INVESTIGATIO ACCVRATIOR PHAENOMENORVM QVAE IN MOTV TERRAE DIVRNO A VIRIBVS COELESTIBVS PRODVCI POSSVNT.

Auctore
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Motum terrae diurnum a viribus Solis et Lunae ita affici vt inde aequinoxiorum praecessio et axis terrestris nutatio exoriatur *Neutonus* iam erat suspicatus, Acutiss. vero *Alembertus* dilucide demonstrauit. Quod specimen sagacitatis humanae eo pluris est aestimandum, quod illo tempore subsidia dynamica, quibus haec inuestigatio innititur, neutiquam adhuc satis erant euoluta, ex quo summas et grauissimas calculi difficultates superari erat necesse. Principium autem harum perturbationum in eo est situm, quod telluris corpus a figura sphaerica recedit, ac diameter aequatoris aliquantum superat axis quantitatem; quodsi enim eius figura perfecte esset sphaerica, motus vertiginis ipsi semel impressus perpetuo eadem celeritate continuaretur, axisque eandem directionem conseruaret; neque vires externae vllam immutationem in hoc motu efficere valerent. Statim autem ac terrae figura a sphaerica discrepat,

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ex viribus Solis ac Lunae nascitur momentum ad axem de situ suo deturbandum tendens, quatenus quidem hae vires oblique in axem agunt. Quocirca in hac inuestigatione, postquam vera terrae figura esset constituta, ex lege attractionis vires Solis et Lunae singula terrae elementa sollicitantes definiri, indeque momenta positionem axis afficientia colligi oportuit; tum vero summa adhuc difficultas in effectu determinando residebat, quem illa virium momenta in situm axis exerere debeant, quod sine profundissima cum Dynamicae tum Analyseos scientia nullo modo praestari poterat.

Cum autem non ita pridem haec dynamicae pars maxime abstrusa, quae in motu corporum gyratorio circa axem mobilem definiendo est occupata, a me sit satis prospero successu ita pertractata, vt in genere corporum quacunque figura praeditorum motus dum a viribus quibuscunque sollicitantur, ad formulas analyticas satis simplices reduci queat; haud incongruum fore arbitror, si ope huius methodi omnes inaequalitates, quibus motus terrae diurnus perturbatur, ita accuratius determinauero, vt non solum verus motus, quo terra cietur, inde innotescat, sed etiam inde aliorum motuum, qui in terram cadere potuissent, siquidem initio aliter fuisset impulsa, indolem perspicere liceat. Quae igitur de hoc argumento sum meditatus, sequentibus propositionibus sum complexurus.

I. Quacunque figura terra sit praedita ad praesens institutum non opus est veram compositionis rationem, qua singulae partes inter se sunt distributae et ordinatae, nosse; verum sufficit vt ex principiis, quae circa motum corporum rigidorum in genere stabiliui, terni axes principales se mutuo in centro grauitatis seu potius inertiae normaliter decussantes notentur, momentaque inertiae respectu singulorum explorata habeantur. Hinc posita terrae massa tota $= M$, si ex centro inertiae I educti sint axes principales IA, IB, IC, eorum respectu momenta inertiae his formulis Maa, Mbb, Mcc designabo. His enim tribus momentis inertiae vniuersa internae structurae ratio, quatenus quidem inde motus determinatio pendet, continetur, ita vt iam taediosissimis illis calculis, quos alias figura et structura terrae exigere solet, supersedere queamus.

II. Quodsi haec tria momenta inertiae inter se essent aequalia, omnes plane axes per centrum terrae ducti pari gauderent proprietate, omnesque aeque pro principalibus haberi possent, ita vt terra, circa quemcunque axem initio gyrari coepisset, hunc motum perpetuo sine vlla axis et celeritatis alteratione esset prosecutura, neque etiam a viribus peregrinis vlla perturbatio esset pertimescenda. Perinde scilicet terra se esset habitura, ac si eius figura perfecte esset sphaerica, omnisque materia aequabiliter circa centrum distributa; vbi imprimis est no-

tandum

tandum hanc insignem proprietatem etiam in figuras maxime irregulares competere posse, dummodo terna illa momenta inertiae fuerint inter se aequalia, hoc autem in figuris maxime irregularibus euenire posse minime dubitare licet.

III. Eatenus ergo tantum motus terrae diurnus perturbationes pati potest, quatenus terna eius momenta inertiae principalia inter se non sunt aequalia. Verum etiamsi terra initio fuisset sphaerica statim atque circa certum axem gyrari coepisset, ob fluiditatem circa aequatorem intumescere debuisset, vnde respectu axis gyrationis momentum inertiae incrementum accepisset. In hoc autem negotio fluiditatis rationem minime habere licet, cum principia dynamica neutiquam adhuc ad hunc scopum sint euoluta; ob quem defectum vtique cogimur terram tanquam corpus solidum spectare, cuius figura nullis viribus sollicitantibus cedere queat. Quamobrem quae de eius motu diurno sum traditurus, ita sunt accipienda, vt ob maris mobilitatem, qua actioni virium quodammodo obsequitur, aliquam correctionem admittere intelligantur.

IV. Terram igitur tanquam eiusmodi corpus solidum considero, cuius trium axium principalium vnus puta IA cum eius axe proprie sic dicto, qui ab altero polo ad alterum per centrum porrigitur conueniat, et cuius respectu momentum inertiae sit $= Maa$. Bini ergo reliqui axes principales in ipſum aequatorem cadent, et quoniam omnium meridianorum par eſſe ratio videtur, momenta inertiae eorum reſpectu tanquam inter ſe aequalia ſpectari poterunt, ita vt ſit $cc = bb$. Hoc autem admiſſo omnes diametri aequatoris axium principalium proprietate aeque erunt praediti ita vt terra circa vnumquemque eorum libere gyrari poſſet. Vnde cum puncta B et C in aequatore pro lubitu accipi queant, dum 90° a ſe inuicem diſtent, alterum B in eo meridiano, qui pro primo habetur, aſſumere licebit; ita in ſuperficie terrae arcus AB primum meridianorum deſignabit.

V. Quoniam motus terrae diurnus potiſſimum ab eo motu qui terrae initio fuerit impreſſus, pendet, quanta varietas inde proficiſci potuerit, ante omnia perpendi conueniet, ac primo quidem mentem ab actione Solis et Lunae abſtrahendo. Iam ſatis perſpicuum eſt ſi terrae in rerum principio motus gyratorius vel circa axem vel quempiam diametrum aequatoris fuiſſet impreſſus, hunc motum ita perpetuo vniformiter duraturum fuiſſe, vt axis gyrationis conſtanter idem coeli punctum reſpexiſſet. Sin autem terra circa aliam quamcunque lineam per centrum tranſeuntem gyrari coepiſſet, tum motus quidem gyratorius vniformis manſiſſet, ſed ipſe axis gyratorius interea circa quodpiam coeli punctum per circulum quendam minorem vniformiter fuiſſet circumlatus, quemadmodum alibi de huius-

huiusmodi corporibus, quae momentorum inertiae principalium bina habent inter se aequalia, fusius demonstraui et hic nouo modo sum demonstraturus.

VI. Statim autem ac vires perturbatrices siue Solis siue Lunae accedunt, vtrumque motus genus ita afficitur, vt vel gyrationis celeritas immutetur, vel axis, circa quem terra gyratur motu magis irregulari feratur. Ad hanc perturbationem inuestigandam quaestionem latiori sensu acceptam tractari conuenit, vt solutio etiam ad eos casus pateat, quibus forte terrae ab initio motus circa axem a principalibus diuersum fuerit impressus. Cum enim a viribus sollicitantibus axis gyrationis de situ suo deturbetur, omnino necesse est, vt etiamsi hae vires abessent gyratio circa axem mobilem definiri queat. Tum vero etiam ad scientiae incrementum non parum conducere videtur, si etiam indolem eorum motuum, qui in terram cadere possent, etiamsi re vera in ea non insint assignare valeamus. Quocirca formulas generales ita sum instructurus, vt etiam ad casum quo bina momenta inertiae principalia non forent aequalia accommodari queant.

VII. Cum ob vires perturbatrices omnia phaenomena ad coelum immotum referri oporteat, sit circulus ♈ ☊ ♎ ecliptica, et E eius polus; et quo inuestigatio aeque ad lunam ac solem pateat, sit O ☊ R orbita astri perturbantis eclipticam secans in nodo ascendente ☊, et nunc quidem hoc astrum Tab. II. Fig. 3.

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haereat in S, per quod punctum ducto latitudinis circulo ESQ erit ♈Q longitudo et QS latitudo astri, tum vero ♈☊ longitudo nodi. Ponamus ergo haec elementa ♈☊ $= \omega$, ang. ♈☊O $= \varepsilon$, arcus ♈☊Q $= q$ et ES $= p$

atque ex trigonometria constat fore cotang. $p =$ tang. ε sin. $(q - \omega)$. Haec proprie ad lunam sunt accommodata, pro sole autem inclinatio seu angulus ♈☊O $= \varepsilon$ euanescens est sumendus, et ob latitudinem nullam arcus ES $= p$ constanter manet 90°.

VIII. Pro actionis autem, quam astrum S in terram exerit, quantitate, sit v eius distantia a centro terrae at e ea distantia in qua astri vis attractrix ipsi grauitati naturali est aequalis ita vt eius vis centrum terrae sollicitans sit ad grauitatem vt $\frac{ee}{vv}$ ad 1. Porro autem sit g altitudo, ex qua grauia in terrae superficie libere delabuntur tempore vnius minuti secundi, quibus positis quantitas actionis astri in terram hac formula $\frac{2gee}{v^3}$ continetur, quam breuitatis gratia littera $V = \frac{2gee}{v^3}$ denotabo. Vbi obseruandum est si corpus ab astro attractum ad distantiam $= v$ in circulo reuolueretur, id singulis minutis secundis angulum esse percursurum, qui sit $= \sqrt{\frac{2gee}{v^3}} = \sqrt{V}$. Ex quo si astrum sit sol $\sqrt{V}$ denotat motum terrae medium vni minuto secundo conuenientem; sin autem astrum fuerit luna, ac massa lunae aequalis statuatur massae telluris, fore $\sqrt{V}$, motum medium lunae vni minuto secundo do conuenientem. Quoties autem maſſa lunae minor fuerit maſſa terrae, motum illum medium in ratione ſubduplicata diminui oportet; vnde ex cognito motu medio valor litterae V facile definitur.

IX. His praemiſſis teneat iam elapſo tempore t in minutis ſecundis exprimendo terra ſitum in figura repraeſentatum, vt axis eius ſeu polus boreus ſit in A, primus meridianus AB, et BC quadrans aequatoris, vbi B et C ſpectantur tanquam bini reliqui axes principales terrae. Ponantur ergo arcus et anguli ſtatum terrae definientes:

1°. Diſtantia poli A a polo eclipticae E ſeu $EA = l$

2°. Longitudo poli A ſeu angulus $\Upsilon EA = \psi$

3°. Situs primi meridiani ſeu angulus $EAB = \phi$.

Hincque colligantur ſequentes arcus:

$$SA = \zeta,\ SB = \eta \ \text{et}\ SC = \theta$$

qui per trigonometriam ſphaericam ita definiuntur vt ſit:

$$\text{coſ.}\,\zeta = \text{ſin.}\,p(\text{ſin.}\,l\,\text{coſ.}(q-\psi) + \text{tang.}\,\varepsilon\,\text{coſ.}\,l\,\text{ſin.}(q-\omega))$$

$$\text{coſ.}\,\eta = \text{ſin.}\,p(-\text{ſin.}\,\phi\,\text{ſin.}(q-\psi) - \text{coſ.}\,l\,\text{coſ.}\,\phi\,\text{coſ.}(q-\psi) + \text{tang.}\,\varepsilon\,\text{ſin.}\,l\,\text{coſ.}\,\phi\,\text{ſin.}(q-\omega))$$

$$\text{coſ.}\,\theta = \text{ſin.}\,p(-\text{coſ.}\,\phi\,\text{ſin.}(q-\psi) + \text{coſ.}\,l\,\text{ſin.}\,\phi\,\text{coſ.}(q-\psi) - \text{tang.}\,\varepsilon\,\text{ſin}\,l\,\text{ſin.}\,\phi\,\text{ſin.}(q-\omega))$$

vbi notetur fore $\text{coſ.}\,\zeta^2 + \text{coſ.}\,\eta^2 + \text{coſ.}\,\theta^2 = 1$.

X. Nunc cum respectu ternorum axium principalium A, B, C momenta inertiae terrae sint Maa, Mbb, Mcc quaerantur primo tres quantitates x, y, z ex his aequationibus:

$$dx + \frac{cc - bb}{aa} dt (yz - 3V \operatorname{cos}. \eta \operatorname{cos}. \theta) = 0$$

$$dy + \frac{aa - cc}{bb} dt (xz - 3V \operatorname{cos}. \zeta \operatorname{cos}. \theta) = 0$$

$$dz + \frac{bb - aa}{cc} dt (xy - 3V \operatorname{cos}. \zeta \operatorname{cos}. \eta) = 0$$

tum vero reliqua elementa situm terrae determinantia ex his aequationibus colligi oportet:

$$dl = -dt(y \operatorname{sin}. \Phi + z \operatorname{cos}. \Phi)$$

$$d\Phi = x dt - \frac{dt(y \operatorname{cos}. \Phi - z \operatorname{sin}. \Phi)}{\operatorname{tang}. l}$$

$$d\psi = \frac{dt(y \operatorname{cos}. \Phi - z \operatorname{sin}. \Phi)}{\operatorname{sin}. l}$$

quarum aequationum ratio ex iis, quae de motu corporum gyratorio circa axem mobilem alibi tradidi, haud difficulter deducitur.

XI. Quoniam vero momenta inertiae pro axibus B et C aequalia statuimus vt sit $cc = bb$, primo quidem ob $dx = 0$ habebimus $x = f$ quantitati nempe constanti; tum vero ponendo breuitatis gratia $\frac{aa - bb}{bb} = \alpha$ seu $\frac{aa}{bb} = 1 + \alpha$, quinque sequentes aequationes resolui oportet:

$$dy + \alpha dt (fz - 3V \operatorname{cos}. \zeta \operatorname{cos}. \theta) = 0$$

$$dz - \alpha dt (fy - 3V \operatorname{cos}. \zeta \operatorname{cos}. \eta) = 0$$

$$dl = -dt(y \operatorname{sin}. \Phi + z \operatorname{cos}. \Phi)$$

$d\Phi$

$$d\Phi = f dt - \frac{dt(y\,cof.\,\Phi - z\,fin.\,\Phi)}{tang.\,l}$$
$$d\psi = \frac{dt(y\,cof.\,\Phi - z\,fin.\,\Phi)}{fin.\,l}$$

ex quibus binis poſterioribus eliminatis y et z colligitur:

$$d\Phi + d\psi\,cof.\,l = f dt$$

ita vt formula $d\Phi + d\psi\,cof.\,l$ ſit temporis elemento proportionalis.

XII. Quo nunc has aequationes diſtinctius euoluam, a ſimplicioribus ad difficiliora ita ordine aſcendam, vt primo vires perturbatrices tanquam euaneſcentes ſpectans motum terrae diurnum, quomodo tum ſe eſſet habiturus, definiam, deinde aſtrum perturbans in ipſa ecliptica motu vniformi circa terram reuolui aſſumam, vt eandem perpetuo ſeruet diſtantiam v, et V ſit quantitas conſtans. Deinde aſtrum etiam vniformiter ad diſtantiam conſtantem ſed in orbita ad eclipticam parumper inclinata circumferri ponam. Denique vero aſtro motum inaequabilem ſecundum leges *Keplerianas* tribuam, orbitam vero eius in ipſam eclipticam incidentem conſiderabo; qui bini poſteriores effectus coniuncti ad motum lunae verum accommodari poſſunt.

I. Euolutio caſus, quo vires perturbatrices euaneſcunt.

XIII. Poſito ergo pro hoc caſu $V = 0$, binae priores aequationes fiunt:

$$dy + \alpha f z dt = 0 \quad \text{et} \quad dz - \alpha f y dt = 0$$

vnde colligitur $y dy + z dz = 0$, et $yy + zz = bb$, ad quam aequationem cum reliquis commodissime construendam, notari conuenit, in coelo dari punctum fixum L, circa quod polus A vniformiter gyretur, dum interea terra circa hunc polum etiam vniformiter reuoluitur. Hoc ergo punctum L in calculum introducentes ponamus:

$$\Upsilon EL = \gamma, \; EL = m, \; LA = n, \text{ existente } EA = l$$

tum vero etiam angulos in triangulo sphaerico EAL

$$ELA = \lambda, \; EAL = \mu \text{ et } AEL = \nu$$

vt sint elementa γ, m et n constantia, angulus vero $ELA = \lambda$ tempori proportionalis. Vocetur porro etiam angulus $LAB = \sigma$, eritque

$$\Upsilon EA = \psi = \gamma - \nu \text{ et } EAB = \Phi = \sigma - \mu.$$

XIV. His positis ex trigonometricis habetur:

$$\text{cos.}\, l = \text{cos.}\, m \,\text{cos.}\, n + \text{sin.}\, m \,\text{sin.}\, n \,\text{cos.}\, \lambda$$

vnde ob m et n constantes differentiatio praebet:

$$dl \,\text{sin.}\, l = d\lambda \,\text{sin.}\, m \,\text{sin.}\, n \,\text{sin.}\, \lambda$$

et quia $\frac{\text{sin.}\,\lambda}{\text{sin.}\,l} = \frac{\text{sin.}\,\mu}{\text{sin.}\,m} = \frac{\text{sin.}\,\nu}{\text{sin.}\,n}$ erit

$$dl = d\lambda \,\text{sin.}\, n \,\text{sin.}\, \mu = d\lambda \,\text{sin.}\, m \,\text{sin.}\, \nu.$$

Deinde cum sit simili modo:

$$\text{cos.}\, m = \text{cos.}\, l \,\text{cos.}\, n + \text{sin.}\, l \,\text{sin.}\, n \,\text{cos.}\, \mu \quad \text{et}$$

$$\text{cos.}\, n = \text{cos.}\, l \,\text{cos.}\, m + \text{sin.}\, l \,\text{sin.}\, m \,\text{cos.}\, \nu$$

erit

erit quoque differentiando:

$$0 = -dl \sin. l \cos. n + dl \cos. l \sin. n \cos. \mu - d\mu \sin. l \sin. n \sin. \mu$$

$$0 = -dl \sin. l \cos. m + dl \cos. l \sin. m \cos. \nu - d\nu \sin. l \sin. m \sin. \nu$$

vnde colligitur:

$$d\mu = \frac{dl(\cos. l \sin. n \cos. \mu - \sin. l \cos. n)}{\sin. l \sin. n \sin. \mu} = \frac{d\lambda}{\sin. l}(\cos. l \sin. n \cos. \mu - \sin. l \cos. n)$$

$$d\nu = \frac{dl(\cos. l \sin. m \cos. \nu - \sin. l \cos. m)}{\sin. l \sin. m \sin. \nu} = \frac{d\lambda}{\sin. l}(\cos. l \sin. m \cos. \nu - \sin. l \cos. m).$$

XV. Ex trigonometria iam recordemur esse:

$$\cos. \mu = \frac{\sin. n \cos. m - \sin. m \cos. n \cos. \lambda}{\sin. l}; \quad \cos. \nu = \frac{\sin. m \cos. n - \sin. n \cos. m \cos. \lambda}{\sin. l}$$

similique modo etiam

$$\cos. \mu = \frac{\sin. l \cos. m - \cos. l \sin. m \cos. \nu}{\sin. n}; \quad \cos. \nu = \frac{\sin. l \cos. n - \cos. l \sin. n \cos. \mu}{\sin. m}$$

vnde concinnius posteriora differentialia ita definiuntur:

$$d\mu = \frac{-d\lambda \sin. m \cos. \nu}{\sin. l} \quad \text{et} \quad d\nu = \frac{-d\lambda \sin. n \cos. \mu}{\sin. l}$$

sicque pro elementis prius adhibitis habebimus:

$$d\psi = \frac{d\lambda \sin. n \cos. \mu}{\sin. l} \quad \text{et} \quad d\phi = d\sigma + \frac{d\lambda \sin. m \cos. \nu}{\sin. l}$$

vbi loco $\cos. \mu$ et $\cos. \nu$ priores valores scribi conueniet, vt omnia ad elementa m, n et λ reuocentur:

XVI. Nunc igitur statuamus:

$$y = h \cos. \sigma \quad \text{et} \quad z = -h \sin. \sigma$$

qui valores in prioribus aequationibus substituti dant:

$$-h d\sigma \sin. \sigma - \alpha f h dt \sin. \sigma = 0 \text{ seu } d\sigma = -\alpha f dt.$$

Tum vero ob

$$y\,\text{ſin.}\,\Phi + z\,\text{coſ.}\,\Phi = b\,\text{ſin.}\,(\Phi-\sigma) = -b\,\text{ſin.}\,\mu \quad \text{et}$$
$$y\,\text{coſ.}\,\Phi - z\,\text{ſin.}\,\Phi = b\,\text{coſ.}\,(\Phi-\sigma) = b\,\text{coſ.}\,\mu$$

ternae poſteriores aequationes has induent formas:

$$dl = d\lambda\,\text{ſin.}\,n\,\text{ſin.}\,\mu = b\,dt\,\text{ſin.}\,\mu, \text{ ſeu } d\lambda = \frac{b\,dt}{\text{ſin.}\,n}$$
$$d\Phi = -\alpha f dt + \frac{d\lambda\,\text{ſin.}\,m\,\text{coſ.}\,\nu}{\text{ſin.}\,l} = f dt - \frac{b\,dt\,\text{coſ.}\,\mu}{\text{tang.}\,l}$$
$$d\psi = \frac{b\,dt\,\text{coſ.}\,\mu}{\text{ſin.}\,l} = \frac{b\,dt\,(\text{ſin.}\,n\,\text{coſ.}\,m - \text{ſin.}\,m\,\text{coſ.}\,n\,\text{coſ.}\,\lambda)}{\text{ſin.}\,l^2}.$$

Antepenultima autem ob $d\lambda = \frac{b\,dt}{\text{ſin.}\,n}$ per dt diuiſa dat

$$(1+\alpha)f = \frac{b\,\text{ſin.}\,m\,\text{coſ.}\,\nu}{\text{ſin.}\,l\,\text{ſin.}\,n} + \frac{b\,\text{coſ.}\,l\,\text{coſ.}\,\mu}{\text{ſin.}\,l} = \frac{b(\text{coſ.}\,l\,\text{coſ.}\,\mu\,\text{ſin.}\,n + \text{ſin.}\,m\,\text{coſ.}\,\nu)}{\text{ſin.}\,l\,\text{ſin.}\,n}.$$

Verum cum ſit $\text{coſ.}\,\mu = \frac{\text{coſ.}\,m - \text{coſ.}\,l\,\text{coſ.}\,n}{\text{ſin.}\,l\,\text{ſin.}\,n}$ et $\text{coſ.}\,\nu = \frac{\text{coſ.}\,n - \text{coſ.}\,l\,\text{coſ.}\,m}{\text{ſin.}\,l\,\text{ſin.}\,m}$ obtinetur:

$$(1+\alpha)f = \frac{b\,(\text{coſ.}\,n - \text{coſ.}\,l^2\,\text{coſ.}\,n)}{\text{ſin.}\,l^2\,\text{ſin.}\,n} = \frac{b\,\text{coſ.}\,n}{\text{ſin.}\,n}$$

ita vt etiam quantitas $b = (1+\alpha)f\,\text{tang.}\,n$ determinetur, ſitque adeo conſtans, vt rei natura poſtulat.

XVII. Ex dato ergo coeli puncto L, ad quod motus terrae eſt referendus, ab eoque axis diſtantia $LA = n$, motus terrae ita ſe habebit, vt primo axis A circa illud punctum L vniformiter reuoluatur celeritate angulari $\frac{d\lambda}{dt} = \frac{b}{\text{ſin.}\,n} = \frac{(1+\alpha)f}{\text{coſ.}\,n}$, quippe qua ſingulis minutis ſecundis angulus $= \frac{(1+\alpha)f}{\text{coſ.}\,n}$ abſoluitur ita vt hoc motu angulus ELA continuo increſcat. Tum vero interea ipſa terra circa axem A gyrabitur

tur celeritate angulari $\frac{d\sigma}{dt} = -\alpha f$, qua angulus LAB continuo imminuetur; vbi notandum, quantitatem f a motu terrae primum impresso perinde atque arcum n pendere, litteram α vero rationem compositionis terrae inuoluere. In motu autem quem terra reuera est initio adepta, arcus $LA = n$ euanescit, sicque sine viribus perturbantibus, ipse arcus LA cum polo A fixus maneret, totusque motus ex angulo $LAB = \Phi$ ita definiretur, vt ob $b = 0$ esset celeritas angularis $\frac{d\Phi}{dt} = f$, vnde patet quantitatem f exprimere celeritatem angularem motus terrae diurni.

XVIII. Si terra eiusmodi motum vertiginis accepisset vt arcus $LA = n$ valorem haberet notabilem, polus A maiori celeritate circa coeli punctum L reuolueretur, siquidem constans f eundem retineret valorem, ipsa vero terra lentissime interea circa axem A in plagam contrariam conuerteretur, ob fractionem α minimam, ita vt hoc motu neglecto terra circa axem per coeli punctum L transeuntem reuolui videretur, spatio saltem temporis non nimis magno tempore autem labente quia sensim alia terrae puncta coeli punctum L subeunt, terra alium axem gyrationis recipere videbitur, ex quo etiam aequator et locorum latitudines mutationem patientur, sicque tam coeli quam terrae phaenomena ingentibus perturbationibus forent obnoxia, etiamsi nullae vires perturbatrices externae accederent.

XIX.

XIX. Hoc casu si pro quouis tempore t situm terrae respectu eclipticae definire velimus; primum ad hoc tempus angulum $\lambda = \frac{(1+\alpha)f}{\cos. n} t + \text{Const.}$ colligi oportet, pro quo facile tabula conderetur. Tum vero hoc angulo inuento statim habebitur axis A a polo eclipticae E distantia $EA = l$ ope formulae $\cos. l = \cos. m \cos. n + \sin. m \sin. n \cos. \lambda$ vbi m et n sunt certi anguli dati. Porro pro situ arcus EA quaeratur angulus ν vt sit $\text{cotang.}\, \nu = \frac{\sin. m \cos. n - \cos. m \sin. n \cos. \lambda}{\sin. n \sin. \lambda}$, eritque angulus $\Upsilon EA = \psi = \gamma - \nu$, denotante γ angulum quendam constantem. Denique pro situ primi meridiani AB seu angulo $LAB = \phi = \sigma - \mu$, primo angulus σ tempori proportionalis ex formula $\sigma = \text{Const.} - \alpha f t$ facile colligitur, tum vero angulus μ ex hac formula capiatur:

$$\text{cotang.}\, \mu = \frac{\sin. n \cos. m - \cos. n \sin. m \cos. \lambda}{\sin. m \sin. \lambda}$$

sicque status terrae ad tempus propositum erit determinatus omnem autem hunc calculum commode tabulis complecti liceret. Hoc casu expedito ad vires perturbantes progredior, quae tractatio praeparationem postulat, reliquis casibus quos suscepi praemittendam.

Praeparatio ad effectus virium perturbantium euoluendos.

XX. Quoniam hic effectus est minimus, ex casu praecedente facile intelligitur, quomodo hanc tracta-

tractationem aptiſſime ſuſcipi conueniat. Vniuerſus ſcilicet motus quoque ad coeli quoddam punctum L referatur, quod autem non amplius tanquam fixum ſpectetur, ſed ita pro variabili habeatur, vt tam angulus $\Upsilon E L = \gamma$ quam arcus $EL = m$ et $LA = n$ a viribus perturbantibus parumper immutari cenſeantur, neque etiam anguli $ELA = \lambda$ et $LAB = \sigma$ incrementa tempori exacte proportionalia capere ſunt cenſenda; quamobrem quoque quantitas b tanquam variabilis erit tractanda. Ponamus deinde vt ſupra; $EA = l$, angulos $EAL = \mu$, $AEL = \nu$ tum vero angulos $\Upsilon EA = \psi$ et $EAB = \Phi$ vt ſit $\psi = \gamma - \nu$ et $\Phi = \sigma - \mu$.

XXI. Statuamus ergo etiam nunc quoque

$$y = b \operatorname{cof.} \sigma \quad \text{et} \quad z = - b \operatorname{fin.} \sigma$$

et quia quantitatem b quoque vt variabilem ſpectamus, binae priores aequationes differentiales fient:

$$db \operatorname{cof.}\sigma - bd\sigma \operatorname{fin.}\sigma - \alpha f b dt \operatorname{fin.}\sigma - 3\alpha V dt \operatorname{cof.}\zeta \operatorname{cof.}\theta = 0$$
$$-db \operatorname{fin.}\sigma - bd\sigma \operatorname{cof.}\sigma - \alpha f b dt \operatorname{cof.}\sigma + 3\alpha V dt \operatorname{cof.}\zeta \operatorname{cof.}\eta = 0$$

vnde per combinationem elicimus:

$$-bd\sigma - \alpha f b dt - 3\alpha V dt \operatorname{cof.}\zeta (\operatorname{fin}\sigma \operatorname{cof.}\theta - \operatorname{cof.}\sigma \operatorname{cof.}\eta) = 0$$
$$db - 3\alpha V dt \operatorname{cof.}\zeta (\operatorname{cof.}\sigma \operatorname{cof.}\theta + \operatorname{fin.}\sigma \operatorname{cof.}\eta) = 0.$$

Ponamus breuitatis gratia

$$\operatorname{cof.}\sigma \operatorname{cof.}\eta - \operatorname{fin.}\sigma \operatorname{cof.}\theta = P \quad \text{et} \quad \operatorname{cof.}\sigma \operatorname{cof.}\theta + \operatorname{fin.}\sigma \operatorname{cof.}\eta = Q$$

vt habeamus has aequationes:

$$d\sigma = -\alpha f dt + \frac{3\alpha VP dt \operatorname{cof.}\zeta}{b} \quad \text{et} \quad db = 3\alpha V Q dt \operatorname{cof.}\zeta,$$

Ve-

Verum ex formulis ſupra §. IX. ob $\Phi = \sigma - \mu$ ſeu $\sigma - \Phi = \mu$ colligimus :

$$P = \text{ſin.}\,p(\text{ſin.}\,\mu\,\text{ſin.}(q-\psi) - \text{coſ.}\,l\,\text{coſ.}\,\mu\,\text{coſ.}(q-\psi) + \text{tang.}\,\varepsilon\,\text{ſin.}\,l\,\text{coſ.}\,\mu\,\text{ſin.}(q-\omega))$$

$$Q = \text{ſin.}\,p(-\text{coſ.}\,\mu\,\text{ſin.}(q-\psi) - \text{coſ.}\,l\,\text{ſin}\,\mu\,\text{coſ.}(q-\psi) + \text{tang.}\,\varepsilon\,\text{ſin.}\,l\,\text{ſin.}\,\mu\,\text{ſin.}(q-\omega))$$

exiſtente $\text{coſ.}\,\zeta = \text{ſin.}\,p(\text{ſin.}\,l\,\text{coſ.}(q-\psi) + \text{tang.}\,\varepsilon\,\text{coſ.}\,l\,\text{ſin.}(q-\omega))$.

Reliquae vero aequationes erunt ;

$$dl = b\,dt\,\text{ſin.}\,\mu$$

$$d\Phi = d\sigma - d\mu = f\,dt - \frac{b\,dt\,\text{coſ.}\,\mu}{\text{tang.}\,l}$$

$$d\psi = d\gamma - d\nu = \frac{b\,dt\,\text{coſ.}\,\mu}{\text{ſin.}\,l}.$$

XXII. Haec iam differentialia dl, $d\mu$ et $d\nu$ ad differentialia $d\lambda$, dm et dn reduci oportet ; ac primo quidem cum ſit $\text{coſ.}\,l = \text{coſ.}\,m\,\text{coſ.}\,n + \text{ſin.}\,m\,\text{ſin.}\,n\,\text{coſ.}\,\lambda$ erit differentiando :

$$dl\,\text{ſin.}\,l = dm\,\text{ſin.}\,m\,\text{coſ.}\,n + dn\,\text{coſ.}\,m\,\text{ſin.}\,n + d\lambda\,\text{ſin.}\,m\,\text{ſin.}\,n\,\text{ſin.}\,\lambda - dm\,\text{coſ.}\,m\,\text{ſin.}\,n\,\text{coſ.}\,\lambda - dn\,\text{ſin.}\,m\,\text{coſ.}\,n\,\text{coſ.}\,\lambda$$

quae forma ob

$$\text{ſin.}\,n\,\text{coſ.}\,m - \text{ſin.}\,m\,\text{coſ.}\,n\,\text{coſ.}\,\lambda = \text{ſin.}\,l\,\text{coſ.}\,\mu \text{ et } \text{ſin.}\,m\,\text{coſ.}\,n - \text{ſin.}\,n\,\text{coſ.}\,m\,\text{coſ.}\,\lambda = \text{ſin.}\,l\,\text{coſ.}\,\nu$$

tranſit in hanc ſimpliciorem :

$$dl\,\text{ſin.}\,l = dm\,\text{ſin}\,l\,\text{coſ.}\,\nu + dn\,\text{ſin.}\,l\,\text{coſ.}\,\mu + d\lambda\,\text{ſin.}\,m\,\text{ſin.}\,n\,\text{ſin.}\,\lambda$$

ſeu

ſeu cum ſit $\frac{\text{ſin.}\,\lambda}{\text{ſin.}\,l} = \frac{\text{ſin.}\,\mu}{\text{ſin.}\,m} = \frac{\text{ſin.}\,\nu}{\text{ſin.}\,n}$ in hanc

$$dl = dm\,\text{coſ.}\,\nu + dn\,\text{coſ.}\,\mu + d\lambda\,\text{ſin.}\,n\,\text{ſin.}\,\mu.$$

XXIII. Deinde vero ſimili modo elicitur:

$$dm = dl\,\text{coſ.}\,\nu + dn\,\text{coſ.}\,\lambda + d\mu\,\text{ſin.}\,n\,\text{ſin.}\,\lambda$$

vbi ſi loco dl valor modo inuentus ſubſtituatur, prodit

$$dm = dm\,\text{coſ.}\,\nu^2 + dn\,\text{coſ.}\,\mu\,\text{coſ.}\,\nu + d\lambda\,\text{ſin.}\,n\,\text{ſin.}\,\mu\,\text{coſ.}\,\nu + d\mu\,\text{ſin.}\,n\,\text{ſin.}\,\lambda + dn\,\text{coſ.}\,\lambda$$

ſeu ob $\text{coſ.}\,\lambda + \text{coſ.}\,\mu\,\text{coſ.}\,\nu = \text{coſ.}\,l\,\text{ſin.}\,\mu\,\text{ſin.}\,\nu$

$$dm\,\text{ſin.}\,\nu^2 = dn\,\text{coſ.}\,l\,\text{ſin.}\,\mu\,\text{ſin.}\,\nu + d\lambda\,\text{ſin.}\,n\,\text{ſin.}\,\mu\,\text{coſ.}\,\nu + d\mu\,\text{ſin.}\,n\,\text{ſin.}\,\lambda.$$

Eſt vero $\text{ſin.}\,n\,\text{ſin.}\,\mu = \text{ſin.}\,m\,\text{ſin.}\,\nu$ et $\text{ſin.}\,n\,\text{ſin.}\,\lambda = \text{ſin.}\,l\,\text{ſin.}\,\nu$, vnde per $\text{ſin.}\,\nu$ diuidendo habetur:

$$dm\,\text{ſin.}\,\nu = dn\,\text{coſ.}\,l\,\text{ſin.}\,\mu + d\lambda\,\text{ſin.}\,m\,\text{coſ.}\,\nu + d\mu\,\text{ſin.}\,l$$

ita vt ſit $d\mu = \frac{dm\,\text{ſin.}\,\nu}{\text{ſin.}\,l} - \frac{dn\,\text{coſ.}\,l\,\text{ſin.}\,\mu}{\text{ſin.}\,l} - \frac{d\lambda\,\text{ſin.}\,m\,\text{coſ.}\,\nu}{\text{ſin.}\,l}$.

Verum quia elementum dl tam commode exprimitur, vt ſit $dl = b\,dt\,\text{ſin.}\,\mu$ praeſtabit hunc valorem ſtatim introduci vnde prodit

$$d\mu = \frac{-b\,dt\,\text{ſin.}\,\mu\,\text{coſ.}\,\nu + dm - dn\,\text{coſ.}\,\lambda}{\text{ſin.}\,n\,\text{ſin.}\,\lambda}$$ ſimilique modo
$$d\nu = \frac{-b\,dt\,\text{ſin.}\,\mu\,\text{coſ.}\,\mu + dn - dm\,\text{coſ.}\,\lambda}{\text{ſin.}\,m\,\text{ſin.}\,\lambda}$$

XXIV. Ex prima ergo aequatione adipiſcimur:

$$b\,dt\,\text{ſin.}\,\mu = dm\,\text{coſ.}\,\nu + dn\,\text{coſ.}\,\mu + d\lambda\,\text{ſin.}\,n\,\text{ſin.}\,\mu.$$

vnde fit $d\lambda = \frac{b\,dt}{\text{ſin.}\,n} - \frac{dm\,\text{coſ.}\,\nu - dn\,\text{coſ.}\,\mu}{\text{ſin.}\,n\,\text{ſin.}\,\mu}$.

 at

at tertia aequatio praebet

$$d\gamma = \frac{b\,dt\,\text{cos}.\mu}{\text{sin}.l} - \frac{b\,dt\,\text{sin}.\mu\,\text{cos}.\mu}{\text{sin}.m\,\text{sin}.\lambda} + \frac{dn - dm\,\text{cos}.\lambda}{\text{sin}.m\,\text{sin}.\lambda}$$

seu $d\gamma = \frac{dn - dm\,\text{cos}.\lambda}{\text{sin}.m\,\text{sin}.\lambda}$

secunda vero aequatio ad $d\Phi$ spectans dat

$$d\sigma = f\,dt - \frac{b\,dt\,\text{cos}.\mu\,\text{cos}.l}{\text{sin}.l} - \frac{b\,dt\,\text{sin}.\mu\,\text{cos}.\nu}{\text{sin}.n\,\text{sin}.\lambda} + \frac{dm - dn\,\text{cos}.\lambda}{\text{sin}.n\,\text{sin}.\lambda}$$

vbi notandum terminos elemento $b\,dt$ affectos, cum sint

$$-\frac{b\,dt\,(\text{sin}.\mu\,\text{cos}.\nu + \text{sin}.\nu\,\text{cos}.\mu\,\text{cos}.l)}{\text{sin}.l\,\text{sin}.\nu}$$

ob $\text{cot}.n = \frac{\text{sin}.\mu\,\text{cos}.\nu + \text{sin}.\nu\,\text{cos}.\mu\,\text{cos}.l}{\text{sin}.l\,\text{sin}.\nu}$ abire in $-\frac{b\,dt\,\text{cos}.n}{\text{sin}.n}$

ita vt fit

$$d\sigma = f\,dt - \frac{b\,dt\,\text{cos}.n}{\text{sin}.n} + \frac{dm - dn\,\text{cos}.\lambda}{\text{sin}.n\,\text{sin}.\lambda} = -\alpha f\,dt + \frac{3\alpha VP\,dt\,\text{cos}.\zeta}{b}$$

hincque $(1+\alpha)f\,dt - \frac{b\,dt\,\text{cos}.n}{\text{sin}.n} + \frac{dm - dn\,\text{cos}.\lambda}{\text{sin}.n\,\text{sin}.\lambda} = \frac{3\alpha VP\,dt\,\text{cos}.\zeta}{b}$.

XXV. Nunc igitur sumamus $b = (1+\alpha)f\,\text{tang}.n$ vt viribus euanescentibus, arcus m et n prodeant constantes, et postrema aequatio dabit

$$dm - dn\,\text{cos}.\lambda = \frac{3\alpha VP\,dt\,\text{cos}.n\,\text{sin}.\lambda\,\text{cos}.\zeta}{(1+\alpha)f}$$

arcus vero n ex hac integratione est definiendus

$$\text{tang}.n = \frac{3\alpha}{(1+\alpha)f}\int VQ\,dt\,\text{cos}.\zeta$$

seu $dn = \frac{3\alpha VQ\,dt\,\text{cos}.\zeta\,\text{cos}.n^2}{(1+\alpha)f}$

vnde fit $dm = \frac{3\alpha V\,dt\,\text{cos}.n\,\text{cos}.\zeta}{(1+\alpha)f}(P\,\text{sin}.\lambda + Q\,\text{cos}.n\,\text{cos}.\lambda)$

hinc-

hincque porro

$$dm\,\mathrm{cof.}\nu + dn\,\mathrm{cof.}\mu = \frac{3\alpha V dt\,\mathrm{cof.}n\,\mathrm{cof.}\zeta}{(1+\alpha)f}\,\mathrm{fin.}\lambda(P\,\mathrm{cof.}\nu + Q\,\mathrm{cof.}m\,\mathrm{cof.}n\,\mathrm{fin.}\nu).$$

Inuentis autem variationibus dm et dn, erit

$$d\lambda = \frac{(1+\alpha)f\,dt}{\mathrm{cof.}n} - \frac{dm\,\mathrm{cof.}\nu - dn\,\mathrm{cof.}\mu}{\mathrm{fin.}n\,\mathrm{fin.}\mu} \text{ feu}$$

$$d\lambda = \frac{(1+\alpha)f\,dt}{\mathrm{cof.}n} - \frac{dm(\mathrm{fin.}m\,\mathrm{cof.}n - \mathrm{fin.}n\,\mathrm{cof.}m\,\mathrm{cof.}\lambda)}{\mathrm{fin.}m\,\mathrm{fin.}n\,\mathrm{fin.}\lambda} - \frac{dn(\mathrm{fin.}n\,\mathrm{cof.}m - \mathrm{fin.}m\,\mathrm{cof.}n\,\mathrm{cof.}\lambda)}{\mathrm{fin.}m\,\mathrm{fin.}n\,\mathrm{fin.}\lambda}$$

et anguli ♈EL variatio fimul innotefcit, quae eft

$$d\gamma = \frac{dn - dm\,\mathrm{cof.}\lambda}{\mathrm{fin.}m\,\mathrm{fin.}\lambda} = \frac{3\alpha V dt\,\mathrm{cof.}n\,\mathrm{cof.}\zeta}{(1+\alpha)f\,\mathrm{fin.}m}(Q\,\mathrm{cof.}n\,\mathrm{fin.}\lambda - P\,\mathrm{cof.}\lambda)$$

fubftitutis autem pro dm et dn valoribus reperitur

$$d\lambda = \frac{(1+\alpha)f\,dt}{\mathrm{cof.}n} - \frac{3\alpha V dt\,\mathrm{cof.}n\,\mathrm{cof.}\zeta}{(1+\alpha)f\,\mathrm{fin.}m\,\mathrm{fin.}n}(P(\mathrm{fin.}m\,\mathrm{cof.}n - \mathrm{fin.}n\,\mathrm{cof.}m\,\mathrm{cof.}\lambda)$$
$$+ Q(\mathrm{fin.}n\,\mathrm{cof.}m\,\mathrm{cof.}n\,\mathrm{fin.}\lambda).$$

Denique pro angulo $EAB = \phi$ habebimus:

$$d\phi = f\,dt - \frac{(1+\alpha)f\,dt\,\mathrm{fin.}n\,\mathrm{cof.}\mu\,\mathrm{cof.}l}{\mathrm{cof.}n\,\mathrm{fin.}l} \text{ et } d\psi = \frac{(1+\alpha)f\,dt\,\mathrm{fin.}n\,\mathrm{cof.}\mu}{\mathrm{fin.}l\,\mathrm{cof.}n}$$

ita vt fit $d\phi + d\psi\,\mathrm{cof.}l = f\,dt$, in quibus integrationibus arcus m et n tanquam conftantes fpectare licet, fimul vero erit proxime $d\lambda = \frac{(1+\alpha)f}{\mathrm{cof.}n}dt$, fiquidem perturbatio fuerit minima.

XXVI. Quo haec nunc propius ad praefens inftitutum accommodemus, obferuandum eft in motu vertiginis terrae arcum n quam minimum effe ftatuendum, ita vt ob $d\phi = f\,dt$ iam f denotet angulum vno minuto fecundo confectum, et angulus ψ in integrationibus pro conftanti haberi poterit.

 Dein-

Deinde ob $\cot.\mu = \frac{n \cos. m - \sin. m \cos. \lambda}{\sin. m \sin. \lambda}$, colligitur fore angulum $\mu = 180^\circ - \lambda - \frac{n \sin. \lambda}{\tan g. m}$, ita vt sit.

$$\sin. \mu = \sin. \lambda + \frac{n \sin. \lambda \cos. \lambda}{\tan g. m}; \quad \sin.(\lambda + \mu) = \frac{n \sin. \lambda}{\tan g. m}$$

$$\cos. \mu = -\cos. \lambda + \frac{n \sin. \lambda^2}{\tan g. m}; \quad \cos.(\lambda + \mu) = 1$$

neglectis terminis vbi n ad altiores potestates exsurgit.

Porro ob $\cos. l = \cos. m + n \sin. m \cos. \lambda$ erit $l = m - n \cos. \lambda$ hincque

$$n = \frac{3\alpha}{(1+\alpha)f} \int V Q dt \cos. \zeta; \quad m = \frac{3\alpha}{(1+\alpha)f} \int V dt \cos. \zeta (P \sin. \lambda + Q \cos. \lambda)$$

$$d\lambda = (1+\alpha) f dt - \frac{3\alpha}{(1+\alpha)f} \cdot \frac{V dt \cos. \zeta (P \sin. m - n \cos. m (P \cos. \lambda - Q \sin. \lambda))}{n \sin. m}$$

$$= (1+\alpha) f dt - \frac{3\alpha V P dt \cos. \zeta}{(1+\alpha) n f} - \frac{3\alpha V dt \cos. \zeta \cos. m (Q \sin. \lambda - P \cos. \lambda)}{(1+\alpha) f \sin. m}$$

$$d\gamma = \frac{3\alpha}{(1+\alpha)f} \cdot \frac{V dt \cos. \zeta (Q \sin. \lambda - P \cos. \lambda)}{\sin. m}$$

$$d\phi = f dt + \frac{n (1+\alpha) f dt \cos. m \cos. \lambda}{\sin. m}$$

$$d\psi = \frac{-n (1+\alpha) f dt \cos. \lambda}{\sin. m}.$$

XXVII. Cum sit proxime $\mu = 180^\circ - \lambda$, et $l = m$ in quantitatibus P et Q his valoribus vti licet, ex quo erit:

$$P = \sin. p (\sin. \lambda \sin. (q - \psi) + \cos. m \cos. \lambda \cos. (q - \psi) - \tan g. \varepsilon \sin. m \cos. \lambda \sin. (q - \omega))$$

$$Q = \sin. p (\cos. \lambda \sin. (q - \psi) - \cos. m \sin. \lambda \cos. (q - \psi) + \tan g. \varepsilon \sin. m \sin. \lambda \sin. (q - \omega))$$

existente $\cos. \zeta = \sin. p (\sin. m \cos. (q - \psi) + \tan g. \varepsilon \cos. m \sin. (q - \omega))$

vnde

vnde colligitur :

$$P \sin.\lambda + Q \cos.\lambda = \sin.p \sin.(q-\psi)$$

$$Q \sin.\lambda - P \cos.\lambda = \sin.p(-\cos.m \cos.(q-\psi) + \tang.\varepsilon \sin.m \sin.(q-\omega)).$$

Hinc totum calculum fatis expedite abfoluere licebit; valor tantum anguli λ moram faceffere videtur, ob terminum $\frac{-3\alpha V P dt \cos.\zeta}{(1+\alpha) f n}$, quantitatem minimam n in denominatore inuoluentem; quae fi euanefceret, hic terminus adeo in infinitum excrefceret. Quod cum in motu terrae eueniat, manifeftum eft hanc appropinquandi rationem noftro cafu locum habere non poffe ex quo aliam methodum ingredi conueniet, quam nunc accuratius fum expofiturus.

Alia methodus formulas inuentas euoluendi, ad motum vertiginis terrae magis accommodata.

XXVIII. Refumamus noftras formulas pro motu vertiginis fupra expofitas:

1°. $\frac{dy}{dt} + \alpha f z - 3\alpha V \cos.\zeta \cos.\theta = 0$

2°. $\frac{dz}{dt} - \alpha f y + 3\alpha V \cos.\zeta \cos.\eta = 0$

3°. $dl = -dt(y \sin.\Phi + z \cos.\Phi)$

4°. $d\Phi = f dt - \frac{dt}{\tang.l}(y \cos.\Phi - z \sin.\Phi)$

5°. $d\psi = \frac{dt}{\sin.l}(y \cos.\Phi - z \sin.\Phi)$

vbi

vbi primum obſeruo, ſi vires perturbatrices littera $V = \frac{2g\,e.e}{v^3}$ contentae euaneſcerent, binis prioribus aequationibus ſatisfacere has formulas $y = b\cos.\alpha f t$ et $z = b\sin.\alpha f t$ praeſenti autem caſu quantitatem b euaneſcentem accipi debere. Quare cum ob quantitatem V particulae quaedam ad hos valores y et z accedant, has ipſas particulas inueſtigari conuenit, quibus deinceps reiectis illis membris litteram b inuoluentibus; erit vtendum.

XXIX. Hunc in finem autem opus eſt ante omnia formulas $\cos.\zeta\cos.\eta$ et $\cos.\zeta\cos.\theta$ euolui, vnde facta reductione reperitur:

$$\cos.\zeta\cos.\eta = \tfrac{1}{2}\sin.p^2\left\{\begin{array}{l}-\sin.l\sin.\Phi\sin.2(q-\psi)-\sin.l\cos.l\cos.\Phi-\sin.l\cos.l\cos.\Phi\\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\cos.2(q-\psi)\\ +\tan.\varepsilon\sin.l^2\cos.\Phi\sin.(2q-\psi-\omega)+\tan.\varepsilon\sin.l^2\cos.\Phi\sin.\\ \qquad\qquad(\psi-\omega)+\tan.\varepsilon\cos.l\sin.\Phi\cos.(2q-\psi-\omega)\\ -\tan.\varepsilon\cos.l^2\cos.\Phi\sin.(2q-\psi-\omega)-\tan.\varepsilon\cos.l^2\cos.\Phi\sin.\\ \qquad\qquad(\psi-\omega)-\tan.\varepsilon\cos.l\sin.\Phi\cos.(\psi-\omega)\end{array}\right\}$$

omiſſis terminis quadratum $\tan.\varepsilon^2$ implicantibus vtpote minimis; quam ob cauſam pro $\sin.p^2 = \frac{1}{1+\tan.\varepsilon^2\sin.(q-\omega)^2}$ etiam vnitatem ſcribere licet. Hinc etiam angulo Φ in ſinus et coſinus ſimplices inuoluto, adipiſcimur:

$$\begin{array}{l}4\cos.\zeta\cos.\eta = -\sin.2l\cos.\Phi \begin{array}{l}-\sin.l\\-\sin.l\cos.l\end{array}\cos.(\Phi-2q+2\psi)\begin{array}{l}+\sin.l\\-\sin.l\cos.l\end{array}\cos.(\Phi+2q-2\psi)\\ \qquad\begin{array}{l}-\tan.\varepsilon\cos.2l\\+\tan.\varepsilon\cos.l\end{array}\sin.(\Phi+2q-\psi-\omega)\begin{array}{l}+\tan.\varepsilon\cos.2l\\+\tan.\varepsilon\cos.l\end{array}\sin.(\Phi-2q+\psi+\omega)\\ \qquad\begin{array}{l}+\tan.\varepsilon\cos.2l\\-\tan.\varepsilon\cos.l\end{array}\sin.(\Phi-\psi+\omega)\begin{array}{l}-\tan.\varepsilon\cos.2l\\-\tan.\varepsilon\cos.l\end{array}\sin.(\Phi+\psi-\omega)\end{array}$$

vbi

vbi notandum eſt ψ eſſe longitudinem puncti ſolſtitialis aeſtiui a termino fixo ♈ puta prima ſtella arietis. Quodſi ergo longitudo primae ſtellae arietis a puncto aequinoctiali verno computata ponatur $= x$, erit $\psi + x = 90°$ et $\psi = 90° - x$, indeque $q - \psi = q + x - 90°$, vbi $q + x$ denotabit aſtri longitudinem a puncto aequinoctiali verno computatam quae ex tabulis habetur. Hinc ergo per meros coſinus erit

$$4\cos.\zeta\cos.\eta = -\sin.2l\cos.\Phi + \sin.l(1+\cos.l)\cos.(\Phi - 2q - 2x)$$
$$- \sin.l(1-\cos.l)\cos.(\Phi + 2q + 2x)$$
$$- \tang.\varepsilon(\cos.l - \cos.2l)\cos.(\Phi + 2q + x - \omega) + \tang.\varepsilon$$
$$(\cos.l + \cos.2l)\cos.(\Phi - 2q + \omega - x)$$
$$+ \tang.\varepsilon(\cos.l - \cos.2l)\cos.(\Phi + \omega + x) - \tang.\varepsilon$$
$$(\cos.l + \cos.2l)\cos.(\Phi - \omega - x)$$

vbi $\omega + x$ denotat longitudinem nodi aſcendentis ☊ a puncto aequinoctiali verno computatam.

XXX. Quodſi ergo pro vſu tabularum q denotet ipſam aſtri perturbantis longitudinem a puncto aequinoctiali verno computatam, ſimilique modo ω longitudinem nodi aſcendentis, erit

$$4\cos.\zeta\cos.\eta = -\sin.2l\cos.\Phi + \sin.l(1+\cos.l)\cos.(\Phi - 2q) - \sin.l$$
$$(1 - \cos.l)\cos.(\Phi + 2q)$$
$$+ \tang.\varepsilon(\cos.l - \cos.2l)\cos.(\Phi + \omega) - \tang.\varepsilon(\cos.l$$
$$+ \cos.2l)\cos.(\Phi - \omega)$$
$$- \tang.\varepsilon(\cos.l - \cos.2l)\cos.(\Phi + 2q - \omega) + \tang.\varepsilon(\cos.l$$
$$+ \cos.2l)\cos.(\Phi - 2q + \omega)$$

 vbi

vbi si loco Φ scribatur $\Phi + 90°$ oritur valor alterius formulae $4 \cos. \zeta \cos. \theta$, quam ergo seorsim euolui non est opus. Deinde obseruo si astrum non in circulo aequabiliter circa terram circumferatur, vt longitudinis q incrementa non sint tempori proportionalia, tamen ex cognita inaequalitate motus, hos cosinus, in cosinus aliorum angulorum tempori proportionalium euolui posse, quod etiam de ipsa forma $V = \frac{2gee}{v^3}$ est intelligendum, quae cum illa coniuncta pariter ad cosinus angulorum tempori proportionalium reuocabitur, quoniam ipse angulus Φ celeritatem angularem motus diurni terrae designans, in his integrationibus vt tempori proportionalis spectari potest. Quocirca certum est has formulas ita semper expressum iri vt sit

$$3\alpha V \cos.\zeta \cos.\eta = A \cos.\Phi + B \cos.(\Phi - \beta t) + C \cos.(\Phi - \gamma t) + D \cos.(\Phi - \delta t) + \mathfrak{B} \cos.(\Phi + \beta t) + \mathfrak{C} \cos.(\Phi + \gamma t) + \mathfrak{D} \cos.(\Phi + \delta t) \text{ etc.}$$

$$3\alpha V \cos.\zeta \cos.\theta = -A \sin.\Phi - B \sin.(\Phi - \beta t) - C \sin.(\Phi - \gamma t) - D \sin.(\Phi - \delta t) - \mathfrak{B} \sin.(\Phi + \beta t) - \mathfrak{C} \sin.(\Phi + \gamma t) - \mathfrak{D} \sin.(\Phi + \delta t) \text{ etc.}$$

vbi pro quolibet astro anguli βt, γt, δt etc. quotcunque fuerint, cum coefficientibus facile exhibentur.

XXXI. Statuamus iam $d\Phi = m\,dt$ et $\alpha f = n$, sitque pro quantitatibus y et z non omissis partibus ante commemoratis;

$y = h$

$$y = b\cos.nt + O\cos.\Phi + P\cos.(\Phi - \varepsilon t) + Q\cos.(\Phi - \gamma t) + R\cos.(\Phi - \delta t) + \mathfrak{P}\cos.(\Phi + \varepsilon t) + \mathfrak{Q}\cos.(\Phi + \gamma t) + \mathfrak{R}\cos.(\Phi + \delta t) \text{ etc.}$$

$$z = b\sin.nt - O\sin.\Phi - P\sin.(\Phi - \varepsilon t) - Q\sin.(\Phi - \gamma t) - R\sin.(\Phi - \delta t) - \mathfrak{P}\sin.(\Phi + \varepsilon t) - \mathfrak{Q}\sin.(\Phi + \gamma t) - \mathfrak{R}\sin.(\Phi + \delta t) \text{ etc.}$$

ac facta substitutione in prioribus aequationibus fiet

$$\left.\begin{array}{llll} -nb\sin.nt - mO\sin.\Phi & -(m-\varepsilon)P\sin.(\Phi-\varepsilon t) & -(m+\varepsilon)\mathfrak{P}\sin.(\Phi+\varepsilon t) & \text{etc.} \\ +nb \quad -nO & -nP & -n\mathfrak{P} & \\ \quad\quad +A & +B & +\mathfrak{B} & \end{array}\right\} = 0$$

$$\left.\begin{array}{llll} nb\cos.nt - mO\cos.\Phi & -(m-\varepsilon)P\cos.(\Phi-\varepsilon t) & -(m+\varepsilon)\mathfrak{P}\cos.(\Phi+\varepsilon t) & \text{etc.} \\ -nb \quad -nO & -nP & -n\mathfrak{P} & \\ \quad\quad +A & +B & +\mathfrak{B} & \end{array}\right\} = 0$$

quae cum congruant, consequimur has determinationes:

$$O = \frac{A}{m+n}; \quad P = \frac{B}{n+m-\varepsilon}; \quad Q = \frac{C}{n+m-\gamma}; \quad R = \frac{D}{n+m-\delta} \text{ etc.}$$

$$\mathfrak{P} = \frac{\mathfrak{B}}{n+m+\varepsilon}; \quad \mathfrak{Q} = \frac{\mathfrak{C}}{n+m+\gamma}; \quad \mathfrak{R} = \frac{\mathfrak{D}}{n+m+\delta} \text{ etc.}$$

sicque quotcunque fuerint termini haec integralia facile formantur.

XXXII. His autem valoribus pro y et z inventis colligimus sequentes formas:

$$y\sin.\Phi + z\cos.\Phi = b\sin.(\Phi + nt) + (P - \mathfrak{P})\sin.\varepsilon t + (Q - \mathfrak{Q})\sin.\gamma t + (R - \mathfrak{R})\sin.\delta t \text{ etc.}$$

$$y\cos.\Phi - z\sin.\Phi = b\cos.(\Phi + nt) + O + (P + \mathfrak{P})\cos.\varepsilon t + (Q + \mathfrak{Q})\cos.\gamma t + (R + \mathfrak{R})\cos.\delta t \text{ etc.}$$

ex quibus primo veram poli aequatoris a polo eclipticae diſtantiam $EA = l$ elicimus; cum enim ſit:

$$dl = -b\,dt\,\text{ſin.}(\Phi + nt) - (P - \mathfrak{P})\,dt\,\text{ſin.}\,\beta t - (Q - \mathfrak{Q})\,dt\,\text{ſin.}\,\gamma t - \text{etc.}$$

ſi haec diſtantia media ponatur $= \mathfrak{l}$, erit integrando

$$l = \mathfrak{l} + \frac{b}{m+n}\,\text{coſ.}(\Phi + nt) + \frac{P - \mathfrak{P}}{\beta}\,\text{coſ.}\,\beta t + \frac{Q - \mathfrak{Q}}{\gamma}\,\text{coſ.}\,\gamma t + \frac{R - \mathfrak{R}}{\delta}\,\text{coſ.}\,\delta t \text{ etc.}$$

Deinde pro angulo $EAB = \Phi$, quo motus gyratorii celeritas definitur, accuratius cognoſcendo habemus:

$$\frac{d\Phi}{dt} = f - \frac{b\,\text{coſ.}(\Phi + nt)}{\text{tang.}\,l} - \frac{O}{\text{tang.}\,l} - \frac{(P + \mathfrak{P})}{\text{tang.}\,l}\,\text{coſ.}\,\beta t - \text{etc.}$$

vnde per integrationem elicimus:

$$\Phi = ft - \frac{b\,\text{ſin.}(\Phi + nt)}{(m+n)\,\text{tang.}\,l} - \frac{Ot}{\text{tang.}\,l} - \frac{(P + \mathfrak{P})\,\text{ſin.}\,\beta t}{\beta\,\text{tang.}\,l} - \frac{(Q + \mathfrak{Q})\,\text{ſin.}\,\gamma t}{\gamma\,\text{tang.}\,l} - \text{etc.}$$

Denique pro vera longitudine primae ſtellae arietis $x = 90° - \psi$, colligimus:

$$x = C - \frac{b\,\text{ſin.}(\Phi + nt)}{(m+n)\,\text{ſin.}\,l} - \frac{Ot}{\text{ſin.}\,l} - \frac{(P + \mathfrak{P})\,\text{ſin.}\,\beta t}{\beta\,\text{ſin.}\,l} - \frac{(Q + \mathfrak{Q})\,\text{ſin.}\,\gamma t}{\gamma\,\text{ſin.}\,l} - \frac{(R + \mathfrak{R})\,\text{ſin.}\,\delta t}{\delta\,\text{ſin.}\,l} \text{ etc.}$$

qua aequatione praeceſſio aequinoctiorum cum omnibus inaequalitatibus determinatur.

XXXIII. Hic denotat m celeritatem motus diurni, vbi loco vnius minuti ſecundi aliud quodvis tempus datum accipere licet, dummodo reliquae celeritates ad idem tempus referantur. Sumto ergo tempore vnius diei, erit $m = 360°$ cui etiam littera f aequalis eſt cenſenda, tum vero erit $n = \alpha m$ vbi

vbi notandum, esse $\alpha = \frac{aa - bb}{bb}$ fractionem minimam ita vt n prae m quasi euanescat. Reliquae celeritates angulares nunc etiam ad tempus vnius diei referendae ex motu et vi astri perturbantis peti debent, cuius locum cum vel Sol vel Luna occupare possit, pro vtroque seorsim has anomalias in motu et axe terrae scrutari conueniet. Vnde mox patebit non opus esse ad horum astrorum inaequalitatem motus respicere cum anomaliae ex eorum motu medio resultantes iam adeo sint exiguae, vt quae insuper ex orbitae excentricitate nascerentur, tuto negligi queant.

De perturbatione motus diurni a vi solis producta.

XXXIV. Pro sole ergo inclinatio ε euanescit, ac posito solis motu diurno $= \mu$ est vt supra vidimus $V = \mu\mu$ et celeritas solis $\frac{dq}{dt} = \mu$; existente q longitudine solis media. Cum ergo sit

$$4 \operatorname{cos.} \zeta \operatorname{cos.} \eta = -\operatorname{sin.} 2l \operatorname{cos.} \Phi + \operatorname{sin.} l (1 + \operatorname{cos.} l) \operatorname{cos.} (\Phi - 2q) - \operatorname{sin} l (1 - \operatorname{cos.} l) \operatorname{cos.} (\Phi + 2q)$$

erit $\mathfrak{E} t = 2q$, et $\mathfrak{E} = 2\mu$ tum vero

$$A = -\tfrac{3}{4} \alpha\mu\mu \operatorname{sin.} 2l, \quad B = \tfrac{3}{4} \alpha\mu\mu \operatorname{sin.} l (1 + \operatorname{cos.} l), \quad \mathfrak{B} = -\tfrac{3}{4} \alpha\mu\mu \operatorname{sin.} l (1 - \operatorname{cos.} l)$$

hincque

$$O = \frac{-3\alpha\mu\mu \operatorname{sin.} 2l}{4(1+\alpha)m}; \quad P = \frac{3\alpha\mu\mu \operatorname{sin.} l (1 + \operatorname{cos.} l)}{4((1+\alpha)m - 2\mu)}; \quad \mathfrak{P} = \frac{-3\alpha\mu\mu \operatorname{sin.} l (1 - \operatorname{cos.} l)}{4((1+\alpha)m + 2\mu)}$$

 et

et sequentes litterae Q, $\mathfrak{Q}$ etc. euanescent:

XXXV. Quare primo pro distantia polorum aequatoris et eclipticae $EA = l$, posita hac distantia media $= \mathrm{I}$, erit

$$l = \mathrm{I} + \frac{b}{(1+\alpha)m} \operatorname{cos.}(\Phi + \alpha m t) + \frac{P - \mathfrak{P}}{2\mu} \operatorname{cos.} 2q.$$

Deinde pro angulo $EAB = \Phi$ colligimus

$$\Phi = \left(f + \frac{3\alpha\mu\mu \operatorname{cos.} l^2}{2(1+\alpha)m}\right) t - \frac{(P + \mathfrak{P}) \operatorname{sin.} 2q}{2\mu \operatorname{tang.} l} - \frac{b \operatorname{sin.}(\Phi + \alpha m t)}{(1+\alpha)m \operatorname{tang.} l}$$

vbi iam tempus t in diebus est exprimendum; eritque $f = m - \frac{3\alpha\mu\mu \operatorname{cos.} l^2}{2(1+\alpha)m}$; seu potius denotante f motum terrae diurnum primo impressum is ob vim solis censendus est acceleratus particula $\frac{3\alpha\mu\mu \operatorname{cos.} l^2}{2(1+\alpha)m}$. Denique pro longitudine primae stellae arietis obtinemus:

$$x = C - \frac{b \operatorname{sin.}(\Phi + \alpha m t)}{(1+\alpha) m \operatorname{sin.} l} + \frac{3\alpha\mu\mu \operatorname{cos.} l}{2(1+\alpha)m} t - \frac{(P + \mathfrak{P}) \operatorname{sin.} 2q}{2\mu \operatorname{sin.} l}$$

vnde patet primam stellam arietis quotidie per spatiolum $\frac{3\alpha\mu\mu \operatorname{cos.} l}{2(1+\alpha)m}$ promoueri.

De perturbatione motus diurni a vi lunae producta.

XXXVI. Hic constat pro ε angulum circiter $5°$ capi oportere; quodsi iam q denotet longitudinem lunae mediam, ν eius motum diurnum; ω longitudinem nodi ascendentis et o eius motum diurnum retrogradum et $\frac{dq}{dt} = \nu$ et $\frac{d\omega}{dt} = -o$. Tum vero si massa lunae aequalis esset terrae foret

$V = \nu\nu$;

$V = \nu\nu$; ſi ergo ſtatuatur maſſa terrae ad maſſam lunae vt 1 ad λ, vt poſita terrae maſſa $= M$ futura ſit maſſa lunae $= \lambda M$ erit $V = \lambda\nu\nu$. Conſideremus iam formam:

$$\begin{aligned} 4\cos\zeta\cos\eta = & -\sin 2l\cos\Phi + \sin l(1+\cos l)\cos(\Phi-2q) - \mathrm{tang.}\,\varepsilon(\cos l \\ & \qquad + \cos 2l)\cos(\Phi-\omega) \\ & - \sin l(1-\cos l)\cos(\Phi+2q) + \mathrm{tang.}\,\varepsilon(\cos l \\ & \qquad - \cos 2l)\cos(\Phi+\omega) \\ & + \mathrm{tang.}\,\varepsilon(\cos l+\cos 2l)\cos(\Phi-2q+\omega) \\ & - \mathrm{tang.}\,\varepsilon(\cos l-\cos 2l)\cos(\Phi+2q-\omega) \end{aligned}$$

atque hinc conſequimur hos valores:

$A = -\frac{3}{4}\alpha\lambda\nu\nu\sin 2l$; $\left.\begin{aligned} B &= \tfrac{3}{4}\alpha\lambda\nu\nu\sin l(1+\cos l) \\ \mathfrak{B} &= -\tfrac{3}{4}\alpha\lambda\nu\nu\sin l(1-\cos l) \end{aligned}\right\}\ \beta = 2\nu$

ſicque

$D = -C$ $\qquad \left.\begin{aligned} C &= -\tfrac{3}{4}\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l+\cos 2l) \\ \mathfrak{C} &= +\tfrac{3}{4}\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l-\cos 2l) \end{aligned}\right\}\ \gamma = -\sigma$

et

$\mathfrak{D} = -\mathfrak{C}$ $\qquad \left.\begin{aligned} D &= +\tfrac{3}{4}\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l+\cos 2l) \\ \mathfrak{D} &= -\tfrac{3}{4}\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l-\cos 2l) \end{aligned}\right\}\ \delta = 2\nu+\sigma$

XXXVII. Ex his porro ſequentes valores elicientur:

$$O = \frac{-3\alpha\lambda\nu\nu\sin 2l}{4(1+\alpha)m}; \quad P = \frac{+3\alpha\lambda\nu\nu\sin l(1+\cos l)}{4((1+\alpha)m-2\nu)}$$

$$\mathfrak{P} = \frac{-3\alpha\lambda\nu\nu\sin l(1-\cos l)}{4((1+\alpha)m+2\nu)}$$

$$Q = \frac{-3\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l+\cos 2l)}{4((1+\alpha)m+\sigma)}$$

$$\mathfrak{Q} = \frac{+3\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\cos l-\cos 2l)}{4((1+\alpha)m-\sigma)}$$

R =

$$R = \frac{+3\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\mathrm{cos{.}}\,l + \mathrm{cos{.}}\,2l)}{4((1+\alpha)m - 2\nu - o)}$$

$$\mathfrak{R} = \frac{-3\alpha\lambda\nu\nu\,\mathrm{tang.}\,\varepsilon(\mathrm{cos{.}}\,l - \mathrm{cos{.}}\,2l)}{4((1+\alpha)m + 2\nu + o)}$$

vnde pro obliquitate eclipticae oritur

$$l = \mathrm{l} \ldots + \frac{P-\mathfrak{P}}{2\nu}\mathrm{cof.}\,2q - \frac{Q+\mathfrak{Q}}{o}\mathrm{cof.}\,\omega + \frac{R-\mathfrak{R}}{2\nu+o}\mathrm{cof.}(2q-\omega)$$

pro celeritate rotationis ſeu angulo $EAB = \Phi$ vero

$$\Phi = \ldots\ldots \frac{3\alpha\lambda\nu\nu\,\mathrm{cof.}\,l^2}{2(1+\alpha)m}t - \frac{(P+\mathfrak{P})\,\mathrm{fin.}\,2q}{2\nu\,\mathrm{tang.}\,l} + \frac{(Q+\mathfrak{Q})\,\mathrm{fin.}\,\omega}{o\,\mathrm{tang.}\,l} - \frac{(R+\mathfrak{R})\,\mathrm{fin.}(2q+\omega)}{(2\nu+o)\,\mathrm{tang.}\,l}$$

et pro longitudine $1 * \Upsilon$

$$x = \ldots + \frac{3\alpha\lambda\nu\nu\,\mathrm{cof.}\,l}{2(1+\alpha)m}t - \frac{(P+\mathfrak{P})\,\mathrm{fin.}\,2q}{2\nu\,\mathrm{fin.}\,l} + \frac{(Q+\mathfrak{Q})\,\mathrm{fin.}\,\omega}{o\,\mathrm{fin.}\,l} - \frac{(R+\mathfrak{R})\,\mathrm{fin.}(2q+\omega)}{(2\nu+o)\,\mathrm{fin.}\,l}$$

ſicque a vi lunae motus diurnus primum impreſſus terrae augetur particula $\frac{3\alpha\lambda\nu\nu\,\mathrm{cof.}\,l^2}{2(1+\alpha)m}$.

Euolutio numerica harum formularum.

XXXVIII. Primo cum l denotet diſtantiam polorum aequatoris et eclipticae, erit nunc quidem eius valor medius $= 23°, 29'$, quo in his formulis vti poterimus. Deinde ex tabulis aſtronomicis colligimus:

motum ſolis diurnum medium $\mu = 3548''$
motum lunae diurnum medium $\nu = 47435$
motum nodorum diurnum medium $o = 190\frac{1}{2}$
inclinationem orbitae lunae mediam $\varepsilon = 5°$

et

et pro motu diurno medio ipſius terrae circa axem ſumamus $(1+\alpha)m = 360° = 1296000''$ quandoquidem in terminis minimis valores proxime veros adhibuiſſe ſufficit. Hinc pro formulis ex vi ſolis natis erit:

$$O = -7'',2849\,\alpha\,\text{ſin.}\,2l; \quad \frac{P}{2\mu} = +0,001032\,\alpha\,\text{ſin.}\,l(1+\text{coſ.}\,l)$$
$$0,8624235 \qquad\qquad 7,0137954$$

$$\frac{\mathfrak{P}}{2\mu} = -0,001021\,\alpha\,\text{ſin.}\,l(1-\text{coſ.}\,l)$$
$$7,0090374.$$

Pro formulis autem ex vi lunae natis:

$$O = -1302'',13\,\alpha\lambda\,\text{ſin.}\,l; \quad \frac{P}{2\nu} = +0,014809\,\alpha\lambda\,\text{ſin.}\,l(1+\text{coſ.}\,l)$$
$$3.1146541 \qquad\qquad 8,1705403$$

$$\frac{\mathfrak{P}}{2\nu} = -0,012777\,\alpha\lambda\,\text{ſin.}\,l(1-\text{coſ.}\,l)$$
$$8,1064437$$

$$\frac{Q}{\rho} = -0,59792\,\alpha\lambda(\text{coſ.}\,l+\text{coſ.}\,2l)$$
$$9,7766439$$

$$\frac{\mathfrak{Q}}{\rho} = +0,59811\,\alpha\lambda(\text{coſ.}\,l-\text{coſ.}\,2l)$$
$$9,7767779$$

$$\frac{R}{2\nu+\rho} = +0,00129\,\alpha\lambda(\text{coſ.}\,l+\text{coſ.}\,2l)$$
$$7,1116918$$

$$\frac{\mathfrak{R}}{2\nu+\rho} = -0,00112\,\alpha\lambda(\text{coſ.}\,l-\text{coſ.}\,2l)$$
$$7,0478673.$$

XXXIX. Si porro loco l valorem $23°,29'$ ſubſtituamus hi valores ita ſe habebunt

 pro

pro vi solis

$$O = -5'',3249\,\alpha$$
$$0,7263152$$
$$\frac{P}{2\mu} = +0,0007886\,\alpha$$
$$\frac{\mathfrak{P}}{2\mu} = -0,0000337\,\alpha$$
$$\frac{P-\mathfrak{P}}{2\mu} = +0,0008223\,\alpha$$
$$\frac{P+\mathfrak{P}}{2\mu} = +0,0007549\,\alpha$$

pro vi lunae

$$O = -951'',80\,\alpha\lambda$$
$$2,9785458$$
$$\frac{P}{2\nu} = +0,011313\,\alpha\lambda$$
$$\frac{\mathfrak{P}}{2\nu} = -0,000422\,\alpha\lambda$$
$$\frac{P-\mathfrak{P}}{2\nu} = +0,011735\,\alpha\lambda$$
$$\frac{P+\mathfrak{P}}{2\nu} = +0,010891\,\alpha\lambda$$

$$\frac{Q}{o} = -0,95643\,\alpha\lambda$$
$$\frac{\mathfrak{Q}}{o} = +0,14043\,\alpha\lambda$$
$$\frac{Q-\mathfrak{Q}}{o} = -1,09686\,\alpha\lambda$$
$$\frac{Q+\mathfrak{Q}}{o} = -0,81600\,\alpha\lambda$$
$$\frac{R}{2\nu+o} = +0,00207\,\alpha\lambda$$
$$\frac{\mathfrak{R}}{2\nu+o} = -0,00026\,\alpha\lambda$$
$$\frac{R-\mathfrak{R}}{2\nu+o} = +0,00233\,\alpha\lambda$$
$$\frac{R+\mathfrak{R}}{2\nu+o} = +0,00181\,\alpha\lambda.$$

XL. Quodsi iam longitudinem solis littera p designemus, vt a longitudine lunae q distinguatur, ternae nostrae formulae pro motu terrae diurno inventae ita se habebunt tempore t in diebus expresso:

$$z = I + k\,\mathrm{cos.}(\phi+\alpha m t) + 0,0008223\,\alpha\,\mathrm{cos.}\,2p + 1,09686\,\alpha\lambda\,\mathrm{cos.}\,\omega$$
$$+ 0,011735\,\alpha\lambda\,\mathrm{cos.}\,2q + 0,00233\,\alpha\lambda\,\mathrm{cos.}(2q-\omega)$$

$x = C$

$$x = C - \frac{k\,\text{ſin.}(\Phi + \alpha m t)}{\text{ſin.}\,l} + \frac{5'',3249\,\alpha t}{\text{ſin.}\,l} - \frac{0,0007549\,\alpha}{\text{ſin.}\,l}\,\text{ſin.}\,2p$$
$$+ \frac{951'',80\,\alpha\lambda t}{\text{ſin.}\,l} - \frac{0,010891\,\alpha\lambda}{\text{ſin.}\,l}\,\text{ſin.}\,2q$$
$$- \frac{0,81600\,\alpha\lambda}{\text{ſin.}\,l}\,\text{ſin.}\,\omega$$
$$- \frac{0,00181\,\alpha\lambda}{\text{ſin.}\,l}\,\text{ſin.}(2q + \omega)$$

$$\Phi = ft - \frac{k\,\text{ſin.}(\Phi + \alpha m t)}{\text{tang.}\,l} + \frac{5'',3249\,\alpha t}{\text{tang.}\,l} - \frac{0,0007549\,\alpha}{\text{tang.}\,l}\,\text{ſin.}\,2p$$
$$+ \frac{951'',80\,\alpha\lambda t}{\text{tang.}\,l} - \frac{0,010891\,\alpha\lambda}{\text{tang.}\,l}\,\text{ſin.}\,2q$$
$$- \frac{0,81600\,\alpha\lambda}{\text{tang.}\,l}\,\text{ſin.}\,\omega$$
$$- \frac{0,00181\,\alpha\lambda}{\text{tang.}\,l}\,\text{ſin.}(2q + \omega).$$

XLI. Quodſi hic coefficientes ſinuum et coſinuum in minuta ſecunda conuertamus reperiemus:

$$l = \mathrm{l} + k\,\text{coſ.}(\Phi + \alpha m t) + 170\,\alpha\,\text{coſ.}\,2p + 226230\,\alpha\lambda\,\text{coſ.}\,\omega$$
$$+ 2421\,\alpha\lambda\,\text{coſ.}\,2q + 480\,\alpha\lambda\,\text{coſ.}(2q + \omega)$$

$$x = C - \frac{k\,\text{ſin.}(\Phi + \alpha m t)}{\text{ſin.}\,l} + 13'',363\,\alpha t - 391''\,\alpha\,\text{ſin.}\,2p$$
$$+ 2388'',5\,\alpha\lambda t - 5637\,\alpha\lambda\,\text{ſin.}\,2q$$
$$- 422383\,\alpha\lambda\,\text{ſin.}\,\omega$$
$$- 937\,\alpha\lambda\,\text{ſin.}(2q + \omega)$$

$$\Phi = ft - \frac{k\,\text{ſin.}(\Phi + \alpha m t)}{\text{tang.}\,l} + 12'',256\,\alpha t - 358''\,\alpha\,\text{ſin.}\,2p$$
$$+ 2190,7\,\alpha\lambda t - 5170\,\alpha\lambda\,\text{ſin.}\,2q$$
$$- 387400\,\alpha\lambda\,\text{ſin.}\,\omega$$
$$- 859\,\alpha\lambda\,\text{ſin.}(2q + \omega)$$

 vbi

vbi loco $\frac{b}{(1+\alpha)m}$ ſcripſi k. In ſtatu autem, quo terra verſatur haec conſtans k euaneſcit, quod niſi eueniret, motus quidam oſcillatorius ipſi diurno foret admixtus, cuius oſcillationes abſoluerentur tot diebus, quoties fractio α in vnitate continetur.

De obliquitate eclipticae eiusque variatione.

XLII. Poſito I pro obliquitate eclipticae media, ea erit maxima, ſi longitudines ſolis $=p$ et Lunae $=q$ vel ſint o vel 6 ſignorum, ſimulque nodus aſcendens in ipſum punctum aequinoctiale vernum incidat, vt ſit $\omega=0$; tum enim erit maxima eclipticae obliquitas $=I+170\alpha+229131\alpha\lambda$ min. ſec. Minima autem reperietur, ſi nodus deſcendens incidit in principium arietis vt ſit $\omega=180^\circ$, ſimul vero Sol et Luna in punctis ſolſtitialibus verſentur; tum autem erit minima obliquitas $=I-170\alpha-228171\alpha\lambda$ min. ſec. ſicque tota variatio, quatenus a viribus Solis et Lunae efficitur erit $340\alpha+457302\alpha\lambda$ min. ſec. quae ex obſeruationibus aeſtimatur quaſi $18''$.

XLIII. Quo autem hinc veros valores quantitatum α et λ definire queamus perpendamus promotionem mediam primae ſtellae arietis, quae interteruallo vnius diei fit per spatiolum $13\frac{1}{3}\alpha + 2388\frac{1}{2}\alpha\lambda$ min. sec. hincque interuallo vnius anni per $4870\alpha + 872400\alpha\lambda$ min. sec. quod ex obseruationibus aestimatur $50\frac{1}{3}''$. Prior autem valor $18''$ ob paruitatem non tam certus videtur, vt nulla correctione egeat. Factis ergo aliquot hypothesibus pro nutatione numeri λ indeque fractionis α valor ita prodit

si nutatio	$18''$	$18''\frac{1}{3}$	$18''\frac{2}{3}$	$19''$
erit	$\lambda = \frac{1}{104}$;	$\frac{1}{97}$;	$\frac{1}{91}$;	$\frac{1}{85}$
et	$\alpha = \frac{1}{263}$;	$\frac{1}{275}$;	$\frac{1}{288}$;	$\frac{1}{300}$

vnde patet vt phaenomenis satisfiat, massam lunae vix maiori terrae parti quam $\frac{1}{85}$ aequari posse. Neque ergo sententia *Neutoni* subsistere potest, qui lunae massam parti quadragesimae terrae aequalem aestimauit; et sententia Cel. *Dan. Bernoulli* multo propius ad veritatem accedere est censenda, qua lunae tantum pars terrae septuagesima tribuitur. Ac si nutationem axis terrae non vltra $19''$ per obseruationes statuere licet, massa lunae adhuc est minor, neque partem octogesimam quintam terrae superare potest.

XLIV. Ponamus ergo $\alpha = \frac{1}{300}$ et $\lambda = \frac{1}{85}$, hincque $\alpha\lambda = \frac{1}{25500}$ et variationes in obliquitate eclipticae ita a longitudine solis p, longitudine lunae q et longitudine nodi ascendentis ω pendebunt, vt sit

$$l = I + 0{,}57 \operatorname{cos} 2p + 0{,}095 \operatorname{cos} 2q + 8{,}87 \operatorname{cos}\omega + 0{,}019 \operatorname{cos}(2q + \omega)$$

 coeffi-

coefficientibus in minutis fecundis expreffis. Cum igitur fecunda et quarta aequatio ne decimam quidem minuti fecundi partem conficiant, iis omiffis erit

$$l = \text{I} + 0,57 \operatorname{cof.} 2p + 8,87 \operatorname{cof.} \omega$$

quarum aequationum prior cofinui duplae longitudinis folis eft proportionalis vixque femiminutum fecundum fuperat pofterior vero cofinui longitudinis nodi afcendentis eft proportionalis, et fere ad 9″ afcendere poteft, quod egregie cum obferuationibus confentire videtur.

De praeceffione aequinoctiorum feu longitudine primae ftellae arietis.

XLV. Hic primo confideranda eft huius ftellae longitudo media, quae ad quoduis tempus ex praeceffione annua facile determinatur. Sit ergo $\mathfrak{x}$ longitudo media ad datum quoduis tempus, ac pro eius longitudine vera inuenienda, pofitis ad hoc tempus longitudine folis $= p$, lunae $= q$ et nodi afcendentis $= \omega$ erit eius longitudo vera:

$$x = \mathfrak{x} - 1,30 \operatorname{fin.} 2p - 0,22 \operatorname{fin.} 2q - 16,56 \operatorname{fin.} \omega$$

omiffa poftrema aequatione, vtpote partem trigefimam minuti fecundi non fuperante. Hinc patet fi nodus afcendens fuerit in ♈ 0°, longitudinem mediam imminui $16\frac{1}{2}$ min. fec., fin autem fit in ♎ 0°, tantundem augeri, tum vero fi fol verfetur in ♉ 15°, vel ♏ 15°, eam minui $1\frac{1}{3}$ fec. tantumdem vero

vero augeri, si versetur in ♌ 15° vel ♒ 15°; correctio a loco lunae pendens negligi potest. Ex quo perspicitur longitudinem veram stellarum fixarum a media vsque ad 18″ discrepare posse.

De inaequalitate in ipso motu diurno terrae a viribus solis ac lunae producta.

XLVI. Haec inaequalitas ab angulo Φ pendet, quem videmus non exacte tempori esse proportionalem; cum sit reuera:

$$\Phi = 360°t - 1,20 \sin. 2p - 0,20 \sin. 2q - 15,20 \sin. \omega - 0,03 \sin. (2q + \omega).$$

Est autem Φ angulus EAB, quo primus meridianus terrae AB a circulo coelesti AE, qui est colurus solstitiorum ab occidente in orientem recedit, quod etiam de quouis alio meridiano terrestri, et coluro aequinoctiorum est intelligendum. Ita si secundum motum aequabilem colurus aequinoctiorum, seu punctum aequinoctiale vernum iam per nostrum meridianum, occasum versus angulo f processisset eius vera elongatio a nostro meridiano esset $\Phi = f - 1,20 \sin. 2p - 0,20 \sin. 2q - 15,20 \sin. \omega$ omissa vltima inaequalitate vt insensibili.

XLVII. Quoniam culminatio puncti aequinoctialis verni in ephemeridibus quotidie assignari solet,

let, nunc quidem cognofcimus illis temporis momentis punctum aequinoctiale vernum fi fumma harum aequationum fit negatiua ad meridianum nondum appuliffe, fed ab eo etiamnunc effe remotum tot minutis fecundis, quot aequationes illae praebent. Sin autem tota aequatio fiat pofitiua, indicio id eft punctum aequinoctionale vernum iam per meridianum tranfiiffe, totidemque minutis fecundis ab eo occidentem verfus effe remotum. Illo igitur cafu ferius culminabit temporis interuallo, quo per motum diurnum aequatio illa conficitur, hoc vero cafu, iam ante tantum temporis interuallum culminauit.

XLVIII. Manifeftum autem eft hanc motus diurni inaequalitatem tantum in punctis aequinoctialibus et folftitialibus cerni, cum ea proxime fit aequalis inaequalitati in praeceffione aequinoctiorum; ita vt in ftellis fixis nulla huiusmodi inaequalitas locum fit habitura, fed interualla temporum, quibus eadem ftella fixa ad meridianum appellit, tuto pro aequalibus haberi queant. Refpectu ergo ftellarum fixarum motus vertiginis terrae perfecte eft aequabilis, neque vllam perturbationem a viribus folis et lunae patitur, ficque illa motus irregularitas vnice ab inaequabili aequinoctiorum praeceffione proficifci eft cenfenda, neque ergo variatio illa in longitudine ftellarum fixarum effecta vllam variationem in earum culminatione gignit; ex quo neceffe necesse est, vt punctorum aequinoctialium culminatio totam illam irregularitatem persentiscat.

XLIX. Hi effectus a viribus solis ac lunae in motu terrae diurno producti probe sunt distinguendi ab iis, quos a viribus planetarum in terram agentibus nasci olim demonstraui, qui etiamsi quoque puncta aequinoctialia et obliquitatem eclipticae afficiant, tamen ex fonte prorsus diuerso promanant, dum iis ipsum planum eclipticae immutatur, aequatore manente inuariato. Atque ex his binis causis coniunctis omnes irregularitates, quibus stellae fixae obnoxiae videntur, explicari oportet; quae phaenomena nunc quidem ab Astronomis eo maiori cura sunt obseruanda, cum ad ea perpetuo omnes illae minimae aberrationes in coelo, ad quas maxime sunt attenti, referri debeant.