E286

DE MOTV VIBRATORIO FILI FLEXILIS, CORPVSCVLIS QVOTCVNQVE ONVSTI,

Auctore

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1.

Considero hic filum perfecte flexile, simulque omni inertia destitutum, quod in datis interuallis sit oneratum ponduscuIis quibuscunque A, B, C, D, etc. concipio autem filum hoc in terminis I et O firmiter fixum, et extensum data quadam vi; ita vt in statu naturali situm teneat rectilineum IO, in quo acquiescat. Quodsi autem a causa quacunque de hoc situ deturbetur, ita vt singula ponduscula A, B, C, D etc. ad datas distantias a recto IO depellantur, subitoque dimittantur, totum filum certo quodam motu agitabitur, quem hic inuestigare constitui. Ne autem solutio huius quaestionis vires analyseos penitus superet, tam ponduscula, quam interualla, ad quae a recta IO fuerint depulsa, tanquam infinite parua spectabo, vnde hoc commodum sim assecuturus, vt viae, quas singula ponduscula motu suo percurrent, sint rectae ad IO normales, ac tensio in omnibus fili partibus mansura sit perpetuo eadem. Tab. I. Fig. 1.

2.

2. Facta hac hypothesi, interualla ponduſculorum etiam durante motu nullam mutationem recipient, quae cum ſint data et conſtantia vocentur:

$IA = a$; $AB = b$; $BC = c$; $CD = d$; $DE = e$; $EF = f$ etc. eritque etiam

$IP = a$; $PQ = b$; $QR = c$; $RS = d$; $ST = e$; $TV = f$; etc. maiusculae autem litterae A, B, C, D, etc. ipſas maſſas ſingulorum corpuſculorum exprimant. Deinde ſit vis, qua filum tenditur $= K$; et graecae litterae α, β, γ, δ, etc. denotent interualla, ad quae initio ſingula corpuscula A, B, C, D etc. a linea recta IO fuerint diducta. Quibus poſitis, quaeſtio huc redit, vt elapſo ab iſto initio tempore quocunque, quod ſit ω min. ſec. ſtatus et motus fili determinetur.

3. Ponamus ergo, hoc tempore filum cum corpuſculis in eum ſitum perueniſſe, quem figura oſtendit; et deſignemus iam ſingulorum corpuſculorum ab axe IO diſtantias:

$AP = p$; $BQ = q$; $CR = r$; $DS = s$; $ET = t$; etc. quas prae interuallis a, b, c, d, tanquam minimas ſpectare licebit. Cum igitur tenſio in ſingulis fili partibus ſit eadem $= K$, quodlibet corpuſculum a tanta vi vtrinque ſollicitabitur, et quatenus hae vires ſibi non ſunt e diametro oppoſitae, eatenus inde vis naſcetur, qua vnumquodque corpuſculum recta ad axem pelletur, vel ab eo repelletur. In has ergo ſingulas vires ante omnia erit inquirendum, quoniam ab iis corpuſcula motus ſui determinationem, hoc eſt, ſiue accele-

accelerationem, siue retardationem, nanciscuntur, quandoquidem per hypothesin certum est, singula corpuscula A, B, C, D, etc. perpetuo per rectas A P, B Q, C R etc. ad axem normales agitari.

4. Si igitur secundum regulas cognitas has vires colligamus, deprehendemus:

Corpusculum	Vrgeri in directione	Vi
A	A P	$K\left(\frac{p}{a}+\frac{p-q}{b}\right)$
B	B Q	$K\left(\frac{q-p}{b}+\frac{q-r}{c}\right)$
C	C R	$K\left(\frac{r-q}{c}+\frac{r-s}{d}\right)$
D	D S	$K\left(\frac{s-r}{d}+\frac{s-t}{e}\right)$
E	E T	$K\left(\frac{t-s}{e}+\frac{t-v}{f}\right)$
F	F V	$K\left(\frac{v-t}{f}+\frac{v-x}{g}\right)$
G	G X	$K\left(\frac{v-x}{g}+\frac{x}{h}\right)$.

Hic scilicet posui, corpusculum septimum G esse vltimum; manifestum autem est, quotcunque fuerint ponduscula, quomodo has formulas construi oporteat.

5. Exprimunt autem hae formulae vires motrices, quibus singula corpuscula axem I O versus incitantur; earum ergo quaelibet per massam ponduscli diuisa praebebit accelerationem eius. Verum ex distantia cuiusque corpusculi ab axe, quae in genere sit $=z$, cum tempore generatim expresso t collata, oritur quoque per regulas mechanicas acceleratio $=-\frac{2ddz}{dt^2}$, sumto elemento temporis constante. Sed haec formula non est ad mensuram temporis in minutis secundis expri-

mendi, quam hic aſſumſimus, accommodata; ſed petita eſt ex ea ratione, qua tempus per ſpatium ad celeritatem applicatum, celeritas autem per radicem quadratam altitudinis debitae, exhiberi ſolet. Quare ſi k denotet altitudinem, ex qua graue vno minuto ſecundo libere deſcendit, referet expreſſio $2\sqrt{k}$ vnum minutum ſecundum, eritque propterea $t : \omega = 2\sqrt{k} : 1$, ſicque $t = 2\omega\sqrt{k}$ et $dt^2 = 4kd\omega^2$, vnde acceleratio ad noſtrum ſcopum accommodata prodit $= \frac{-ddz}{2kd\omega^2}$.

6. Quodſi iam has ſingulas accelerationes cum iis, quae ex ſollicitationibus ſunt erutae, conferamus, obtinebimus ſequentes aequationes: ſiue

$$
\begin{array}{l|l}
\frac{K}{A}\left(\frac{p}{a} + \frac{p-q}{b}\right) = \frac{-ddp}{2kd\omega^2} & \frac{p}{a} + \frac{p-q}{b} + \frac{A\,ddp}{2Kkd\omega^2} = 0 \\
\frac{K}{B}\left(\frac{q-p}{b} + \frac{q-r}{c}\right) = \frac{-ddq}{2kd\omega^2} & \frac{q-p}{b} + \frac{q-r}{c} + \frac{B\,ddq}{2Kkd\omega^2} = 0 \\
\frac{K}{C}\left(\frac{r-q}{c} + \frac{r-s}{d}\right) = \frac{-ddr}{2kd\omega^2} & \frac{r-q}{c} + \frac{r-s}{d} + \frac{C\,ddr}{2Kkd\omega^2} = 0 \\
\frac{K}{D}\left(\frac{s-r}{d} + \frac{s-t}{e}\right) = \frac{-dds}{2kd\omega^2} & \frac{s-r}{d} + \frac{s-t}{e} + \frac{D\,dds}{2Kkd\omega^2} = 0 \\
\frac{K}{E}\left(\frac{t-s}{e} + \frac{t-v}{f}\right) = \frac{-ddt}{2kd\omega^2} & \frac{t-s}{e} + \frac{t-v}{f} + \frac{E\,ddt}{2Kkd\omega^2} = 0 \\
\frac{K}{F}\left(\frac{v-t}{f} + \frac{v-x}{g}\right) = \frac{-ddv}{2kd\omega^2} & \frac{v-t}{f} + \frac{v-x}{g} + \frac{F\,ddv}{2Kkd\omega^2} = 0 \\
\frac{K}{G}\left(\frac{x-v}{g} + \frac{x}{b}\right) = \frac{-ddx}{2kd\omega^2} & \frac{x-v}{g} + \frac{x}{b} + \frac{G\,ddx}{2Kkd\omega^2} = 0.
\end{array}
$$

6. Totidem igitur quouis caſu impetramus huiusmodi aequationes differentio-differentiales, quod ponduſculis filum intra terminos I et O fuerit oneratum, quarum reſolutio, ob variabilium permixtionem, ſummopere difficilis primo intuitu videatur. Quoniam vero in omnibus his aequationibus variabiles vnicam tantum dimenſionem obtinent, manifeſtum eſt, ſingulas iſtas aequa-

aequationes per eiusmodi constantes multiplicari posse, vt si omnes in vnam summam colligantur, prodeat huiusmodi aequatio:

$$\mathfrak{A}p + \mathfrak{B}q + \mathfrak{C}r + \mathfrak{D}s + \mathfrak{E}t + \mathfrak{F}v + \mathfrak{G}x + \frac{n}{d\omega^2}(\mathfrak{A}ddp + \mathfrak{B}ddq + \mathfrak{C}ddr + \mathfrak{D}dds + \mathfrak{E}ddt + \mathfrak{F}ddv + \mathfrak{G}ddx) = 0,$$

cuius integratio iam nulli amplius difficultati est obnoxia, cum sit:

$$\mathfrak{A}p + \mathfrak{B}q + \mathfrak{C}r + \mathfrak{D}s + \mathfrak{E}t + \mathfrak{F}v + \mathfrak{G}x = \text{Const.} \cos. \omega n.$$

8. At si hos multiplicatores, qui ad huiusmodi aequationem integrabilem perducant, inuestigemus, eos non vno modo, sed adeo semper tot modis, quot fuerint corpuscula, definiri deprehendemus; sicque tandem etiam totidem aequationes integrales diuersas adipiscemur. Ex tot autem aequationibus deinceps valores singularum applicatarum p, q, r, s etc. elicere poterimus, quorum quilibet huiusmodi formam sortietur:

$$\mathfrak{A} \cos \mathfrak{a}\omega + \mathfrak{B} \cos \mathfrak{b}\omega + \mathfrak{C} \cos. \mathfrak{c}\omega + \mathfrak{D} \cos. \mathfrak{d}\omega + \text{etc.}$$

vbi $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc. sunt constantes arbitrariae, ex statu fili initiali, quando ponitur tempus $\omega = 0$, definiendae, et pro singulis applicatis p, q, r etc. peculiares obtinebunt valores. At vero litterae $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, etc. in omnibus erunt eaedem, ac per totidem radices aequationis cuiuspiam tot dimensionum, quot fuerint pondufcula, exhibebuntur.

9 Hinc aliam eumque multo faciliorem nanciscimur methodum, cunctas superiores aequationes differentia-

rentiales secundi gradus quasi vno actu resoluendi. Cum enim, si omnes coefficientes $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, etc. praeter vnum in vna forma integrali euanescant, iidem in reliquis omnibus euanescere debeant, statuamus statim mutata harum litterarum significatione:

$$p = \mathfrak{A} \operatorname{cos.} n\omega; \quad q = \mathfrak{B} \operatorname{cos.} n\omega; \quad r = \mathfrak{C} \operatorname{cos.} n\omega; \quad s = \mathfrak{D} \operatorname{cos.} n\omega; \text{ etc.}$$

quibus valoribus substitutis, non solum relatio inter hos coefficientes $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, etc. determinabitur, sed etiam valor litterae n per aequationem tot dimensionum, quot fuerint corpuscula, definietur, vnde etiam totidem valores diuersos recipiet. His autem inuentis singulae expressiones completae reddentur, et huiusmodi formas induent:

$$p = \mathfrak{A} \operatorname{cos.} n\omega + \mathfrak{A}' \operatorname{cos.} n'\omega + \mathfrak{A}'' \operatorname{cos.} n''\omega + \mathfrak{A}''' \operatorname{cos.} n'''\omega + \text{ect.}$$

$$q = \mathfrak{B} \operatorname{cos.} n\omega + \mathfrak{B}' \operatorname{cos.} n'\omega + \mathfrak{B}'' \operatorname{cos.} n''\omega + \mathfrak{B}''' \operatorname{cos.} n'''\omega + \text{etc.}$$

etc.

10. Pro quouis enim alio valore litterae n, alios quoque valores litterae $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, etc. sortientur, qui si debito modo in has aequationes introducantur, obtinebimus valores integrales completos pro singulis applicatis p, q, r, s, etc. qui propterea ad quoduis tempus statum fili praebebunt, ex cuius variatione instantanea simul eius motus innotescet. Praeterea vero totidem adhuc manebunt coefficientes arbitrarii, quot fuerint corpuscula, quos denique ita definire licebit, vt initio $\omega = 0$ distantiae singulorum corpusculorum

lorum

lorum ab axe sint, statui, filo inducto consentaneae: tum vero hae formulae iam ita sunt comparatae, vt initio motus singulorum corpusculorum euanescat; seu motus tum a quiete incipiat; alioquin enim etiam sinus angulorum $n\omega$, $n'\omega$ etc. introduci potuissent. Exposita autem methodo solutionis in genere, conueniet eam pro quolibet corpusculorum numero accuratius euolui.

Problema I.

11. Si filum in terminis I et O fixum, et a data vi $= K$ tensum, vnico corpusculo A sit oneratum, determinare eius motum, postquam de statu naturali vtcunque fuerit deturbatum. Fig. 2.

Solutio.

Cum igitur sit $IA = IP = a$; $AO = PO = b$; et elapso tempore ω min. sec. ponatur distantia $PA = p$, quae initio fuerat $= \alpha$; habebimus hanc vnicam aequationem differentio-differentialem:

$$\frac{p}{a} + \frac{p}{b} + \frac{A\,ddp}{2Kk\,d\omega^2} = 0$$

Statuamus ergo $p = \mathfrak{A}\,\text{cof.}\,n\omega$, eritque $\frac{ddp}{d\omega^2} = -nn\mathfrak{A}\,\text{cof.}\,np$ vnde fit $\frac{1}{a} + \frac{1}{b} = \frac{Ann}{2Kk}$, et $n = \sqrt{\frac{2Kk}{A}\left(\frac{1}{a} + \frac{1}{b}\right)}$. Quare aequatio integralis quaesita habebitur:

$$p = \mathfrak{A}\,\text{cof.}\,\omega\sqrt{\frac{2Kk}{A}\left(\frac{1}{a} + \frac{1}{b}\right)}$$

quae vt praebeat $p = \alpha$, posito $\omega = 0$, poni oportet $\mathfrak{A} = \alpha$, sicque erit pro casu proposito:

$$p = \alpha\,\text{cof.}\,\omega\sqrt{\frac{2Kk}{A}\left(\frac{1}{a} + \frac{1}{b}\right)}$$

 vnde

vnde ad quoduis tempus ω min. ſec. ab initio elapſum locus corpuſculi A cognoſcitur.

Coroll. 1.

12. Hinc ſtatim innoteſcit tempus, quo corpuſculum A ab initio motus primum in rectam I O perueniet, quod eueniet, quando fit $p = 0 = \alpha \operatorname{coſ.} \frac{1}{2}\pi$, denotante π ſemiperipheriam circuli, cuius radius eſt 1, vt $\frac{1}{2}\pi$ ſit menſura anguli recti: erit ergo hoc tempus:

$$= \frac{\pi}{2\sqrt{\frac{2Kk}{A}\left(\frac{1}{a}+\frac{1}{b}\right)}} \text{ min. ſec.}$$

quod ſimul eſt tempus dimidiae vibrationis fili.

Coroll. 2.

13. Si enim tempus capiatur duplo maius, corpus A perueniet ad parem diſtantiam α ab axe in altera parte, integramque vibrationem confeciſſe eſt cenſendum. Quare tempus ſingularum vibrationum, quae inter ſe erunt iſochronae, erit:

$$\frac{\pi}{\sqrt{\left(\frac{2Kk}{A}\left(\frac{1}{a}+\frac{1}{b}\right)\right)}} = \frac{\pi\sqrt{Aab}}{\sqrt{2Kk(a+b)}} \text{ min. ſecund.}$$

Coroll. 3.

14. Hoc ergo tempus erit maximum, ſi corpuſculum A medium locum in filo I O tenuerit. Si enim ponamus totam longitudinem $IO = a + b = l$, et $a = \frac{l+u}{2}$; $b = \frac{l-u}{2}$, erit tempus vibrationis $= \frac{\pi\sqrt{A(ll-uu)}}{2\sqrt{2Kkl}}$, vnde

unde patet, quo magis corpusculum a fili puncto medio remoueatur, eo rapidiores fore vibrationes; ipsum autem tempus maximum, quo $u=0$ fit $=\frac{\pi}{1}\sqrt{\frac{Al}{2Kk}}$ min. secund.

Problema 2.

15. Si filum in terminis I et O fixum, et data vi K tensum, duobus ponduscculis A et B fuerit oneratum, determinare eius motum, postquam de statu suo naturali recto IO vtcunque fuerit deturbatum. Fig. 3.

Solutio.

Hic igitur habemus $IA=a$; $AB=b$; $BO=c$, et si elapso tempore ω min. secund. ponamus distantias $PA=p$, et $QB=q$, quae initio fuerant α et β, sequentes duae aequationes differentio-differentiales resoluendae occurrunt:

$$\frac{p}{a}+\frac{p-q}{b}+\frac{a\,ddp}{2Kk\,d\omega^2}=0$$

$$\frac{q-p}{b}+\frac{q}{c}+\frac{B\,ddq}{2Kk\,d\omega^2}=0$$

Statuamus ergo:

$$p=\mathfrak{A}\,\text{cos}.\,n\omega \quad \text{et} \quad q=\mathfrak{B}\,\text{cos}.\,n\omega, \text{ eritque}$$

$$\frac{ddp}{d\omega^2}=-nn\mathfrak{A}\,\text{cos}.\,n\omega \quad \text{et} \quad \frac{ddq}{d\omega^2}=-nn\mathfrak{B}\,\text{cos}.\,n\omega.$$

Hi valores substituti praebent:

$$\frac{\mathfrak{A}}{a}+\frac{\mathfrak{A}-\mathfrak{B}}{b}=\frac{nnA\mathfrak{A}}{2Kk} \quad \text{et} \quad \frac{\mathfrak{B}-\mathfrak{A}}{b}+\frac{\mathfrak{B}}{c}=\frac{nnB\mathfrak{B}}{2Kk}$$

ideoque

$$\frac{\mathfrak{B}}{\mathfrak{A}}=1+\frac{b}{a}-\frac{nnAb}{2Kk} \quad \text{et} \quad \frac{\mathfrak{A}}{\mathfrak{B}}=1+\frac{b}{c}-\frac{nnBb}{2Kk}$$

unde

vnde per multiplicationem oritur, ponendo $\frac{nn}{2Kk} = z$:

$$1 = (1 + \frac{b}{a} - Abz)(1 + \frac{b}{c} - Bbz) \text{ seu}$$

$$0 = \frac{1}{a} + \frac{1}{c} + \frac{b}{ac} - A\,z)\,(1 + \frac{b}{c}) - Bz(1 + \frac{b}{a}) + ABbzz,$$

quae reducitur ad hanc formam:

$$zz - \frac{z}{A}(\frac{1}{a} + \frac{1}{b}) - \frac{z}{B}(\frac{1}{b} + \frac{1}{c}) + \frac{1}{AB}(\frac{1}{ab} + \frac{1}{ac} + \frac{1}{bc}) = 0,$$

vnde elicitur:

$$z = \frac{1}{2A}(\frac{1}{a} + \frac{1}{b}) + \frac{1}{2B}(\frac{1}{b} + \frac{1}{c})) \pm$$

$$\sqrt{(\frac{1}{4AA})\frac{1}{a} + \frac{1}{b})^2 + \frac{1}{4BB}(\frac{1}{b} + \frac{1}{c})^2 + \frac{1}{2AB}(\frac{1}{bb} - \frac{1}{ab} - \frac{1}{ac} - \frac{1}{bc})},$$

quo valore inuento, est $n = \sqrt{2Kkz}$. Tum vero habebitur

$$\frac{\mathfrak{B}}{\mathfrak{A}} = \frac{1}{2} + \frac{b}{2a} - \frac{Ab}{2B}(\frac{1}{b} + \frac{1}{c}) \mp$$

$$Ab\sqrt{(\frac{1}{4AA}(\frac{1}{a} + \frac{1}{b})^2 + \frac{1}{4BB}(\frac{1}{b} + \frac{1}{c})^2 - \frac{1}{2Ab}(\frac{1}{ab} + \frac{1}{ac} + \frac{1}{bc} - \frac{1}{bb}))}.$$

Cum igitur hoc modo duo inueniantur valores ipsius n, qui sint n et n', et pro vtroque relatio inter $\mathfrak{A}$ et $\mathfrak{B}$ definiatur, vnde prodeant valores $\mathfrak{A}$, $\mathfrak{B}$ et $\mathfrak{A}'$, $\mathfrak{B}'$, obtinebimus hinc sequentes valores completos pro applicatis p et q:

$$p = \mathfrak{A}\,\text{cos.}\,n\omega + \mathfrak{A}'\,\text{cos.}\,n'\omega$$

$$q = \mathfrak{B}\,\text{cos.}\,n\omega + \mathfrak{B}'\,\text{cos.}\,n'\omega$$

ob statum autem initialem esse oportet:

$$\mathfrak{A} + \mathfrak{A}' = \alpha, \text{ et } \mathfrak{B} + \mathfrak{B}' = \beta$$

alter autem tam valorum $\mathfrak{A}$ et $\mathfrak{B}$, quam $\mathfrak{A}'$ et $\mathfrak{B}'$, erat indefinitus, vnde ii hinc determinabuntur.

Coroll. 1.

Coroll. 1.

16. Si ponamus breuitatis gratia:

$\frac{1}{2A}(\frac{1}{a}+\frac{1}{b})=P$; $\frac{1}{2B}(\frac{1}{b}+\frac{1}{c})=Q$; $\frac{1}{AB}(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc})=R$ vt fit $zz-2(P+Q)z+R=0$; et geminus ipsius z valor:

$$z=P+Q\pm\sqrt{((P+Q)^2-R)},$$

erit $\mathfrak{B}=\mathfrak{A}Ab(2P-z)$ seu $\mathfrak{A}=\mathfrak{B}Bb(2Q-z)$.

Coroll. 2.

17. Verum cum fit $4PQ=\frac{1}{AB}(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}+\frac{1}{bb})$ $=R+\frac{1}{ABbb}$, ideoque $R=4PQ-\frac{1}{ABbb}$, habebitur:

$$z=P+Q\pm\sqrt{((P-Q)^2+\frac{1}{ABbb})},$$

ex qua forma patet, ambos valores ipsius z semper esse reales, ex priori autem esse positiuos; vnde vterque valor ipsius $n=\sqrt{2Kkz}$ erit realis.

Coroll. 3.

18. Ponamus porro ad abbreuiandum $\sqrt{((P-Q)^2+\frac{1}{ABbb})}=S$, ac distinguendo geminos valores, obtinebimus:

$z=P+Q+S$; $\quad z'=P+Q-S$

$n=\sqrt{2Kk(P+Q+S)}$; $\quad n'=\sqrt{2Kk(P+Q-S)}$

$\mathfrak{B}=\mathfrak{A}Ab(P-Q-S)$; $\quad \mathfrak{B}'=\mathfrak{A}'Ab(P-Q+S)$

ac motus fili his duabus aequationibus continebitur:

$$p=\mathfrak{A}\cos.n\omega+\mathfrak{A}'\cos.n'\omega$$
$$q=\mathfrak{B}\cos.n\omega+\mathfrak{B}'\cos.n'\omega.$$

Coroll. 4.

19. Vt autem motus ad ſtatum initialem datum accommodetur, fieri debet $\mathfrak{A}+\mathfrak{A}'=\alpha$ et $\alpha Ab(P-Q)+(\mathfrak{A}'-\mathfrak{A})AbS=\beta$, vnde erit $\mathfrak{A}'-\mathfrak{A}=\frac{\beta}{AbS}-\frac{\alpha(P-Q)}{S}$. Quare hinc nanciſcimur vtriusque conſtantis $\mathfrak{A}$ et $\mathfrak{A}'$ determinationem:

$\mathfrak{A}=\frac{1}{2}\alpha+\frac{\alpha(P-Q)}{2S}-\frac{\beta}{2AbS}$ et $\mathfrak{A}'=\frac{1}{2}\alpha-\frac{\alpha(P-Q)}{2S}+\frac{\beta}{2AbS}$, item

$\mathfrak{B}=\frac{1}{2}\beta-\frac{\beta(P-Q)}{2S}-\frac{\alpha}{2BbS}$ et $\mathfrak{B}'=\frac{1}{2}\beta-\frac{\beta(P-Q)}{2S}+\frac{\alpha}{2BbS}$.

Coroll. 5.

20. Si ſtatus initialis ita fuerit comparatus, vt vel $\mathfrak{A}$ vel $\mathfrak{A}'$ fuerit $=0$, tum etiam vel $\mathfrak{B}$ vel $\mathfrak{B}'$ euaneſcet, motusque continebitur

vel in his formulis:	vel in his formulis:
$p=\mathfrak{A}\,\text{coſ.}\,n\omega$	$p=\mathfrak{A}'\,\text{coſ.}\,n'\omega$
$q=\mathfrak{B}\,\text{coſ.}\,n\omega$	$q=\mathfrak{B}'\,\text{coſ.}\,n'\omega$

vtroque ergo caſu vibrationes orientur regulares oſcillationibus penduli ſimplicis conformes, ac tempus vnius vibrationis erit,

Caſu priori $=\frac{\pi}{n}$ min. ſecund.; poſteriori $=\frac{\pi}{n'}$ min. ſec.

Coroll. 6.

21. Sin autem ſtatus initialis fuerit eiusmodi, vt neque $\mathfrak{A}$ neque $\mathfrak{A}'$ euaneſcat, vibrationes orientur irregulares, et quaſi ex vtroque genere ſimplici mixtae; neque filum vnquam ad eundem ſitum reuertetur, niſi numeri n et n' rationem inter ſe teneant rationa-

tionalem. Ponatur huiusmodi ratio $n:n' = \mu:\nu$, ac fiet $\frac{P+Q+S}{P+Q-S} = \frac{\mu\mu}{\nu\nu}$; seu $(\mu\mu - \nu\nu)(P+Q) = (\mu\mu + \nu\nu)S$, ideoque $4(\mu^4 + \nu^4)PQ - 4\mu\mu\nu\nu(PP+QQ) = \frac{(\mu\mu+\nu\nu)^2}{ABbb}$, hincque

$$P = \frac{\mu^4+\nu^4}{2\mu\mu\nu\nu}Q \pm \frac{(\mu\mu+\nu\nu)}{2\mu\mu\nu\nu}\sqrt{\left((\mu\mu-\nu\nu)^2 QQ - \frac{\mu\mu\nu\nu}{ABbb}\right)}.$$

Coroll. 7.

22. Vt ergo vibrationes euadant regulares, status initialis ita debet esse comparatus, vt sit

vel $\frac{\alpha}{\beta} = \frac{-1}{Ab(S-P+Q)} = -Bb(S+P-Q)$

vel $\frac{\alpha}{\beta} = \frac{1}{Ab(S+P-Q)} = +Bb(S-P-Q)$

est enim $SS-(P-Q)^2 = \frac{1}{ABbb}$, ideoque $S > (P-Q)$.

Coroll. 8.

23. Si omnia interualla corpusculorum fuerint inter se aequalia, seu $a=b=c$; erit $P = \frac{1}{Aa}$, $Q = \frac{1}{Ba}$, et $R = \frac{3}{ABaa}$, vnde fit $z = \frac{1}{a}\left(\frac{1}{A}+\frac{1}{B}\right) \pm \frac{1}{a}\sqrt{\left(\frac{1}{AA} - \frac{1}{AB} + \frac{1}{BB}\right)}$, seu $z = \frac{A+B \pm \sqrt{(AA-AB+BB)}}{ABa}$, hincque

$$n = \sqrt{\frac{2Kk}{ABa}(A+B+\sqrt{(AA-AB+BB)})};\quad n' = \sqrt{\frac{2Kk}{ABa}(A+B-\sqrt{(AA-AB+BB)})}$$

$$\mathfrak{B} = \mathfrak{A}.\frac{B-A-\sqrt{(AA-AB+BB)}}{B};\quad \mathfrak{B}' = \mathfrak{A}'.\frac{B-A+\sqrt{(AA-AB+BB)}}{B}$$

et pro statu initiali adimplendo

$$\mathfrak{A} = \tfrac{1}{2}\alpha + \frac{\alpha(B-A)-\beta B}{2\sqrt{(AA-AB+BB)}};\quad \mathfrak{A}' = \tfrac{1}{2}\alpha - \frac{\alpha(B-A)+\beta B}{2\sqrt{(AA-AB+BB)}}$$

$$\mathfrak{B} = \tfrac{1}{2}\beta - \frac{\beta(B-A)-\alpha A}{2\sqrt{(AA-AB+BB)}};\quad \mathfrak{B}' = \tfrac{1}{2}\beta - \frac{\beta(B-A)+\alpha A}{2\sqrt{(AA-AB+BB)}}.$$

Motus vero his aequationibus exprimetur:

$$p = \mathfrak{A}\,\text{cof.}\,\omega + \mathfrak{A}'\,\text{cof.}\,n'\omega$$
$$q = \mathfrak{B}\,\text{cof.}\,\omega + \mathfrak{B}'\,\text{cof.}\,n'\omega$$

Coroll. 9.

24. Vt fiat hoc cafu $n:n' = \mu:\nu$, oportet effe:

$$\frac{4(\mu^4+\nu^4)}{AB} - 4\mu\mu\nu\nu\left(\frac{1}{AA}+\frac{1}{BB}\right) = \frac{(\mu\mu+\nu\nu)^2}{AB}, \text{ feu}$$
$$\frac{B}{A} = \frac{3\mu^4 - 2\mu\mu\nu\nu + 3\nu^4 \pm \sqrt{(9\mu^8 - 12\mu^6\nu\nu - 42\mu^4\nu^4 - 12\mu\mu\nu^6 + 9\nu^8)}}{8\mu\mu\nu\nu}$$

vnde, fi fit $\mu = 2$, et $\nu = 1$, feu $n:n' = 2:1$, fiet

$$\frac{B}{A} = \frac{43 \pm \sqrt{1825}}{32} = \frac{43 \pm 5\sqrt{73}}{32}.$$

Coroll. 10.

25. Si praeterea ambo corpora A et B fint aequalia, erit $z = \frac{2\pm 1}{A\alpha}$, vnde fequentes obtinentur determinationes:

$$n = \sqrt{\tfrac{6Kk}{A\alpha}} \;;\quad n' = \sqrt{\tfrac{2Kk}{A\alpha}}$$
$$\mathfrak{B} = -\mathfrak{A} \;;\quad \mathfrak{B}' = +\mathfrak{A}'$$
$$\mathfrak{A} = \tfrac{1}{2}(\alpha-\beta) \;;\quad \mathfrak{A}' = \tfrac{1}{2}(\alpha+\beta)$$
$$\mathfrak{B} = \tfrac{1}{2}(\beta-\alpha) \;;\quad \mathfrak{B}' = \tfrac{1}{2}(\alpha+\beta)$$

et pro motu:

$$p = \tfrac{1}{2}(\alpha-\beta)\,\text{cof.}\,\omega\sqrt{\tfrac{6Kk}{A\alpha}} + \tfrac{1}{2}(\alpha+\beta)\,\text{cof.}\,\omega\sqrt{\tfrac{2Kk}{A\alpha}}$$
$$q = -\tfrac{1}{2}(\alpha-\beta)\,\text{cof.}\,\omega\sqrt{\tfrac{6Kk}{A\alpha}} + \tfrac{1}{2}(\alpha+\beta)\,\text{cof.}\,\omega\sqrt{\tfrac{2Kk}{A\alpha}}.$$

Problema 3.

Fig. 4. 26. Si filum in terminis I et O fixum et data vi $=K$ tenfum tribus ponduſculis A, B, et C fuerit onera-

oneratum, determinare eius motum, postquam de statu suo naturali recto IO vtcunque fuerit deturbatum.

Solutio.

Hic igitur habemus $IA = a$; $AB = b$; $BC = c$ et $CO = d$, ac si, elapso tempore ω min. sec., ponamus distantias:

$PA = p$; $QB = q$ et $RC = r$

quae initio fuerant respectiue α, β, γ, sequentes tres resoluendae sunt aequationes:

$$\frac{p}{A}\left(\frac{1}{a}+\frac{1}{b}\right)-\frac{q}{A}\cdot\frac{1}{b}+\frac{ddp}{2Kkd\omega^2}=0$$

$$\frac{q}{B}\left(\frac{1}{b}+\frac{1}{c}\right)-\frac{p}{B}\cdot\frac{1}{b}-\frac{r}{B}\cdot\frac{1}{c}+\frac{ddq}{2Kkd\omega^2}=0$$

$$\frac{r}{C}\left(\frac{1}{c}+\frac{1}{d}\right)-\frac{q}{C}\cdot\frac{1}{c}+\frac{ddr}{2Kkd\omega^2}=0.$$

Quodsi ergo statuamus $p=\mathfrak{A}\,\text{cos}.\,n\omega$; $q=\mathfrak{B}\,\text{cos}.\,n\omega$; $r=\mathfrak{C}\,\text{cos}.\,n\omega$; habebimus, ponendo $\frac{nn}{2Kk}=z$, has aequationes:

$$\frac{\mathfrak{A}}{A}\left(\frac{1}{a}+\frac{1}{b}\right)-\frac{\mathfrak{B}}{Ab}=\mathfrak{A}z$$

$$\frac{\mathfrak{B}}{B}\left(\frac{1}{b}+\frac{1}{c}\right)-\frac{\mathfrak{A}}{Bb}-\frac{\mathfrak{C}}{Bc}=\mathfrak{B}z$$

$$\frac{\mathfrak{C}}{C}\left(\frac{1}{c}+\frac{1}{d}\right)-\frac{\mathfrak{B}}{Cc}=\mathfrak{C}z$$

vnde eruimus:

$$\mathfrak{A}=\frac{\mathfrak{B}}{b\left(\frac{1}{a}+\frac{1}{b}\right)-Abz};\quad \mathfrak{C}=\frac{\mathfrak{B}}{c\left(\frac{1}{c}+\frac{1}{d}\right)-Ccz}$$

qui in secunda substituti praebent:

$$\frac{1}{b}+\frac{1}{c}-\frac{1}{bb\left(\frac{1}{a}+\frac{1}{b}\right)-Abbz}-\frac{1}{cc\left(\frac{1}{c}+\frac{1}{d}\right)-Cccz}=Bz$$

 qua

qua aequatione ordinata oritur:

$$z^3 \left.\begin{matrix} -\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right) \\ -\frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right) \\ -\frac{1}{C}\left(\frac{1}{c}+\frac{1}{d}\right) \end{matrix}\right\} zz \left.\begin{matrix} +\frac{1}{AB}\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right) \\ +\frac{1}{AC}\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{c}+\frac{1}{d}\right) \\ +\frac{1}{Bc}\left(\frac{1}{bc}+\frac{1}{bd}+\frac{1}{cd}\right) \end{matrix}\right\} z - \frac{1}{ABC}\left(\frac{1}{abc}+\frac{1}{abd}+\frac{1}{acd}+\frac{1}{bcd}\right)=0$$

quae aequatio, ſi ponatur breuitatis gratia:

$$\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right)=P;\ \frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right)=Q;\ \frac{1}{C}\left(\frac{1}{c}+\frac{1}{d}\right)=R$$

transmutatur in ſequentem formam:

$$(z-P)(z-Q)(z-R)=\frac{z-R}{ABbb}+\frac{z-P}{BC\,c}$$

vnde terni eliciuntur valores ipſius z, iique ſemper reales et poſitiui, qui ſint z, z', et z'', ex quibus ſequentes terni valores porro eruuntur:

$$n=\sqrt{2Kkz};\quad n'=\sqrt{2Kkz'}\quad n''=\sqrt{2Kkz''}$$

$$\mathfrak{A}=\frac{\mathfrak{B}}{Ab(P-z)};\quad \mathfrak{A}'=\frac{\mathfrak{B}'}{Ab(P-z')}\quad \mathfrak{A}''=\frac{\mathfrak{B}''}{Ab(P-z'')}$$

$$\mathfrak{C}=\frac{\mathfrak{B}}{Cc(R-z)};\quad \mathfrak{C}'=\frac{\mathfrak{B}'}{Cc(R-z')}\quad \mathfrak{C}''=\frac{\mathfrak{B}''}{Cc(R-z'')}$$

quibus valoribus inuentis, motus his formulis definietur:

$$p=\mathfrak{A}\,\text{coſ.}\,n\omega+\mathfrak{A}'\,\text{coſ.}\,n'\omega+\mathfrak{A}''\,\text{coſ.}\,n''\omega$$

$$q=\mathfrak{B}\,\text{coſ.}\,n\omega+\mathfrak{B}'\,\text{coſ.}\,n'\omega+\mathfrak{B}''\,\text{coſ.}\,n''\omega$$

$$r=\mathfrak{C}\,\text{coſ.}\,n\omega+\mathfrak{C}'\,\text{coſ.}\,n'\omega+\mathfrak{C}''\,\text{coſ.}\,n''\omega$$

quae, vt ad ſtatum propoſitum initialem accommodentur, fiat:

$$\mathfrak{A}+\mathfrak{A}'+\mathfrak{A}''=\alpha;\ \mathfrak{B}+\mathfrak{B}'+\mathfrak{B}''=\beta \text{ et } \mathfrak{C}+\mathfrak{C}'+\mathfrak{C}''=\gamma$$

ſicque motus quaeſitus erit determinatus.

Coroll.

Coroll. I.

27. Tota ergo ſolutio reducitur ad reſolutionem huius aequationis cubicae

$$z^3 \left.\begin{matrix} -\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right) \\ -\frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right) \\ -\frac{1}{C}\left(\frac{1}{c}+\frac{1}{d}\right) \end{matrix}\right\} z^2 \left.\begin{matrix} +\frac{1}{AB}\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right) \\ +\frac{1}{AC}\left(\frac{1}{ac}+\frac{1}{bc}+\frac{1}{ad}+\frac{1}{bd}\right) \\ +\frac{1}{BC}\left(\frac{1}{bc}+\frac{1}{bd}+\frac{1}{cd}\right) \end{matrix}\right\} z - \frac{1}{ABC}\left(\frac{1}{abc}+\frac{1}{abd}+\frac{1}{add}+\frac{1}{bcd}\right) = 0$$

dum casu problematis praecedentis, vbi filum duobus tantum ponduſculis erat onuſtum, haec aequatio quadratica ſolutionem continebat:

$$zz \left.\begin{matrix} -\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right) \\ -\frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right) \end{matrix}\right\} z + \frac{1}{AB}\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right) = 0.$$

Coroll. 2

28. Tres ſcilicet huius aequationis cubicae radices, generaliter modo expoſito coniunctae, generalem problematis ſuppeditant ſolutionem, quae ad omnes caſus, quicunque flatus filo fuerit inductus initio, pateat. Vnde patet, ſi terni numeri n, n', n'' fuerint incommenſurabiles inter ſe, motum fili admodum fore irregularem, neque certas periodos eſſe habiturum.

Coroll. 3

29. Tres autem dantur caſus, quibus filum ad vibrationes regulares et iſochronas excitari poteſt, qui conditionibus his locum habebunt:

Caſus I

	Casus I	Casus II	Casus III
si sit	$\alpha = \mathfrak{A}$	$\alpha = \mathfrak{A}'$	$\alpha = \mathfrak{A}''$
	$\beta = \mathfrak{B}$	$\beta = \mathfrak{B}'$	$\beta = \mathfrak{B}''$
	$\gamma = \mathfrak{C}$	$\gamma = \mathfrak{C}'$	$\gamma = \mathfrak{C}''$
tum erit	$p = \mathfrak{A}\,\text{cos}.\,n\,\omega$	$p = \mathfrak{A}'\,\text{cos}\,n'\omega$	$p = \mathfrak{A}''\,\text{cos}.\,n''\omega$
	$q = \mathfrak{B}\,\text{cos}\,n\,\omega$	$q = \mathfrak{B}'\,\text{cos}.\,n'\omega$	$q = \mathfrak{B}''\,\text{cos}.\,n''\omega$
	$r = \mathfrak{C}\,\text{cos}\,n\,\omega$	$r = \mathfrak{C}'\,\text{cos}.\,n'\omega$	$r = \mathfrak{C}''\,\text{cos}.\,n''\omega$
temp. vibrat.	$= \frac{\pi}{n}$ min. sec.	$= \frac{\pi}{n'}$ min. sec.	$= \frac{\pi}{n''}$ min. sec.

Coroll. 4.

30. Inuentis autem his tribus casibus, quibus vibrationes isochronae euadunt, ex iis omnes reliqui casus motuum irregularium per compositionem definiri poterunt; vbi notari meretur, vtcunque hi motus appareant irregulares, eos tamen ex combinatione vibrationum isochronarum oriri.

Coroll. 5.

31. Quando autem tria corpora A, B, C, tam ratione massae, quam distantiarum, ita fuerint comparata, vt numeri n, n', n'' inde resultent commensurabiles, irregularitas motus eatenus euanescit, quod in motu percipientur periodi, quibus filum in eundem statum restituitur.

Coroll. 6.

32. Si corpusculorum interualla a, b, c, d inter se fuerint aequalia, aequatio cubica resoluenda abibit in hanc formam:

$$z^3 - \left(\frac{2}{A} + \frac{2}{B} + \frac{2}{C}\right)\frac{zz}{a} + \left(\frac{3}{AB} + \frac{4}{AC} + \frac{3}{BC}\right)\frac{z}{aa} - \frac{4}{ABCa^3} = 0$$

quae

quae si corpuscula insuper sint inter se aequalia, fit:

$$z^3 - \frac{6zz}{Aa} + \frac{10z}{A^2a^2} - \frac{4}{A^3a^3} = 0$$

cuius tres radices sunt:

$$\text{I. } z = \frac{2}{Aa}; \quad \text{II. } z' = \frac{2-\sqrt{2}}{Aa}; \quad \text{III. } z'' = \frac{2+\sqrt{2}}{Aa}.$$

Coroll. 7.

33. In hoc ergo postremo casu, quo $A = B = C$ et $a = b = c = d$ erit pro motu fili generatim determinando ob $P = \frac{2}{Aa} = Q = R$:

$$n = \sqrt{\frac{4Kk}{Aa}}; \quad n' = \sqrt{\frac{2(2-\sqrt{2})Kk}{Aa}}; \quad n'' = \sqrt{\frac{2(2+\sqrt{2})Kk}{Aa}}$$

$$\mathfrak{A} = \frac{\mathfrak{B}}{0Aa} \qquad \mathfrak{A}' = \frac{\mathfrak{B}'}{\sqrt{2}} \qquad \mathfrak{A}'' = -\frac{\mathfrak{B}''}{\sqrt{2}}$$

$$\mathfrak{C} = \frac{\mathfrak{B}}{0Aa} \qquad \mathfrak{C}' = \frac{\mathfrak{B}'}{\sqrt{2}} \qquad \mathfrak{C}'' = -\frac{\mathfrak{B}''}{\sqrt{2}}$$

seu $\mathfrak{B} = 0$ $\qquad \mathfrak{B}' = \mathfrak{A}'\sqrt{2} \qquad \mathfrak{B}'' = -\mathfrak{A}''\sqrt{2}$

et $\mathfrak{C} = -\mathfrak{A} \qquad \mathfrak{C}' = \mathfrak{A}' \qquad \mathfrak{C}'' = \mathfrak{A}''.$

Ac pro statu initiali dato habebitur:

$$\mathfrak{A} + \mathfrak{A}' + \mathfrak{A}'' = \alpha; \quad 0 + \mathfrak{A}'\sqrt{2} - \mathfrak{A}''\sqrt{2} = \beta; \quad -\mathfrak{A} + \mathfrak{A}' + \mathfrak{A}'' = \gamma$$

vnde obtinetur:

$$\mathfrak{A} = \frac{\alpha-\gamma}{2}; \quad \mathfrak{A}' = \frac{\alpha+\beta\sqrt{2}+\gamma}{4}; \quad \mathfrak{A}'' = \frac{\alpha-\beta\sqrt{2}+\gamma}{4}$$

$$\mathfrak{B} = 0 \quad \mathfrak{B}' = \frac{\alpha+\beta\sqrt{2}+\gamma}{2\sqrt{2}}; \quad \mathfrak{B}'' = \frac{-\alpha+\beta\sqrt{2}-\gamma}{2\sqrt{2}}$$

$$\mathfrak{C} = \frac{-\alpha+\gamma}{2}; \quad \mathfrak{C}' = \frac{\alpha+\beta\sqrt{2}+\gamma}{4}; \quad \mathfrak{C}'' = \frac{\alpha-\beta\sqrt{2}+\gamma}{4}$$

Consequenter motus definietur per has formulas:

$$p = \frac{\alpha-\gamma}{2}\cos.\omega\sqrt{\frac{4Kk}{Aa}} + \frac{\alpha+\beta\sqrt{2}+\gamma}{4}\cos.\omega\sqrt{\frac{2(2-\sqrt{2})Kk}{Aa}} + \frac{\alpha-\beta\sqrt{2}+\gamma}{4}\cos\omega\sqrt{\frac{2(2+\sqrt{2})Kk}{Aa}}$$

$$q = +\frac{\alpha+\beta\sqrt{2}+\gamma}{2\sqrt{2}}\cos\omega\sqrt{\frac{2(2-\sqrt{2})Kk}{Aa}} - \frac{\alpha+\beta\sqrt{2}-\gamma}{2\sqrt{2}}\cos\omega\sqrt{\frac{2(2+\sqrt{2})Kk}{Aa}}$$

$$r = \frac{-\alpha+\gamma}{2}\cos.\omega\sqrt{\frac{4Kk}{Aa}} + \frac{\alpha+\beta\sqrt{2}+\gamma}{4}\cos.\omega\sqrt{\frac{2(2-\sqrt{2})Kk}{Aa}} + \frac{\alpha-\beta\sqrt{2}+\gamma}{4}\cos.\omega\sqrt{\frac{2(2+\sqrt{2})Kk}{Aa}}.$$

Problema 4.

Fig. 5. 34. Si filum, in terminis I et O fixum et data vi $=K$ tenſum, quatuor corpuſculis A, B, C, D fuerit oneratum, determinare eius motum, poſtquam de ſtatu ſuo naturali recto IO vtcunque fuerit depulſum.

Solutio.

Habemus ergo $IA=a$, $AB=b$, $BC=c$, $CD=d$ et $DO=e$, ac ſi elapſo tempore ω min. ſec. ponamus diſtantias

$$PA=p;\ QB=q;\ RC=r \text{ et } SD=s$$

quae initio fuerant reſpectiue α, β, γ, δ, ſequentes quatuor reſolui debent aequationes:

$$\frac{p}{A}\left(\frac{1}{a}+\frac{1}{b}\right)-\frac{q}{A}\cdot\frac{1}{b}+\frac{ddp}{2Kkd\omega^2}=0$$

$$\frac{q}{B}\left(\frac{1}{b}+\frac{1}{c}\right)-\frac{p}{B}\cdot\frac{1}{b}-\frac{r}{B}\cdot\frac{1}{c}+\frac{ddq}{2Kkd\omega^2}=0$$

$$\frac{r}{C}\left(\frac{1}{c}+\frac{1}{d}\right)-\frac{q}{C}\cdot\frac{1}{c}-\frac{s}{C}\cdot\frac{1}{d}+\frac{ddr}{2Kkd\omega^2}=0$$

$$\frac{s}{D}\left(\frac{1}{d}+\frac{1}{e}\right)-\frac{r}{D}\cdot\frac{1}{d}+\frac{dds}{2Kkd\omega^2}=0.$$

Quodſi iam ſtatuamus

$$p=\mathfrak{A}\,\text{coſ.}\,n\omega;\ q=\mathfrak{B}\,\text{coſ.}\,n\omega;\ r=\mathfrak{C}\,\text{coſ.}\,n\omega;\ s=\mathfrak{D}\,\text{coſ.}\,n\omega$$

ac ad abbreuiationem ponamus $\frac{nn}{2Kk}=z$, nec non
$\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right)=P$; $\frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right)=Q$; $\frac{1}{C}\left(\frac{1}{c}+\frac{1}{d}\right)=R$ et $\frac{1}{D}\left(\frac{1}{d}+\frac{1}{e}\right)=S$
orientur ſequentes aequationes:

$$\mathfrak{A}P-\frac{\mathfrak{B}}{Ab}=\mathfrak{A}z$$

$$\mathfrak{B}Q-\frac{\mathfrak{A}}{Bb}-\frac{\mathfrak{C}}{Bc}=\mathfrak{B}z$$

$$\mathfrak{C}R-\frac{\mathfrak{B}}{Cc}-\frac{\mathfrak{D}}{Cd}=\mathfrak{C}z$$

$$\mathfrak{D}S-\frac{\mathfrak{C}}{Dd}=\mathfrak{D}z$$

ex quibus elicitur:

$$\mathfrak{B} = \mathfrak{A}Ab(P-z);$$

$$\mathfrak{C} = \mathfrak{A}ABbc(P-z)(Q-z) - \frac{\mathfrak{A}c}{b}$$

$$\mathfrak{D} = \mathfrak{A}ABCbcd(P-z)(Q-z)(R-z) - \frac{\mathfrak{A}Ccd}{b}(R-z) - \frac{\mathfrak{A}Abd}{c}(P-z),$$

qui valores in vltima aequatione ſubſtituti praebent:

$$ABCbcd(P-z)(Q-z)(R-z)(S-z) - \frac{Ccd}{b}(R-z)(S-z)$$
$$-\frac{Abd}{c}(P-z)(S-z) - \frac{ABbc}{Pd}(P-z)(Q-z) + \frac{c}{Dbd} = 0$$

quae per $ABCbcd$ diuiſa abit in hanc formam:

$$(P-z)(Q-z)(R-z)(S-z) - \frac{(R-z)(S-z)}{ABbb} - \frac{(P-z)(S-z)}{BCcc} - \frac{(P-z)(Q-z)}{CDdd}$$
$$+ \frac{1}{ABCDbbdd} = 0,$$

cuius indoles clarius perſpicietur ex hac forma:

$$1 - \frac{1}{ABbb(P-z)(Q-z)} - \frac{1}{BCcc(Q-z)(R-z)} - \frac{1}{CDdd(R-z)(S-z)}$$
$$+ \frac{1}{ABCDbbdd(P-z)(Q-z)(R-z)(S-z)} = 0.$$

Verum ſi illa aequatio, reſtituendis pro P, Q, R, S valoribus, penitus euoluatur, obtinebitur ſequens aequatio biquadratica:

$$z^4 \left.\begin{matrix} -\frac{1}{A}\left(\frac{1}{a}+\frac{1}{b}\right) \\ -\frac{1}{B}\left(\frac{1}{b}+\frac{1}{c}\right) \\ -\frac{1}{C}\left(\frac{1}{c}+\frac{1}{d}\right) \\ -\frac{1}{D}\left(\frac{1}{d}+\frac{1}{e}\right) \end{matrix}\right\} z^3 \left.\begin{matrix} +\frac{1}{AB}\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right) \\ +\frac{1}{AC}\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{c}+\frac{1}{d}\right) \\ +\frac{1}{AD}\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{d}+\frac{1}{e}\right) \\ +\frac{1}{BC}\left(\frac{1}{bc}+\frac{1}{bd}+\frac{1}{cd}\right) \\ +\frac{1}{BD}\left(\frac{1}{b}+\frac{1}{c}\right)\left(\frac{1}{d}+\frac{1}{e}\right) \\ +\frac{1}{CD}\left(\frac{1}{cd}+\frac{1}{ce}+\frac{1}{de}\right) \end{matrix}\right\} zz \left.\begin{matrix} -\frac{1}{ABC}\left(\frac{1}{abc}+\frac{1}{abd}+\frac{1}{acd}+\frac{1}{bcd}\right) \\ -\frac{1}{ABD}\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right)\left(\frac{1}{d}+\frac{1}{e}\right) \\ -\frac{1}{ACD}\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{cd}+\frac{1}{ce}+\frac{1}{de}\right) \\ -\frac{1}{BCD}\left(\frac{1}{bcd}+\frac{1}{bce}+\frac{1}{bde}+\frac{1}{cde}\right) \end{matrix}\right\} z$$
$$+\frac{1}{ABCD}\left(\frac{1}{abcd}+\frac{1}{abce}+\frac{1}{abde}+\frac{1}{acde}+\frac{1}{bcde}\right) = 0.$$

Inuentis autem huius aequationis quaternis radicibus z, z', z'' et z''', ex illis totidem valores numeri n habebunbebuntur per formulam $n = \sqrt{2Kkz}$; ac sumtis quoque quaternis arbitrariis $\mathfrak{A}$, $\mathfrak{A}'$, $\mathfrak{A}''$, $\mathfrak{A}'''$, ex vnoquoque reliqui $\mathfrak{B}$, $\mathfrak{C}$ et $\mathfrak{D}$ respondentes reperientur ope formularum:

$$\mathfrak{B} = \mathfrak{A}\,A\,b\,(P-z)$$

$$\mathfrak{C} = \mathfrak{A}\,A\,B\,bc\left((P-z)(Q-z) - \tfrac{1}{AB\,bb}\right)$$

$$\mathfrak{D} = \mathfrak{A}\,ABC\,bcd\left((P-z)(Q-z)(R-z) - \tfrac{1}{ABbb}(R-z) - \tfrac{1}{BC\,cc}(P-z)\right)$$

ac tandem formulae pro motu fili erunt:

$$p = \mathfrak{A}\,\mathrm{cos}.n\omega + \mathfrak{A}'\,\mathrm{cos}.n'\omega + \mathfrak{A}''\,\mathrm{cos}\,n''\omega + \mathfrak{A}'''\,\mathrm{cos}\,n'''\omega$$

$$q = \mathfrak{B}\,\mathrm{cos}.n\omega + \mathfrak{B}'\,\mathrm{cos}\,n'\omega + \mathfrak{B}''\,\mathrm{cos}.n''\omega + \mathfrak{B}'''\,\mathrm{cos}.n'''\omega$$

$$r = \mathfrak{C}\,\mathrm{cos}.n\omega + \mathfrak{C}'\,\mathrm{cos}.n'\omega + \mathfrak{C}''\,\mathrm{cos}.n''\omega + \mathfrak{C}'''\,\mathrm{cos}.n'''\omega$$

$$s = \mathfrak{D}\,\mathrm{cos}.n\omega + \mathfrak{D}'\,\mathrm{cos}.n'\omega + \mathfrak{D}''\,\mathrm{cos}.n''\omega + \mathfrak{D}'''\,\mathrm{cos}.n'''\omega$$

quatuor autem constantes arbitrariae $\mathfrak{A}$, $\mathfrak{A}'$, $\mathfrak{A}''$, $\mathfrak{A}'''$ ex statu initiali ita definiri debent, vt fiat:

$$\mathfrak{A} + \mathfrak{A}' + \mathfrak{A}'' + \mathfrak{A}''' = \alpha$$

$$\mathfrak{B} + \mathfrak{B}' + \mathfrak{B}'' + \mathfrak{B}''' = \beta$$

$$\mathfrak{C} + \mathfrak{C}' + \mathfrak{C}'' + \mathfrak{C}''' = \gamma$$

$$\mathfrak{D} + \mathfrak{D}' + \mathfrak{D}'' + \mathfrak{D}''' = \delta$$

Coroll. 1.

35. Iam igitur quatuor existunt casus, quibus vibrationes erunt isochronae, quarum tempora erunt:

$\frac{\pi}{n}$; $\frac{\pi}{n'}$; $\frac{\pi}{n''}$; $\frac{\pi}{n'''}$ min. sec.

atque ex his casibus, tanquam motibus simplicibus, reliqui omnes per compositionem oriuntur.

Coroll.

Coroll. 2.

36. Ex his iam lex istarum formularum, si filum pluribus corpusculis fuerit onustum, non difficulter perspicitur. Si enim quinque habeantur corpuscula, adiecto valore $\frac{1}{E}(\frac{1}{e}+\frac{1}{f})=\Gamma$, aequatio principalis resoluenda ita se habebit:

$$1-\frac{1}{ABbb(P-z)(Q-z)}-\frac{1}{BCcc(Q-z)(R-z)}-\frac{1}{CDdd(R-z)(S-z)}-\frac{1}{DEee(S-z)(T-z)}$$
$$+\frac{1}{ABCDb^2d^2(P-z)(Q-z)(R-z)(S-z)}+\frac{1}{BCDEccee(Q-z)(R-z)(S-z)(T-z)}=0$$

Ac si sex fuerint corpuscula, posito $\frac{1}{F}(\frac{1}{f}+\frac{1}{g})=V$, erit

$$1-\frac{1:ABbb}{(P-z)(Q-z)}-\frac{1:BCcc}{(Q-z)(R-z)}-\frac{1:CDdd}{(R-z)(S-z)}-\frac{1:DEee}{(S-z)(T-z)}-\frac{1:EFff}{(T-z)(V-z)}$$
$$+\frac{1:ABCDbbdd}{(P-z)(Q-z)(R-z)(S-z)}+\frac{1:BCDEccee}{(Q-z)(R-z)(S-z)(T-z)}+\frac{1\cdot CDEFddff}{(R-z)(S-z)(T-z)(V-z)}$$
$$-\frac{1.\ ABCDEFbbddff}{(P-z)(Q-z)(R-z)(S-z)(T-z)(V-z)}=0.$$

Coroll. 3.

37. Hae autem formulae in genere nimis sunt complicatae, quam vt quidquam ad cognitionem motus inde concludi queat. Concipiamus ergo interualla a, b, c, d; etc. inter se aequalia, ac pro quouis corpusculorum numero aequationes, ex quibus valores ipsius z elici oportet, ita se habebunt:

Pro vno corpusculo

$$z-\frac{2}{Aa}=0.$$

Pro duobus corpusculis

$$zz-\frac{1}{a}(\frac{2}{A}+\frac{2}{B})z+\frac{1}{aa}\cdot\frac{3}{AB}=0$$

Pro tribus corpusculis

$$z^3-\frac{1}{a}(\frac{2}{A}+\frac{2}{B}+\frac{2}{C})zz+\frac{1}{aa}(\frac{3}{AB}+\frac{4}{AC}+\frac{3}{BC})z-\frac{1}{a^3}\cdot\frac{4}{ABC}=0$$

 Pro

Pro quatuor corpusculis

$$z^4 - \frac{1}{a}\left(\frac{2}{A} + \frac{2}{B} + \frac{2}{C} + \frac{2}{D}\right)z^3 + \frac{1}{aa}\left(\frac{3}{AB} + \frac{4}{AC} + \frac{4}{AD} + \frac{3}{BC} + \frac{4}{BD} + \frac{3}{CD}\right)z^2$$
$$- \frac{1}{a^3}\left(\frac{4}{ABC} + \frac{6}{ABD} + \frac{6}{ACD} + \frac{4}{BCD}\right)z + \frac{1}{a^4}\cdot\frac{5}{ABCD} = 0$$

Pro quinque corpusculis

$$z^5 - \frac{1}{a}\left(\frac{2}{A} + \frac{2}{B} + \frac{2}{C} + \frac{2}{D} + \frac{2}{E}\right)z^4$$
$$+ \frac{1}{aa}\left(\frac{3}{AB} + \frac{4}{AC} + \frac{4}{AD} + \frac{4}{AE} + \frac{3}{BC} + \frac{4}{BD} + \frac{4}{BE} + \frac{3}{CD} + \frac{4}{CE} + \frac{3}{DE}\right)z^3$$
$$- \frac{1}{a^3}\left(\frac{4}{ABC} + \frac{6}{ABD} + \frac{6}{ABE} + \frac{6}{ACD} + \frac{8}{ACE} + \frac{6}{ADE} + \frac{4}{BCD} + \frac{6}{BCE} + \frac{6}{BDE} + \frac{4}{CDE}\right)z^2$$
$$+ \frac{1}{a^4}\left(\frac{5}{ABCD} + \frac{8}{ABCE} + \frac{9}{ABDE} + \frac{8}{ACDE} + \frac{5}{BCDE}\right)z - \frac{1}{a^5}\cdot\frac{6}{ABCDE} = 0.$$

Coroll. 4.

38. Si non solum interualla corpusculorum a, b, c, d etc. sed etiam ipsa corpuscula A, B, C, D etc. inter se aequalia assumamus, aequationes sequentes prodibunt:

num. corp.	
I	$z - \frac{2}{Aa} = 0$
II	$z^2 - \frac{4z}{Aa} + \frac{3}{AAaa} = 0$
III	$z^3 - \frac{6zz}{Aa} + \frac{10z}{AAaa} - \frac{4}{A^3a^3} = 0$
IV	$z^4 - \frac{8z^3}{Aa} + \frac{21z^2}{A^2a^2} - \frac{20z}{A^3a^3} + \frac{5}{A^4a^4} = 0$
V	$z^5 - \frac{10z^4}{Aa} + \frac{36z^3}{A^2a^2} - \frac{56z^3}{A^3a^3} + \frac{35z^2}{A^4a^4} - \frac{6}{A^5a^5} = 0$
VI	$z^6 - \frac{12z^5}{Aa} + \frac{55z^4}{A^2a^2} - \frac{120z^3}{A^3a^3} + \frac{126z^2}{A^4a^4} - \frac{56z}{A^5a^5} + \frac{7}{A^6a^6} = 0$

vnde pro corpusculorum numero quocunque m concluditur, fore:

$$z^{2m} - \frac{2m}{1} \cdot \frac{z^{2m-1}}{Aa} + \frac{(2m-1)(2m-2)}{1 \cdot 2} \cdot \frac{z^{2m-2}}{A^2a^2} - \frac{(2m-2)(2m-3)(2m-4)}{1 \cdot 2 \cdot 3} \cdot \frac{z^{2m-3}}{A^3a^3}$$
$$+ \frac{(2m-3)(2m-4)(2m-5)(2m-6)}{1 \cdot 2 \cdot 3 \cdot 4} \frac{z^{2m-4}}{A^4a^4} - \frac{(2m-4)(2m-5)(2m-6)(2m-7)(2m-8)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \cdot \frac{z^{2m-5}}{A^5a^5}$$
$$+ \text{etc.} = 0$$

Coroll. 5.

39. Hoc autem casu coefficientes $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ etc. ex primo $\mathfrak{A}$ arbitrario pro quouis valore ipsius z ita definientur, vt sit:

$$\frac{\mathfrak{B}}{\mathfrak{A}} = Aa\,(\frac{z}{Aa}) - z)$$
$$\frac{\mathfrak{C}}{\mathfrak{A}} = A^2a^2(\frac{2}{Aa} - z)^2 - 1$$
$$\frac{\mathfrak{D}}{\mathfrak{A}} = A^3a^3(\frac{2}{Aa} - z)^3 - 2Aa(\frac{2}{Ab} - z)$$
$$\frac{\mathfrak{E}}{\mathfrak{A}} = A^4a^4(\frac{2}{Aa} - z)^4 - 3A^2a^2(\frac{2}{Aa} - z)^2 + 1$$

siue

$$\frac{\mathfrak{B}}{\mathfrak{A}} = 2 - Aaz$$
$$\frac{\mathfrak{C}}{\mathfrak{A}} = 3 - 4Aaz + A^2a^2zz$$
$$\frac{\mathfrak{D}}{\mathfrak{A}} = 4 - 10Aaz + 6A^2a^2zz - A^3a^3z^3$$
$$\frac{\mathfrak{E}}{\mathfrak{A}} = 5 - 20Aaz + 21A^2a^2zz - 8A^3a^3z^3 + A^4a^4z^4$$

etc.

quarum formularum progressio ex superioribus facillime colligitur.

Coroll. 6.

40. Ponamus pro eodem casu breuitatis gratia $Aaz = y$, ac pro quouis corpusculorum numero aequationes resoluendae ita se habebunt:

Pro

Pro vno corpusculo

$y - 2 = 0$, cuius radix est $y = 2$

Pro duobus corpusculis

$yy - 4y + 3 = 0$, cuius radices sunt $y = 1$; $y' = 3$

Pro tribus corpusculis

$y^3 - 6yy + 10y - 4 = 0$, cuius radices sunt

$y = 2$; $y' = 2 + \sqrt{2}$; $y'' = 2 - \sqrt{2}$

Pro quatuor corpusculis

$y^4 - 8y^3 + 21y^2 - 20y + 5 = 0$, cuius radices sunt

$y = \frac{3+\sqrt{5}}{2}$; $y' = \frac{3-\sqrt{5}}{2}$; $y'' = \frac{5+\sqrt{5}}{2}$; $y''' = \frac{5-\sqrt{5}}{2}$.

Coroll. 7.

41. Si has formulas bene perpendamus, eas per quadrata sinuum, denotante ϱ angulum rectum, sequenti modo exhiberi posse deprehendemus:

Pro vno corpusculo

$y = 4(\sin. \tfrac{1}{2}\varrho)^2$

Pro duobus corpusculis

$y = 4(\sin. \tfrac{1}{3}\varrho)^2$; $y' = 4(\sin. \tfrac{2}{3}\varrho)^2$

Pro tribus corpusculis

$y = 4(\sin. \tfrac{1}{4}\varrho)^2$; $y' = 4(\sin. \tfrac{2}{4}\varrho)^2$; $y'' = 4(\sin. \tfrac{3}{4}\varrho)^2$

Pro quatuor corpusculis

$y = 4(\sin. \tfrac{1}{5}\varrho)^2$; $y' = 4(\sin. \tfrac{2}{5}\varrho)^2$; $y'' = 4(\sin. \tfrac{3}{5}\varrho)^2$;
$y''' = 4(\sin. \tfrac{4}{5}\varrho)^2$

quarum formularum progressio per se est manifesta.

Coroll.

Coroll. 8.

42. Inuentis autem pro quouis casu valoribus ipsius y, ob $z = \frac{y}{Aa}$, erit $n = \sqrt{\frac{2Kk}{Aa}}y$, et pro reliquis coefficientibus:

$$\mathfrak{B} = \mathfrak{A}(2-y)$$
$$\mathfrak{C} = \mathfrak{A}(3-4y+yy)$$
$$\mathfrak{D} = \mathfrak{A}(4-10y+6yy-y^3)$$
$$\mathfrak{E} = \mathfrak{A}(5-20y+21yy-8y^3+y^4)$$
$$\mathfrak{F} = \mathfrak{A}(6-35y+56yy-36y^3+10y^4-y^5)$$
etc.

Cum autem y habeat huiusmodi formam $y = 4(\text{sin.}\,\Phi)^2$, erit:

$$\mathfrak{B} = \mathfrak{A}.\,2\,\text{cos.}\,2\,\Phi = \mathfrak{A}.\frac{\text{sin.}\,4\Phi}{\text{sin.}\,2\Phi}$$
$$\mathfrak{C} = \mathfrak{A}.(2\,\text{cos.}\,4\Phi+1) = \mathfrak{A}.\frac{\text{sin.}\,6\Phi}{\text{sin.}\,2\Phi}$$
$$\mathfrak{D} = \mathfrak{A}.(2\,\text{cos.}\,6\Phi+2\,\text{cos.}\,2\Phi) = \mathfrak{A}.\frac{\text{sin.}\,8\Phi}{\text{sin.}\,2\Phi}$$
$$\mathfrak{E} = \mathfrak{A}.(2\,\text{cos.}\,8\Phi+2\,\text{cos.}\,4\Phi+1) = \mathfrak{A}.\frac{\text{sin.}\,10\Phi}{\text{sin.}\,2\Phi}$$
$$\mathfrak{F} = \mathfrak{A}.(2\,\text{cos.}\,10\Phi+2\,\text{cos.}\,6\Phi+2\,\text{cos.}\,2\Phi) = \mathfrak{A}.\frac{\text{sin.}\,12\Phi}{\text{sin.}\,2\Phi}$$
$$\mathfrak{G} = \mathfrak{A}.(\text{cos.}\,12\Phi+2\,\text{cos.}\,8\Phi+2\,\text{cos.}\,4\Phi+1) = \mathfrak{A}.\frac{\text{sin.}\,14\Phi}{\text{sin.}\,2\Phi}$$

vnde sequens problema poterit in genere resolui.

Problema 5.

43. Si filum, terminis I et O fixum et data vi K tensum, onustum sit quotcunque corpusculis A, B, C etc. aequalibus et paribus interuallis a se inuicem distinctis, definire motum eius, postquam de statu suo naturali vtcunque fuerit depulsum.

 Solutio.

Solutio.

Sit numerus corpusculorum $= m$; massa vniuscuiusque $= A$, et binorum interuallum $= a$, erit totius fili massa $= mA$, et longitudo $IO = (m+1)a$. Reducta sint initio corpuscula A, B, C etc. ad distantias ab axe α, β, γ, etc. elapso autem tempore ω min. secund. peruenerint ad distantias $PA = p$; $QB = q$; $RC = r$, etc. His positis, si angulus rectus denotetur signo ϱ, et i sumatur pro numero quocunque integro positiuo; valor quilibet ipsius y erit $y = A(\sin.\frac{i}{m+1}\varrho)^2$, vnde fit $n = 2\sin.\frac{i}{m+1}\varrho \sqrt{\frac{2Kk}{Aa}}$; et ob $\Phi = \frac{i}{m+1}\varrho$, erit:

$$\mathfrak{B} = \mathfrak{A}\sin.\frac{4i}{m+1}\varrho : \sin.\frac{2i}{m+1}\varrho$$
$$\mathfrak{C} = \mathfrak{A}\sin.\frac{6i}{m+1}\varrho : \sin.\frac{2i}{m+1}\varrho$$
$$\mathfrak{D} = \mathfrak{A}\sin.\frac{8i}{m+1}\varrho : \sin.\frac{2i}{m+1}\varrho$$
etc.

Ponatur iam $\mathfrak{A} = \mathfrak{a}\sin.\frac{2i}{m+1}\varrho$, ac pro motu habebuntur hae formulae:

$$p = \mathfrak{a}\sin.\frac{2i}{m+1}\varrho.\ \mathrm{cos.}(2\omega\sin.\frac{i}{m+1}\varrho.\sqrt{\frac{2Kk}{Aa}}) + \text{etc.}$$
$$q = \mathfrak{a}\sin.\frac{4i}{m+1}\varrho.\ \mathrm{cos.}(2\omega\sin.\frac{i}{m+1}\varrho.\sqrt{\frac{2Kk}{Aa}}) + \text{etc.}$$
$$r = \mathfrak{a}\sin.\frac{6i}{m+1}\varrho.\ \mathrm{cos.}(2\omega\sin.\frac{i}{m+1}\varrho.\sqrt{\frac{2Kk}{Aa}}) + \text{etc.}$$
etc.

Scilicet ex quouis valore ipsius i formentur tales expressiones, eaeque coniunctae praebebunt valores generales pro applicatis p, q, r etc. At pro i successiue sumi debent numeri 1, 2, 3, 4 vsque ad m.

Coroll.

Coroll. 1.

44. Si breuitatis gratia ponatur angulus $\frac{1}{m+1}\varrho = \Phi$ et angulus $2\omega\sqrt{\frac{2Kk}{Aa}} = \psi$, habebuntur, fubftituendo pro i fucceffiue numeros 1, 2, 3. 4 etc. fequentes expreffiones pro applicatis:

$$p = \mathfrak{a} \text{ fin. } 2\Phi . \text{ cof. } \psi \text{ fin. } \Phi + \mathfrak{b} \text{ fin. } 4\Phi . \text{ cof. } \psi \text{ fin. } 2\Phi + \mathfrak{c} \text{ fin. } 6\Phi . \text{ cof. } \psi \text{ fin. } 3\Phi + \text{etc.}$$

$$q = \mathfrak{a} \text{ fin. } 4\Phi . \text{ cof. } \psi \text{ fin. } \Phi + \mathfrak{b} \text{ fin. } 8\Phi \text{ cof. } \psi \text{ fin } 2\Phi + \mathfrak{c} \text{ fin. } 12\Phi . \text{ cof. } \psi \text{ fin. } 3\Phi + \text{etc.}$$

$$r = \mathfrak{a} \text{ fin. } 6\Phi . \text{ cof. } \psi \text{ fin. } \Phi + \mathfrak{b} \text{ fin. } 12\Phi . \text{ cof. } \psi \text{ fin. } 2\Phi + \mathfrak{c} \text{ fin. } 18\Phi . \text{ cof. } \psi \text{ fin. } 3\Phi + \text{etc.}$$

$$s = \mathfrak{a} \text{ fin. } 8\Phi . \text{ cof. } \psi \text{ fin. } \Phi + \mathfrak{b} \text{ fin. } 16\Phi . \text{ cof. } \psi \text{ fin. } 2\Phi + \mathfrak{c} \text{ fin. } 24\Phi . \text{ cof. } \psi \text{ fin. } 3\Phi + \text{etc.}$$

etc.

Coroll. 2.

45. Ratio autem harum formularum clarius apparebit, fi eas ad quemuis corpufculorum numerum accommodemus. Maneat ergo breuitatis gratia angulus $2\omega\sqrt{\frac{2Kk}{Aa}} = \psi$, eritque pro cafu vnius corpufculi, ob $\Phi = \frac{1}{2}\varrho$,

$$p = \mathfrak{a} \text{ fin. } \varrho . \text{ cof. } \psi \text{ fin } \tfrac{1}{2}\varrho .$$

Coroll. 3.

46. Pro cafu autem duorum corpufculorum, vbi $\Phi = \frac{1}{3}\varrho$, habebimus:

$$p = \mathfrak{a} \text{ fin. } \tfrac{2}{3}\varrho . \text{ cof. } \psi \text{ fin. } \tfrac{1}{3}\varrho + \mathfrak{b} \text{ fin. } \tfrac{2}{3}\varrho . \text{ cof. } \psi \text{ fin. } \tfrac{2}{3}\varrho$$

$$q = \mathfrak{a} \text{ fin. } \tfrac{2}{3}\varrho . \text{ cof. } \psi \text{ fin. } \tfrac{1}{3}\varrho - \mathfrak{b} \text{ fin. } \tfrac{2}{3}\varrho . \text{ cof. } \psi \text{ fin } \tfrac{2}{3}\varrho .$$

Coroll. 4.

47. Pro casu trium corpusculorum, ob $\Phi = \frac{1}{4}\rho$, habebimus:

$$p = \mathfrak{a}\sin.\tfrac{2}{4}\rho.\cos.\psi\sin.\tfrac{1}{4}\rho + \mathfrak{b}\sin.\tfrac{4}{4}\rho.\cos.\psi\sin.\tfrac{2}{4}\rho$$
$$+ \mathfrak{c}\sin.\rho.\cos.\psi\sin.\tfrac{3}{4}\rho$$
$$q = \mathfrak{a}\sin.\tfrac{4}{4}\rho.\cos.\psi\sin.\tfrac{1}{4}\rho + \mathfrak{b}\sin.\tfrac{3}{4}\rho.\cos.\psi\sin.\tfrac{2}{4}\rho$$
$$- \mathfrak{c}\sin.\tfrac{4}{4}\rho.\cos.\psi\sin.\tfrac{3}{4}\rho$$
$$r = \mathfrak{a}\sin.\tfrac{2}{4}\rho.\cos.\psi\sin.\tfrac{1}{4}\rho - \mathfrak{b}\sin.\tfrac{4}{4}\rho.\cos.\psi\sin.\tfrac{2}{4}\rho$$
$$+ \mathfrak{c}\sin.\tfrac{2}{4}\rho.\cos.\psi\sin.\tfrac{3}{4}\rho.$$

Coroll. 5.

48. Pro casu quatuor corpusculorum, ob $\Phi = \frac{1}{5}\rho$, habebimus:

$$p = \mathfrak{a}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{1}{5}\rho + \mathfrak{b}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{2}{5}\rho$$
$$+ \mathfrak{c}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{3}{5}\rho + \mathfrak{d}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{4}{5}\rho$$
$$q = \mathfrak{a}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{1}{5}\rho + \mathfrak{b}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{2}{5}\rho$$
$$- \mathfrak{c}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{3}{5}\rho - \mathfrak{d}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{4}{5}\rho$$
$$r = \mathfrak{a}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{1}{5}\rho - \mathfrak{b}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{2}{5}\rho$$
$$- \mathfrak{c}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{3}{5}\rho + \mathfrak{d}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{4}{3}\rho$$
$$s = \mathfrak{a}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{1}{5}\rho - \mathfrak{b}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{2}{5}\rho$$
$$+ \mathfrak{c}\sin.\tfrac{4}{5}\rho.\cos.\psi\sin.\tfrac{3}{5}\rho - \mathfrak{d}\sin.\tfrac{2}{5}\rho.\cos.\psi\sin.\tfrac{4}{5}\rho.$$

Coroll. 6.

49. Quod si vero numerus corpusculorum fuerit infinite magnus, ob $\sin.\Phi = \Phi = \frac{\rho}{m}$, nanciscemur has formulas:

$$p = \frac{2\mathfrak{a}\rho}{m}\cos.\frac{\psi\rho}{m} + \frac{4\mathfrak{b}\rho}{m}\cos.\frac{2\psi\rho}{m} + \frac{6\mathfrak{c}\rho}{m}\cos.\frac{3\psi\rho}{m} + \text{etc.}$$
$$q = \frac{4\mathfrak{a}\rho}{m}\cos.\frac{\psi\rho}{m} + \frac{8\mathfrak{b}\rho}{m}\cos.\frac{2\psi\rho}{m} + \frac{12\mathfrak{c}\rho}{m}\cos.\frac{3\psi\rho}{m} + \text{etc.}$$
$$r = \frac{6\mathfrak{a}\rho}{m}\cos.\frac{\psi\rho}{m} + \frac{12\mathfrak{b}\rho}{m}\cos.\frac{2\psi\rho}{m} + \frac{18\mathfrak{c}\rho}{m}\cos.\frac{3\psi\rho}{m} + \text{etc.}$$

etc.

Coroll.

Coroll. 7.

50. Verum si huius cordae tota longitudo IO ponatur $= l$, et massa totius cordae $= M$ ob $a = \frac{l}{m}$, et $A = \frac{M}{m}$, erit $\psi = 2m\omega\sqrt{\frac{2Kk}{Ml}}$, vnde $\frac{\psi\varrho}{m} = 2\varrho\omega\sqrt{\frac{2Kk}{Ml}}$ $= \pi\omega\sqrt{\frac{2Kk}{Ml}}$; in coefficientibus autem constantibus vtpote arbitrariis omitti poterunt litterae ϱ et m, ita vt sit:

$$p = \mathfrak{a}\,\text{cos}.\,\pi\omega\sqrt{\tfrac{2Kk}{Ml}} + \mathfrak{b}\,\text{cos}.\,2\pi\omega\sqrt{\tfrac{2Kk}{Ml}} + \mathfrak{c}\,\text{cos}.\,3\pi\omega\sqrt{\tfrac{2Kk}{Ml}} + \text{etc.}$$

$$q = 2p;\ r = 3p;\ s = 4p \text{ etc.}$$

quae formula eundem exhibet motum, qui pro corda vniformiter crassa determinari solet.