc.

§. 52. Cum iam sit $\sin \frac{2}{\lambda}\rho^2 = \frac{1}{2}(1 - \cos \frac{4}{\lambda}\rho)$ et
 $\sin \frac{4}{\lambda}\rho^2 = \frac{1}{2}(1 - \cos \frac{8}{\lambda}\rho),$ summae productorum superiorum
 sinuum in sequentes summas cosinuum simplicium conuertentur:

$$\begin{aligned}
 \int \sin \frac{2}{\lambda}\rho^2 &= \frac{1}{2} \int (1 - \cos \frac{4}{\lambda}\rho) \\
 \int \sin \frac{2}{\lambda}\rho^2 \cdot \sin \frac{4}{\lambda}\rho^2 &= \frac{1}{8} \int (2 - \cos \frac{2}{\lambda}\rho - 2\cos \frac{4}{\lambda}\rho + \cos \frac{6}{\lambda}\rho) \\
 \int \sin \frac{2}{\lambda}\rho^2 \cdot \sin \frac{6}{\lambda}\rho^2 &= \frac{1}{32} \int (5 - 4\cos \frac{2}{\lambda}\rho - 4\cos \frac{4}{\lambda}\rho + 4\cos \frac{6}{\lambda}\rho - \cos \frac{8}{\lambda}\rho) \\
 \int \sin \frac{2}{\lambda}\rho^2 \cdot \sin \frac{8}{\lambda}\rho^2 &= \frac{1}{128} \int (14 - 14\cos \frac{2}{\lambda}\rho - 8\cos \frac{4}{\lambda}\rho + 13\cos \frac{6}{\lambda}\rho - 6\cos \frac{8}{\lambda}\rho + \cos \frac{10}{\lambda}\rho) \\
 \int \sin \frac{2}{\lambda}\rho^2 \cdot \sin \frac{10}{\lambda}\rho^2 &= \frac{1}{512} \int (42 - 48\cos \frac{2}{\lambda}\rho - 15\cos \frac{4}{\lambda}\rho + 40\cos \frac{6}{\lambda}\rho - 26\cos \frac{8}{\lambda}\rho + 8\cos \frac{10}{\lambda}\rho - \cos \frac{12}{\lambda}\rho) \\
 \int \sin \frac{2}{\lambda}\rho^2 \cdot \sin \frac{12}{\lambda}\rho^2 &= \frac{1}{2048} \int (132 - 165\cos \frac{2}{\lambda}\rho - 22\cos \frac{4}{\lambda}\rho + 121\cos \frac{6}{\lambda}\rho - 100\cos \frac{8}{\lambda}\rho + 43\cos \frac{10}{\lambda}\rho - 10\cos \frac{12}{\lambda}\rho + \cos \frac{14}{\lambda}\rho)
 \end{aligned}$$

etc.

In

PER MEDIVM ELASTICVM for

In quibus scriebus haec lex obseruatur, vt quisque coefficientis numericus bis summus demta summa coefficientium adiacentium praebeat coefficientem respondentem in serie sequente; in quo computo signa coefficientium non sunt negligenda, ac praeterea termini primi duplo maiores sunt aestimandi, sic est $+ 2 \cdot 40 + 26 + 15 = + 121$, et $- 2 \cdot 48 - 15 - 2 \cdot 42 = - 165$.

§. 53. Omnes hae summae igitur fierent $= 0$, nisi casus ante excepti occurrant, vnde ex his summis soli illi termini relinquuntur, in quibus inest vel cos. 2ρ vel cos. 6ρ , vel cos. 10ρ vel etc. quorum loco poni debet $-\lambda$. Primum autem huiusmodi terminus occurrit in summa $\int \sin \frac{2}{\lambda} \rho^2 \sin \frac{\rho}{\lambda}$; eritque ergo haec summa $\pm \frac{\lambda}{2^{2\lambda-3}}$; sequens autem summa $\int \sin \frac{2}{\lambda} \rho \sin \frac{\rho}{\lambda}$ erit $\mp \frac{(2\lambda-2)\lambda}{2^{2\lambda-4}}$.

Hinc aequatio ita incipiet:

$$0 = \frac{2^{2\lambda-4}\lambda^{2\lambda-4}m^{2\lambda-4}}{1 \cdot 2 \cdot 3 \dots (2\lambda-4)} \cdot \frac{2^{2\lambda-2}\lambda^{2\lambda-2}m^{2\lambda-2}}{1 \cdot 2 \cdot 3 \dots (2\lambda-2)} \cdot \frac{\lambda(2\lambda-2)}{2^{2\lambda-1}} + \text{etc.}$$

$$\text{seu } 0 = 1 - \frac{4\lambda^2 m^2}{(2\lambda-3)(2\lambda-2)} \cdot \frac{2\lambda-2}{4} + \text{etc.}$$

$$\text{seu } 0 = 1 - \frac{\lambda^2 m^2}{2\lambda-3} + \text{etc.}$$

Apparet ergo hanc seriem, si λ statuatur numerus valde magnus, maxime fore diuergentem, ita vt ex ea etiam habetur, vix quicquam concludi queat.

§. 54. Cum igitur hoc modo pro valore numeri m cognoscendo nihil colligere diceat, videamus, cuiusmodi formas aequatio resoluenda §. 47. induat, si loco λ successive substituantur numeri $2, 3, 4, 5$, etc. Ac primo quidem si sit $\lambda = 2$ habebitur haec aequatio:

N 3

○ =

$\circ = \sin. \rho^2 \cos. 4m \sin. \frac{t}{2}$, ergo $\frac{t}{\sqrt{2}} = p$ et $m = \frac{p}{2\sqrt{2}}$, existente $p = \frac{1}{2}\pi = 1$, $57^\circ 79' 63''$. Atque tempus, quo spatium f a pulsu percurretur erit $= \frac{f}{180} \cdot \frac{p}{2\sqrt{g}}$. Sit porro $\lambda = 3$ et orietur haec aequatio :

$$\begin{aligned} \circ &= \sin. \frac{3}{2} \rho^2 \cos. 6m \sin. \frac{1}{2} \rho - \sin. \frac{3}{2} \rho^2 \cos. 6m \sin. \frac{2}{3} \rho \\ &\text{seu } \cos. 3m = \cos. 3m \sqrt{3}. \quad \text{Sit ergo } 3m = 2p + s \text{ et} \\ &3m \sqrt{3} = 2p + s \text{ erit } 3m(1 + \sqrt{3}) = 4p \text{ et } m = \frac{4p}{3(1 + \sqrt{3})} \\ &\text{Ponatur } \lambda = 4 \text{ et prodibit :} \\ \circ &= \sin. \frac{1}{2} \rho^2 \cos. 8m \sin. \frac{1}{4} \rho - \sin. \rho^2 \cos. 8m \sin. \frac{1}{2} \rho + \sin. \frac{1}{2} \rho^2 \cos. 8m \sin. \frac{3}{4} \rho \\ &\text{quae ob } \sin. \rho = 1; \sin. \frac{1}{2} \rho = \frac{1}{2}\sqrt{2}; \sin. \frac{1}{4} \rho = \frac{1}{2}\sqrt{(2-\sqrt{2})} \text{ et} \\ &\sin. \frac{3}{4} \rho = \frac{1}{2}\sqrt{(2+\sqrt{2})}, \text{ transibit in hunc :} \\ \circ &= \cos. 4m\sqrt{(2-\sqrt{2})} - 2 \cos. 4m\sqrt{2} + \cos. 4m\sqrt{(2+\sqrt{2})}. \end{aligned}$$

§. 55. Ascendamus hinc secundum rationem duplam, sitque $\lambda = 8$, erit :

$$\begin{aligned} \circ &= \sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{1}{8} \rho - \sin. \frac{1}{4} \rho^2 \cos. 16m \sin. \frac{1}{4} \rho \\ &+ \sin. \frac{3}{4} \rho^2 \cdot \cos. 16m \sin. \frac{3}{8} \rho - \sin. \rho^2 \cdot \cos. 16m \sin. \frac{1}{2} \rho \\ &+ \sin. \frac{3}{4} \rho^2 \cdot \cos. 16m \sin. \frac{5}{8} \rho - \sin. \frac{1}{2} \rho^2 \cdot \cos. 16m \sin. \frac{7}{8} \rho \\ &- \sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{7}{8} \rho. \end{aligned}$$

quae reducitur ad hanc formam magis ordinatam

$$\begin{aligned} \circ &= -\sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{1}{8} \rho - \sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{1}{4} \rho + \sin. \frac{3}{4} \rho^2 \cdot \cos. 16m \sin. \frac{3}{8} \rho + \sin. \frac{3}{4} \rho^2 \cdot \cos. 16m \sin. \frac{5}{8} \rho \\ &+ \sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{7}{8} \rho - \sin. \frac{1}{4} \rho^2 \cdot \cos. 16m \sin. \frac{1}{2} \rho - \sin. \frac{3}{4} \rho^2 \cdot \cos. 16m \sin. \frac{1}{2} \rho - \sin. \rho^2 \cdot \cos. 16m \sin. \frac{1}{2} \rho. \end{aligned}$$

Simili modo si ponamus $\lambda = 16$, aequatio resultabit, quae sequentem formam induet.

$$\begin{aligned} \sin. \frac{1}{8} \rho^2 \cdot \cos. 32m \sin. \frac{1}{16} \rho - \sin. \frac{1}{8} \rho^2 \cdot \cos. 32m \sin. \frac{1}{8} \rho + \sin. \frac{3}{8} \rho^2 \cdot \cos. 32m \sin. \frac{3}{16} \rho \\ \sin. \frac{1}{8} \rho^2 \cdot \cos. 32m \cos. \frac{1}{16} \rho - \sin. \frac{1}{8} \rho^2 \cdot \cos. 32m \cos. \frac{1}{8} \rho + \sin. \frac{3}{8} \rho^2 \cdot \cos. 32m \cos. \frac{3}{16} \rho \end{aligned}$$

$$\begin{aligned}
 & -\sin. \frac{1}{4} \rho^2 \cdot \cos. 32m \sin. \frac{1}{4} \rho + \sin. \frac{5}{8} \rho^2 \cdot \cos. 32m \sin. \frac{5}{16} \rho - \sin. \frac{3}{4} \rho^2 \cdot \cos. 32m \sin. \frac{3}{8} \rho \\
 & -\sin. \frac{1}{2} \rho^2 \cdot \cos. 32m \cos. \frac{1}{4} \rho + \sin. \frac{5}{8} \rho^2 \cdot \cos. 32m \cos. \frac{5}{16} \rho - \sin. \frac{3}{4} \rho^2 \cdot \cos. 32m \cos. \frac{3}{8} \rho \\
 & + \sin. \frac{7}{8} \rho^2 \cdot \cos. 32m \sin. \frac{7}{16} \rho - \sin. \rho^2 \cdot \cos. 32m \sin. \frac{1}{2} \rho = 0 \\
 & + \sin. \frac{7}{8} \rho^2 \cdot \cos. 32m \cos. \frac{7}{16} \rho
 \end{aligned}$$

§. 56. Huiusmodi ergo aequatio formari debebit in qua sit λ numerus infinitus seu $\lambda = 2^{10}$, ab eiusque resolutione pendebit valor numeri m . Inuentio igitur numeri m accurata, quo celeritas propagationis pulsuum per quodvis medium elasticum definitur, maxime est ardua, neque sine insigni amplificatione doctrinae serierum expectari potest. Interm tamen methodus, qua Celeb. Newtonus ad propagationem pulsuum inuestigandam usus est, non parum est elegans, et pro idonea approximatione haberi potest, quamuis a rigore geometrico valde abhorreat. Per experientiam autem verus valor ipsius m satis prope cognosci poterit. Cum enim in aere sit $g = 27980,000$ ped. Rhen. sonusque uno minuto secundo per intervalum 1100 ped. propagetur, hinc proxime reperietur $m = \frac{\sqrt[5]{14}}{22} = 0,8504$, neque multum differt a sinu anguli 60° . Ad hunc autem valorem satis celeriter conuergere videntur valores ipsius m pro casibus $\lambda = 2$ et $\lambda = 3$ inuenti, ex quorum priori prodit $m = 0,554$, ex posteriori vero $m = 0,766$, unde iam tuto colligere licet esse $m > 0,766$ id quod per experientiam mirifice comprobatur.

§. 57. Quanquam autem hinc verum valorem litterae m elicere non valemus, tamen modum, quo pulsus per medium elasticum propagantur, satis clare perspicimus

mus. Primum enim, cum tempus, quo pulsus per interuallum $=f$ propagatur, inuentum sit $= \frac{mf}{250V\frac{1}{2}g}$ videmus in eodem medio tempus ipsi spatio esse proportionale, sicutque pulsus motu uniformi propagari vti experientia testatur. Deinde celeritas istius motus, quo pulsus progrediuntur, erit vt $\frac{f}{t}$ hoc est vt Vg . Est vero g longitudo columnae eiusdem fluidi, cuius pondus ipsius vi elasticae aequatur. Vnde si vis elastica designetur per E et densitas per D, erit pondus columnae g vt Dg , et cum sit E vt Dg , erit g vt $\frac{E}{D}$. Quare in diuersis fluidis elasticis erunt celeritates, quibus pulsus per ea propagantur in ratione subduplicata composita ex directa elasticitatibus et inuersa densitatum, seu vt $V\frac{E}{D}$.

§. 58. Haec autem aliunde iam satis constant, atque a Newtono firmiter sunt demonstrata: quoniam ad hoc non est opus, vt ipsa singularum particularum fluidi elastici agitatio sit perspecta. Ex hactenus allatis autem simul modum, quo singulae fluidi elastici particulae, dum ipsi in uno loco impulsus infligitur, singulis momentis agitantur. Vidimus scilicet, si unica particula intra parietes P et Q constituatur, eius motum ab impulsu acceptum similem fore motui oscillatorio penduli, atque ideo perinde vibrationes peragere, ac cordam impulsum. Cum autem duo plurae corpuscula intra parietes P et Q collocata concipiuntur, quorum unum duntaxat impellatur, tum nullum corpusculum ad similitudinem penduli amplius agitatur, sed singulorum motus ab hac

hac lege eo magis recedent, quo maior fuerit eorum numerus. Ex quo intelligimus sonum neutquam eo modo, quo nonnulli eximii Viri volunt, per aerem propagari, qui statuunt, cum corda vel aliud instrumentum sonorum impellitur, dari in aere eiusmodi particulas, quae similem motum oscillatorium recipiant, eoque organum auditus excitent. Quae sententia, cum pluribus aliis incommodis laboret, ut in tractatu meo de lumine et coloribus ostendi, nunc etiam nequidem cum vera theoria pulsuum per medium elasticum propagatorum consistere potest: atque hinc eo magis corroboratur ea propagationis pulsuum ratio, quam in eodem scripto fusi exponui.

RECORDED AND INDEXED
BY THE LIBRARY STAFF
OF THE UNIVERSITY OF TORONTO
ON APRIL 22, 1947
BY J. R. H.
RECORDED AND INDEXED
BY THE LIBRARY STAFF
OF THE UNIVERSITY OF TORONTO
ON APRIL 22, 1947
BY J. R. H.

Tom. I.

1

EXAMEN