omnis generis tibiae, quae Organis pneumaticis inseri solent, figuram habeant vel cylindricam vel prismaticam, vt amplitudo vbique sit eadem, haecque sola figura ad Musicam accommoda videtur; quod quidem duplici modo fieri licet, dum eae superne vel apertae sunt, vel clauduntur. At si tibias hyperbolicas claudere velimus, soni multo rudiores edentur, quia tum neutra scala seorsim considerata motum oscillatorium regularem producere valet; ex quo operae pretium haud erit casum, quo tubus hyperbolicus in altero termino apertus in altero vero clausus sumeretur, euolui.

CAPVT VI.

DE

MINIMIS AERIS AGITATIONIBVS IN TVBIS CONICIS.

Problema 92.

128. Si tubus habuerit figuram conicam, aerque in eo contentus vtcunque de statu aequilibrii deturbetur, describere ambas scalas, ex quibus deinceps status aeris ad quoduis tempus definiri queat.

Solutio.

Solutio.

Sit vertex coni in A et recta AB eius axis, in quo sumto puncto quocunque S, positoque intervallo $AS = S$, aëri in S initio eiusmodi inductus sit status, vt densitas fuerit $= Q$ naturali existente $= B$ celeritas vero in directione $AB = \Upsilon$. Cum nunc tubi amplitudo in S sit $\Omega = nnSS$ ex (88) ad hunc casum accommodato habebimus $\alpha = 1$, $\beta = 0$, hincque $M = -\frac{1}{S}$ et $L = \frac{1}{SS}$. Quodsi iam solutionem probl. 84. huc transferamus, et grauitatis actionem negligamus, obtinebimus primo Tab. IX. Fig. 103.

$$0 = \frac{1}{SS}\int SS\,dS\,l\tfrac{Q}{B},$$

tum vero elapso tempore quocunque t status aeris, qui initio fuerat in S ita definitur vt sit spatium translationis

$$\begin{aligned} Ss = \frac{1}{SS}\int SS\,dS\,l\tfrac{Q}{B} - \tfrac{1}{S}\Gamma' : (S+ct) + \tfrac{1}{SS}\Gamma : (S+ct) = v \\ - \tfrac{1}{S}\Delta' : (S-ct) + \tfrac{1}{SS}\Delta : (S-ct) \end{aligned}$$

vnde pro densitate q et celeritate v deducimus:

$$\begin{aligned} \left(\tfrac{dv}{dS}\right) = -\tfrac{2}{S^3}\int SS\,dS\,l\tfrac{Q}{B} - \tfrac{1}{S}\Gamma'' : (S+ct) + \tfrac{2}{SS}\Gamma' : (S+ct) - \tfrac{2}{S^3}\Gamma : (S+ct) \\ + l\tfrac{Q}{B} \quad - \tfrac{1}{S}\Delta'' : (S-ct) + \tfrac{2}{SS}\Delta' : (S-ct) - \tfrac{2}{S^3}\Delta : (S-ct) \end{aligned}$$

ab ob $\frac{d\Omega}{\Omega\, dS} = \frac{2}{S}$ est

$$\begin{aligned} \tfrac{v\,d\Omega}{\Omega\,dS} = \tfrac{2}{S^3}\int SS\,dS\,l\tfrac{Q}{B} - \tfrac{2}{SS}\Gamma' : (S+ct) + \tfrac{2}{S^3}\Gamma : (S+ct) \\ - \tfrac{2}{SS}\Delta' : (S-ct) + \tfrac{2}{S^3}\Delta : (S-ct) \end{aligned}$$

quare cum sit $q = Q\left(1 - \frac{v\,d\Omega}{\Omega\,dS} - \left(\frac{dv}{dS}\right)\right)$ colligimus

$$q = Q\left(1 - l\tfrac{Q}{B} + \tfrac{1}{S}\Gamma'' : (S+ct) + \tfrac{1}{S}\Delta'' : (S-ct)\right)$$

ac denique ob $\upsilon = \left(\frac{dv}{dt}\right)$ erit

$$\upsilon = -\frac{c}{S}\Gamma'':(S+ct) + \frac{c}{SS}\Gamma':(S+ct)$$
$$+\frac{c}{S}\Delta'':(S-ct) - \frac{c}{SS}\Delta':(S-ct).$$

Hinc ergo pro ſtatu initiali ponendo $t = 0$ adipiſcimur:

$$v = \frac{1}{SS}\int SS\,dS\,l\frac{Q}{B} - \frac{1}{S}(\Gamma':S + \Delta':S) + \frac{1}{SS}(\Gamma:S + \Delta:S)$$
$$q = Q\left(1 - l\frac{Q}{B} + \frac{1}{S}(\Gamma'':S + \Delta'':S)\right)$$
$$\upsilon = -\frac{c}{S}(\Gamma'':S - \Delta'':S) + \frac{c}{SS}(\Gamma':S - \Delta':S).$$

Cum igitur tum fuerit $v = 0$, $q = Q$ et $\upsilon = \Upsilon$, ſequentes nanciſcimur aequationes ex quibus naturam binarum functionum Γ et Δ definiri oportet

$$\text{I.}\quad \int SS\,dS\,l\frac{Q}{B} - S(\Gamma':S + \Delta':S) + \Gamma:S + \Delta:S = 0$$
$$\text{II.}\quad S\,l\frac{Q}{B} = \Gamma'':S + \Delta'':S$$
$$\text{III.}\quad \frac{1}{c}SS\Upsilon = -S(\Gamma'':S - \Delta'':S) + \Gamma':S - \Delta':S$$

quarum prima differentiata cum ſecunda conuenit, vti quidem rei natura poſtulat, ita vt duabus tantum conditionibus ſit ſatisfaciendum. Ponamus breuitatis gratia:

$\Gamma':S + \Delta':S = y$ et $\Gamma':S - \Delta':S = z$ vt fiat

$$\text{II.}\quad S\,dS\,l\frac{Q}{B} = dy \quad \text{et} \quad \text{III.}\quad \frac{1}{c}SS\Upsilon\,dS = -S\,dz + z\,dS$$

hincque

$$y = \int S\,dS\,l\frac{Q}{B} \quad \text{et} \quad -\frac{z}{S} = \frac{1}{c}\int \Upsilon\,dS \quad \text{ſeu} \quad z = -\frac{S}{c}\int \Upsilon\,dS,$$

vnde

vnde functiones ita definiuntur, vt fit:

$$\Gamma' : S = \tfrac{1}{2}\int S\, dS\, l\tfrac{Q}{B} - \tfrac{S}{2c}\int \Upsilon\, dS \text{ et}$$

$$\Delta' : S = \tfrac{1}{2}\int S\, dS\, l\tfrac{Q}{B} + \tfrac{S}{2c}\int \Upsilon\, dS$$

hincque porro differentiando

$$\Gamma'' : S = \tfrac{1}{2} S\, l\tfrac{Q}{B} - \tfrac{1}{2c}\int \Upsilon\, dS - \tfrac{1}{2c} S\Upsilon$$

$$\Delta'' : S = \tfrac{1}{2} S\, l\tfrac{Q}{B} + \tfrac{1}{2c}\int \Upsilon\, dS + \tfrac{1}{2c} S\Upsilon$$

integrando vero reperitur:

$$\Gamma : S = \tfrac{1}{2} S\int S dS l\tfrac{Q}{B} - \tfrac{1}{2}\int SS dS\, l\tfrac{Q}{B} - \tfrac{1}{4c} SS\int \Upsilon dS + \tfrac{1}{4c}\int SS\Upsilon dS$$

$$\Delta : S = \tfrac{1}{2} S\int S dS l\tfrac{Q}{B} - \tfrac{1}{2}\int SS dS\, l\tfrac{Q}{B} + \tfrac{1}{4c} SS\int \Upsilon dS - \tfrac{1}{4c}\int SS\Upsilon dS.$$

Construantur ergo super axe ex dato statu initiali duae lineae curuae AQD et $A\Upsilon F$ sumendo applicatas

$$SQ = S\, l\tfrac{Q}{B} \text{ et } S\Upsilon = \tfrac{1}{c}\int \Upsilon\, dS + \tfrac{1}{c} S\Upsilon$$

et functionum Γ et Δ natura inde ita determinabitur vt fit

$$\Gamma'' : S = \tfrac{1}{2} SQ - \tfrac{1}{2} S\Upsilon;\quad \Delta'' : S = \tfrac{1}{2} SQ + \tfrac{1}{2} S\Upsilon$$

hincque porro per areas

$$\Gamma' : S = \tfrac{1}{2} ASQ - \tfrac{1}{2} AS\Upsilon;\quad \Delta' : S = \tfrac{1}{2} ASQ + \tfrac{1}{2} AS\Upsilon$$

tum vero si haec forma $M : ASQ$ denotet eam quantitatem quae exprimit integrale $\int dS.\, ASQ$ habebitur:

$$\Gamma : S = +\tfrac{1}{2} M : ASQ - \tfrac{1}{2} M : AS\Upsilon$$

$$\Delta : S = +\tfrac{1}{2} M : ASQ + \tfrac{1}{2} M : AS\Upsilon.$$

His curuis deſcriptis poſt tempus elapſum $= t$, a puncto S vtrinque abſcindantur interualla $ST = St = ct$ et ex applicatis his punctis reſpondentibus ſtatus aeris qui initio ad S verſabatur nunc ita definitur, vt ſit

$$q = Q\left(1 - \frac{SQ}{S} + \frac{1}{2S}(TM - TN + tm + tn)\right)$$

$$u = \frac{c}{2S}(tm + tn - TM + TN) + \frac{c}{2SS}(ATM - ATN - Atm - Atn)$$

at ſpatiolum translationis $Ss = v$ ita exprimetur:

$$Ss = \frac{1}{SS}\int SS\,dS\,l\frac{Q}{B} - \frac{1}{2S}(ATM - ATN + Atm + Atn)$$
$$+ \frac{1}{2SS}(M:ATM - M:ATN)$$
$$+ \frac{1}{2SS}(M:Atm + M:Atn) \quad \text{ſeu}$$

$$Ss = \frac{1}{SS}\int SS\,dS\,l\frac{Q}{B} - \frac{1}{2S}(ATM - ATN + Atm + Atn) + \frac{1}{2SS}$$
$$(M:ATM - M:ATN + M:Atm + M:Atn)$$

vbi eſt $\int SS\,dS\,l\frac{Q}{B} = \int S.SQ.\,dS = S.ASQ - M:ASQ.$

Coroll 1.

128. Ad quoduis ergo tempus t ſumtis interuallis $ST = St = ct$ aeris qui initio erat in S ſtatus ita definitur vt ſit

I. denſitas $q = Q\left(1 + \frac{1}{2S}(TM + tm - 2SQ) - \frac{1}{2S}(TN - tn)\right)$

II. celeritas $u = \frac{c}{2S}(tm - TM) + \frac{c}{2S}(TN + tn) + \frac{c}{2SS}.tmTM - \frac{c}{2SS}(ATN + Atn)$

III. ſpatiolum $Ss = \frac{1}{2S}(2ASQ - ATM - Atm) + \frac{1}{2S}.tnTN)$

$- \frac{1}{2SS}(2M:ASQ - M:ATM - M:Atm) - \frac{1}{2SS}(M:ATN - M:Atn)$

Coroll.

Coroll. 2.

129. Si diſtantia A S conſideretur vt infinita, tubus conicus abibit in cylindricum; et quia tum fit

$$\tfrac{1}{S}.\,SQ = l\tfrac{Q}{B},\ \tfrac{1}{S}.\,S\Upsilon = \tfrac{\Upsilon}{c};\ \tfrac{1}{S}.\,ASQ = \int dS\,l\tfrac{Q}{B},$$

ideoque

$$\tfrac{1}{SS}.\,ASQ = 0;\ \text{at}\ \tfrac{1}{SS}\,M:ASQ = \int dS\,l\tfrac{Q}{B}$$

tum vero $\frac{1}{S}.\,AS\Upsilon = \frac{1}{c}\int \Upsilon\,dS$ hinc

$$\tfrac{1}{SS}\,AS\Upsilon = 0\ \text{et}\ \tfrac{1}{SS}\,M:AS\Upsilon = \tfrac{1}{c}\int \Upsilon\,dS,$$

hae formulae cum illis, quae ſupra pro tubus cylindricis ſunt inuentae, conuenient.

Scholion 1.

130. Conſtructio prioris ſcalae A Q D, ex denſitate initiali formatae nulli eſt ſubiecta difficultati, at altera ſcala A Υ F non ſimpliciter ex celeritatibus initialibus Υ conſtruitur, ſed inſuper formulam quandam integralem inuoluit, quemadmodum ergo hinc eam conſtrui oporteat, perpendamus. Deſcripta ergo linea curua A v C cuius applicatae S v ipſam celeritatem initialem Υ in loco S exhibeant, ex ea ſcala illa A Υ F ita conſtruitur, vt ſit eius applicata $S\Upsilon = \frac{1}{c}AS.Sv + \frac{1}{c}ASv$ tum vero erit huius curuae area $AS\Upsilon = \frac{1}{c}AS.ASv$, cum ſit vt vidimus $\int dS(\Upsilon S + \int \Upsilon\,dS) = S\int \Upsilon\,dS$. Hanc ergo curuam A v C loco illius indroducendo, habebimus

T N =

$$TN = \tfrac{1}{c} AT . TL + \tfrac{1}{c} ATL;$$

$$tn = \tfrac{1}{c} At . tl + \tfrac{1}{c} . Atl$$

$$ATN = \tfrac{1}{c} . AT . ATL; \; Atn = \tfrac{1}{c} At . Atl$$

vnde fit pro formula celeritatum

$$v = \tfrac{c}{2S}(tm - TM) + \tfrac{c}{2SS} . tm\,TM + \tfrac{1}{2S}(AT . TL + At . tl) - \tfrac{ct}{2SS} . TL\,tl$$

ob $ST = St = ct$

Atque hinc iam perſpicitur, ſi per totum ſpatium Tt tam denſitas initialis fuerit naturalis, quam celeritas nulla, tum etiam fore $q = Q$ et $v = 0$, quod poſterius ex formula priori minus perſpicitur. Si enim intra ſpatium At initio fuerit motus, et curua AvC ibi incluſerit aream quandam, etiamſi deinceps haec curua tota in axem incidat, haec area praebebit applicatas pro curua AYF etiam per totum axem ſequentem tT, vnde neque areae ATN et Atn neque applicatae TN et tn euaneſcent: ſicque dubium foret, vtrum forma

$$\tfrac{c}{2S}(TN + tn) - \tfrac{c}{2SS}(ATN + Atn)$$

hoc caſu euaneſceret; quod autem nunc quidem ſcala AvC in calculum introducta neceſſario euenire debere intelligitur.

Scholion 2.

131. Loco indicis M, quem ſupra areis praefiximus, vt ſcriptio $M: ASQ$ denotet integrale $\int dS . ASQ$ commodius ſigno ſummatorio $\int$ vtemur, vt

vt $\int ASQ$ idem denotet, quod $\int dS.\,ASQ$, quoniam differentiale abſciſſae dS facile mente ſuppletur. Hoc praemiſſo etiam pro ſcala $A\Upsilon F$ loco $M:AS\Upsilon$ ſcribam $\int AS\Upsilon$ quae forma autem ex ſcala celeritatum naturali AvC ita determinatur vt ſit

$$\int AS\Upsilon = \tfrac{1}{c}(AS\int ASv - \int\int ASv)$$

ideoque

$$M:ATN = \tfrac{1}{c}(AT\int ATL - \int\int ATL)$$
$$M:Atn = \tfrac{1}{c}(At\int Atl - \int\int Atl)$$

Quare per ſcalam denſitatum AQD et celeritatum AvC noſtrae determinationes ita ſe habebunt

$$q = Q(1 + \tfrac{1}{2S}(TM + tm - 2SQ) - \tfrac{1}{2cS}(AT.TL - At.tl + tl\,TL)$$
$$\vartheta = \tfrac{c}{2S}(tm - TM) + \tfrac{c}{2SS}TMtm + \tfrac{1}{2S}(AT.TL + At.tl) - \tfrac{ST}{2SS}.tlTL$$
$$Ss = \tfrac{1}{2S}(SQtm - SQTM) - \tfrac{1}{2SS}(\int SQtm - \int SQTM)$$
$$+ \tfrac{1}{2cS}(AT.ATL - At.Atl) - \tfrac{1}{2cSS}(AT.\int ATL - At.\int Atl - \int\int tlTL)$$

quae ad vſum magis videntur accommodatae.

Problema 93.

132. Si in tubo conico ABb in infinitum extenſo alicubi per ſpatiolum minimum GH pulſus excitetur, propagationem huius pulſus in tubo verſus Bb ortam determinare. Tab. IX. Fig. 104.

Solutio.

Cum pulſus in ſpatiolo GH contineatur, in vtroque termino G et H denſitatem naturalem et celeritatem nullam esse oportet. Sit igitur $G\,l\,H$ scala celeritatum, cuius applicata $t\,l$ celeritatem ipsam initialem in loco t exprimat vt sit $t\,l = \Upsilon$ directione ad B versa: pro scala densitatum $G\,m\,H$ vero applicata $t\,m = A\,t.\,l\frac{Q}{B}$ denotante Q densitatem in t initialem, et B naturalem. His positis consideremus locum tubi quemcunque S; existente $AS = S$ et quia ambae scalae extra spatium GH in axem incidunt, aër in S tam diu in aequilibrio perseuerabit, quoad ab initio effluxerit tempus $= \frac{SH}{c}$ min. sec. elapso autem tempore $\frac{SG}{c}$ pulsus iterum cessasse est existimandus ita vt totus pulsus ibi nonnisi per tempus $\frac{GH}{c}$ sit duraturus. Ponamus ergo ab initio elapsum esse tempus $t = \frac{St}{c}$, et quia ad alteram partem B versus aequali spatio $ST = St$ abscisso ibi ambae applicatae TL et TM euanescunt, densitas aeris q et celeritas ȣ versus B tendens in loco S ita exprimentur:

$$q = B\left(1 + \frac{t\,m}{2\,S} + \frac{A\,t.\,t\,l}{2\,c\,S} - \frac{H\,t\,l}{2\,c\,S}\right)$$

quia initio densitas in S erat $= B$, tum vero erit:

$$\text{ȣ} = \frac{c.\,t\,m}{2\,S} + \frac{c.\,H\,t\,m}{2\,S\,S} + \frac{A\,t.\,t\,l}{2\,S} - \frac{S\,t.\,H\,t\,l}{2\,S\,S}.$$

Cum autem interuallum GH minimum assumatur, areae $H\,t\,l$ et $H\,t\,m$ pro euanescentibus haberi possunt; quare ob $t\,l = \Upsilon$ et

$$t\,m = A\,t\,l\frac{Q}{B} = A\,t\left(\frac{Q}{B} - 1\right) \text{ habebimus:}$$

$$q = B\left(1 + \frac{A\,t}{2\,AS}\left(\frac{Q}{B} - 1\right) + \frac{A\,t.\,\Upsilon}{2\,c.\,AS}\right) \text{ seu } l\frac{q}{B} = \frac{A\,t}{2\,AS}\left(l\frac{Q}{B} + \frac{\Upsilon}{c}\right)$$

$$\text{ȣ} = \frac{c.\,A\,t}{2\,AS}\left(\frac{Q}{B} - 1\right) + \frac{A\,t.\,\Upsilon}{2\,AS} = \frac{c.\,A\,t}{2\,AS}\left(l\frac{Q}{B} + \frac{\Upsilon}{c}\right).$$

Coroll.

Coroll. 1.

133. Si areolas H $t\,l$ et H $t\,m$ negligere nolimus, habebimus

$$l\frac{q}{B} = \frac{A\,t}{2\,AS}\left(l\frac{Q}{B} + \frac{T}{c}\right) - \frac{ar.\,H\,t\,l}{2\,c.\,AS} \text{ et}$$

$$\vartheta = \frac{c.\,A\,t}{2\,AS}\left(l\frac{Q}{B} + \frac{T}{c}\right) + \frac{c}{2\,AS^2}\left(\mathrm{Ar.}\,H\,t\,m - \frac{S\,t}{c}\,\mathrm{Ar.}\,H\,t\,l\right)$$

vbi $\frac{S\,t}{c}$ denotat tempus ab initio elapſum t in min. ſec. expreſſum.

Coroll. 2.

134. Areas autem has eo minus negligere licet, quo maius ſpatiolum GH, et quo propius fuerit vertici coni A. quoniam tum ob $t\,l = T$, area H $t\,l$ prae rectangulo A $t.\,t\,l$ neutiquam pro euaneſcente haberi poteſt.

Coroll. 3.

135. Si tempus elapſum t maius fuerit quam $\frac{SG}{c}$, ita vt punctum t vltra G verſus A cadat, vbi initio erat $l\frac{Q}{B} = 0$ et $T = 0$; pro loco S adhuc erit:

$$l\frac{q}{B} = -\frac{Ar.\,G\,l\,H}{2.\,c\,AS} \text{ et } \vartheta = \frac{c}{2\,AS^2}\left(\mathrm{Ar.}\,G\,m\,H - t.\,G\,l\,H\right)$$

tum ergo neque denſitas eſt naturalis neque motus penitus extinctus.

Scholion.

136. Omnino ſingulare eſt hoc phaenomenon, quod etiamſi pulſus in S iam ceſſauiſſe eſt aeſtimandus

dus tamen aeris aequilibrium nondum prorſus ſit reſtitutum, ſed fieri poſſit vt etiamnum tam denſitas q a naturali B diſcrepet, quam aer ipſe motu quodam proferatur. Diſcrimen quidem hoc in valde magnis diſtantiis A S vt nullum ſpectari poteſt; in minoribus vero eo magis eſt notatu dignum, quod perpetuo durare videatur. cum calculus nullam porro mutationem indicet. Verum hic obſeruari oportet, noſtras ſcalas hic non vltra verticem A porrigi, earumque continuationem ex altera parte ipſius A ſimilem pulſum G H ad parem diſtantiam implicare, quo ſi tempus S t pertigerit in S nouus pulſus ſecundarius excitetur, quo finito demum aer ad S prorſus in aequilibrium reſtituatur. Hic enim ſimilis repetitio oriri debet ac ſi tubus eſſet cylindricus et in A clauſus: nunc autem videmus hoc diſcrimen intercedere, quod in caſu tubi conici interea dum pulſus ſecundarius ad S appellit, aerem ibi quandam adhuc agitationem retinere ſecus ac fit in cylindricis. Quare quemadmodum ambae ſcalae vltra coni verticem continuari debeant in ſequente Problemate inueſtigabimus.

Problema 94.

Tab. IX Fig. 105. 137. Si circa verticem coni A aer vtcunque de ſtatu aequilibrii deturbetur ſeu pulſus ibi quicunque excitetur ſcalas ambas quibus ad propagationem definiendam opus eſt, retro vltra verticem A continuare, indeque propagationem in tubo pro quouis loco S determinare.

Solutio.

Solutio.

Extendatur pulſus primo excitatus per ſpatium A D, et in loco quocunque T poſito interuallo A T $= S$, fuerit denſitas aeris $= Q$ naturali exiſtente $= B$, celeritas vero $= \Upsilon$ ſecundum directionem A B: hinc conſtruatur primo curua A M D ſumendo applicatas T M $= S\, l \frac{Q}{B}$, quae ergo vltra D in ipſum axem D B incidit. Deinde conſtruatur etiam curua A N F ſumendo applicatas T N $= \frac{S\Upsilon + \int \Upsilon dS}{c}$, cuius quidem primum membrum in D vbi $\Upsilon = 0$ euaneſcit, alterum vero $\frac{1}{c}\int \Upsilon\, dS$ ibi certum valorem adipiſcetur, cui ſequentes applicatae omnes verſus B erunt aequales, ita vt haec ſcala A N F vltra F abeat in rectam F G ipſi axi A B parallelam; at area huius curuae ita definietur vt ſit A T N $= \frac{S}{c} \int \Upsilon\, dS$: vnde cum puncto T in D promoto fiat applicata D F $= \frac{1}{c}\int \Upsilon\, dS$, erit area A D F $=$ A D . D F perinde ac ſi tota ſcala A N F G eſſet recta G F ipſi axi parallela et ſupra verticem A vsque continuata. His poſitis primo quaeritur quomodo has ſcalas vltra A continuari oporteat, vbi ante omnia eſt obſeruandum aerem in ipſo vertice A nullum plane motum concipere poſſe. Quare ſi tubi punctum S, in quo ſupra generatim celeritatem υ definiuimus, in A transferamus, formula ibi pro celeritate exhibita

$$\upsilon = \frac{c}{2S}(tm - TM + TN + tn) + \frac{c}{2SS}(ATM - Atm - ATN - Atn)$$

hoc caſu nulla eſſe debet. At hoc caſu habebimus $S = 0$, quia in hac formula poſuimus A S $= S$, ex quo neceſſe eſt continuationem quaeſitam ita eſſe

 com-

comparatam vt ſit $tm = TM$ et $tn = -TN$. Quamobrem in noſtra figura ſcalam A M D ſine vlla variatione ſuper axe A d deſcribi conuenit vt ſit ea A md altera vero ſcala A N F G ibi ad contrariam axis partem deſcribi debet vt ſit A nfg. His iam ambabus ſcalis vltra A continuatis conſideremus in tubo locum quemcunque S, ſtatuendo nunc diſtantiam a vertice $AS = S$, vbi cum aer initio fuerit in aequilibrio, ideoque $T = 0$ et $Q = B$ hincque $SQ = 0$, elapſo tempore $= t$, capiamus vtrinque ab S interualla $ST' = St' = ct$, ſintque in punctis T' et t' applicatae ambarum ſcalarum $T'M'$, $T'N'$ et $t'm'$ $t'n'$; atque ex his ſtatus aeris in S, ita definietur vt ſit ibi

densitas $q = B\left(1 + \frac{1}{2S}(T'M' + t'm') - \frac{1}{2S}(T'N' - t'n')\right)$ et

celeritas $v = \frac{c}{2S}(t'm' - T'M') + \frac{c}{2S}(T'N' + t'n')$

$$- \frac{c}{2SS}(At'm' - AT'M') - \frac{c}{2SS}(AT'N' + At'n')$$

vnde primo patet, quamdiu tempus ab initio elapſum t minus fuerit quam $\frac{SD}{c}$, tum ob

$$T'M' = 0, \; t'm' = 0, \text{ et } T'N' = t'n' = DF$$

fore $q = B$ et ob

$$At'm' = AT'M' \text{ et } AT'N' = AT'.DF$$

atque $At'n' = At'.DF$ prodire

$v = \frac{c}{S}.DF - \frac{c.DF}{2SS}(AT' + At') = 0$ ob $AT' + At' = 2AS = 2S$,

tamdiu ergo aer ibi in aequilibrio manebit. Statim autem ac tempus elapſum t maius fit quam $\frac{SD}{c}$,

aer

aer in S agitari incipiet; ponamus ergo elapſum iam eſſe tempus $t = \frac{AS}{c}$, et quia nunc eſt

$T'M' = o$, $t'm' = o$, $T'N' = DF$, $t'n' = o$,
$AT'M' = AMD$ $At'm' = o$, $AT'N' = AT'.DF = 2S.DF$ et $At'n' = o$
erit tum in loco S

denſitas $q = B(1 - \frac{DF}{2S})$ et

celeritas $\mathfrak{u} = \frac{c.DF}{2S} + \frac{c.AMD}{2SS} - \frac{c.DF}{S} = \frac{c.AMD}{2SS} - \frac{c.DF}{2S}$.

Pro tempore autem quouis medio elapſo $t = \frac{ST}{c}$ elicitur

denſitas $q = B(1 + \frac{TM + TN - DF}{2S})$ et

celeritas $\mathfrak{u} = \frac{c}{2S}(TM + TN - DF) + \frac{c}{2SS}(DTM + DFTN - DT.DF)$.

Videamus denique etiam quomodo ſtatus aeris in S ſe ſit habiturus cum effluxerit tempus

$t = \frac{Sd}{c} = \frac{AS + AD}{c}$;

tum vero erit

$T'M' = o$, $AT'M' = AMD : T'N' = DF : AT'N' = AT'.DF = 2S.DF + AD.DF$:

porro

$t'm' = o$, $At'm' = Amd = AMD$, $t'n' = -df = -DF$, $At'n' = -Adf = -AD.DF$,

ideoque

denſitas $q = B(1 - \frac{DF}{S})$ et

celeritas $\mathfrak{u} = -\frac{c.DF}{S}$.

Coroll.

Coroll. 1.

138. Celeritas ergo pulſus, cum ad locum quemuis S peruenerit duabus conſtat partibus, quarum prior decreſcit in ratione diſtantiae A S = S, poſterior vero in ratione duplicata diſtantiae, quae autem poſterior pars prae priori euaneſcit ſtatim ac diſtantia A S = S fit modica.

Coroll. 2.

139. Cum impulſio in organo auditus facta pendeat ſine dubio a celeritate agitationis in quouis pulſu, hinc patet quomodo ſonus aucta diſtantia continuo diminuatur; et quia vis impulſionis quadrato celeritatis proportionalis ſtatuenda videtur, ſoni debilitatio ſequetur rationem duplicatam diſtantiarum.

Scholion. 1.

140. Inſignis hic difficultas occurrit, quod poſtquam pulſus penitus per locum S tranſierit, aer tamen ibi non in aequilibrium reſtituatur, niſi pulſus initialis ita fuerit comparatus, vt interuallum D F euaneſcat. Talis certe commotio poſt tranſitum pulſus nullo modo admitti poteſt, ideoque calculus noſter defectus potius cuiuspiam arguendus videtur, niſi dicere velimus, omnem agitationem aeri primum inductam neceſſario ita ſemper eſſe comparatam, vt interuallum D F inde prodeat euaneſcens. Quoniam vero hoc nulla ratione probari poteſt, origo

go huius incommodi in eo quaerenda videtur, quod cum interuallum D F ex valore integrali $\frac{1}{c}\int \Upsilon\, dS$ sit natum, in hac ipsa integratione iusta constans sit neglecta: dummodo ergo talem constantem introduxissemus, qua hoc integrale per totam scalam celeritatum extensam ad nihilum fuisset reductum, omnis haec difficultas penitus euanuisset, cum inde etiam curua A N F vltra D tota in axem D B incidisset. Nullum ergo est dubium quin hic origo istius defectus calculi sit sita, totumque calculum hoc modo expediri conueniat. Talia autem incommoda in argumento prorsus nouo minime sunt miranda, et sperare licet, cum id diligentius fuerit excultum, tum omnia sponte dissipatum iri. Hic autem continuatio scalarum vltra verticem A merito suspecta videtur, propterea quod in hoc loco sit $S = 0$, iique termini quos destrui oportet infiniti; cui incommodo in sequenti problemate medelam afferre conabor.

Scholion 2.

141. Ex hoc problemate colligere licet, quemadmodum pulsus quicunque in libero aere quaqua versus propagetur; si enim quasi centrum pulsus propagati sit in A, vniuersus aer circumfusus in infinitos conos, quorum vertices in A concurrant distributus concipi potest; et manifestum est per quemlibet eorum pulsum perinde propagatum iri, ac per liberum aerem quoniam qua duo huiusmodi tubi se mutuo tangunt, ibi densitas ac propterea etiam

 pressio

preſſio eſt eadem, ita vt etiam tubis remotis propagatio eandem legem ſit ſecutura. Nunc ergo intelligimus pulſus in libero aere eadem prorſus celeritate propagari atque in tubis cylindricis; at vero ob tuborum amplificationem pulſus continuo debilitari, ita vt celeritas cuiusque agitationis diminuatur in ratione diſtantiarum a pulſu initiali A; latitudinem vero pulſuum, ceteraſque ſoni qualitates in propagatione non alterari.

Problema 95.

Tab. IX. Fig. 106. 142. Si tubus conicus citra verticem I in A a ſit terminatus ibique ſiue apertus ſiue clauſus, ex agitatione initiali aeri in tubo A a B b contento inducta formulas pro motu ſequente definiendo exhibere.

Solutio.

Poſita termini A a vertice coni I diſtantia I A $= a$, ſumatur in cono locus quicunque S vocata diſtantia A S $= s$, vt diſtantia quam ante poſuimus S hic ſit $= a + s$, initio autem in S fuerit denſitas $=$ Q naturali exiſtente $=$ B et celeritas ſecundum S B $=$ T. Elapſo autem tempore quocunque t eiusdem aeris, qui initio fuerat in S, ſit denſitas $= q$, celeritas $= \mathfrak{v}$ et translationis ſpatiolum S $s = v$. Cum iam amplitudo tubi in S ſit $\Omega = nn(a+s)^2$, erit $\frac{d\Omega}{\Omega\, ds} = \frac{2}{a+s}$, hincque ex formulis ſupra inuentis obtinebimus:

$\mathfrak{v} =$

$$v = \frac{1}{(a+s)^2}\int (a+s)^2\, ds\, l\tfrac{Q}{B} - \frac{1}{a+s}\Gamma'\colon(s+ct) + \frac{1}{(a+s)^2}\Gamma\colon(s+ct)$$
$$- \frac{1}{a+s}\Delta'\colon(s-ct) + \frac{1}{(a+s)^2}\Delta\colon(s+ct)$$

$$q = Q\,(1 - l\tfrac{Q}{B} + \frac{1}{a+s}\Gamma''\colon(s+ct) + \frac{1}{a+s}\Delta''\colon(s-ct) \quad \text{et}$$
$$\Upsilon = \frac{-c}{a+s}\Gamma''\colon(s+ct) + \frac{c}{a+s}\Delta''\colon(s-ct) + \frac{c}{(a+s)^2}\Gamma'\colon(s+ct)$$
$$- \frac{c}{(a+s)^2}\Delta'\colon(s-ct)$$

vnde pro ſtatu initiali cognito colligimus:

$$0 = \frac{1}{(a+s)^2}\int (a+s)^2\, ds\, l\tfrac{Q}{B} - \frac{1}{a+s}(\Gamma'\colon s + \Delta'\colon s)$$
$$+ \frac{1}{(a+s)^2}(\Gamma\colon s + \Delta\colon s)$$
$$0 = -l\tfrac{Q}{B} + \frac{1}{a+s}(\Gamma''\colon s + \Delta''\colon s)$$
$$\Upsilon = \frac{-c}{a+s}(\Gamma''\colon s - \Delta''\colon s) + \frac{c}{(a+s)^2}(\Gamma'\colon s - \Delta'\colon s).$$

Ex illa fit

$$\Gamma''\colon s + \Delta''\colon s = (a+s)\,l\tfrac{Q}{B} \quad \text{et} \quad \Gamma'\colon s + \Delta'\colon s = \int (a+s)\, ds\, l\tfrac{Q}{B}$$

atque

$$\Gamma\colon s + \Delta\colon s = \int ds \int (a+s)\, ds\, l\tfrac{Q}{B}$$

ex iſta vero

$$\int \Upsilon ds = \frac{-c}{a+s}(\Gamma'\colon s - \Delta'\colon s), \quad \text{ſeu} \quad c(\Gamma'\colon s - \Delta'\colon s) = -(a+s)\int \Upsilon ds$$

hincque

$$c\,(\Gamma''\colon s - \Delta''\colon s) = -\int \Upsilon\, ds - (a+s)\Upsilon \quad \text{et} \quad c\,(\Gamma\colon s - \Delta\colon s)$$
$$= -\int (a+s)\, ds \int \Upsilon\, ds.$$

Qui valores in prima aequatione ſubſtituti praebent:

$$0 = \int (a+s)^2 ds\, l\tfrac{Q}{B} - (a+s)\int (a+s)\, ds\, l\tfrac{Q}{B} + \int ds \int (a+s)\, ds\, l\tfrac{Q}{B}$$

 vnde

vnde ambiguitas ob conſtantem per integrationem ingreſſam oriunda tollitur, cum debeat eſſe:

$$\int ds \int (a+s)\, ds\, l\tfrac{Q}{B} = (a+s)\int (a+s)\, ds\, l\tfrac{Q}{B} - \int (a+s)^2\, ds\, l\tfrac{Q}{B}$$

quae duo integralia ſimplicia ita capiantur, vt poſito $s=0$ euaneſcant. Functiones ergo Γ et Δ ita definiüntur vt ſit

$$\Gamma'' : s = \tfrac{1}{2}(a+s)\, l\tfrac{Q}{B} - \tfrac{(a+s)T - \int T\, ds}{2c}$$

$$\Delta'' : s = \tfrac{1}{2}(a+s)\, l\tfrac{Q}{B} + \tfrac{(a+s)T + \int T\, ds}{2c}$$

tum vero $\Gamma' : s = \tfrac{1}{2}\int (a+s)\, ds\, l\tfrac{Q}{B} - \tfrac{(a+s)\int T\, ds}{2c}$

$\Delta' : s = \tfrac{1}{2}\int (a+s)\, ds\, l\tfrac{Q}{B} + \tfrac{(a+s)\int T\, ds}{2c}$ et

$$\Gamma : s = \tfrac{1}{2}(a+s)\int (a+s)\, ds\, l\tfrac{Q}{B} - \tfrac{1}{2}\int (a+s)^2\, ds\, l\tfrac{Q}{B} - \tfrac{\int (a+s)\, ds \int T\, ds}{2c}$$

$$\Delta : s = \tfrac{1}{2}(a+s)\int (a+s)\, ds\, l\tfrac{Q}{B} + \tfrac{1}{2}\int (a+s)^2\, ds\, l\tfrac{Q}{B} + \tfrac{\int (a+s)\, ds \int T\, ds}{2c}$$

Statuamus ergo in S duas applicatas

$$SQ = (a+s)\, l\tfrac{Q}{B} \quad \text{et} \quad SV = \tfrac{(a+s)T + \int T\, ds}{c}$$

deſcriptisque binis ſcalis CQc et DVd, erit

$$\Gamma'' : s = \tfrac{1}{2}SQ - \tfrac{1}{2}SV; \qquad \Delta'' : s = \tfrac{1}{2}SQ + \tfrac{1}{2}SV$$

$$\Gamma' : s = \tfrac{1}{2}ACSQ - \tfrac{1}{2}ADSV; \quad \Delta' : s = \tfrac{1}{2}ACSQ + \tfrac{1}{2}ADSQ$$

Pro tempore ergo elapſo t ſumtis interuallis $ST = St = ct$ denſitas q et celeritas ȣ ita definiuntur vt ſit:

$$q = Q\left(1 + \tfrac{1}{2(a+s)}(TM - TN + tm + tn - 2SQ)\right)$$

$$\text{ȣ} = \tfrac{c}{2(a+s)}(tm + tn - TM + TN) + \tfrac{c}{2(a+s)^2}(ACTM - ADTN - ACtm - ADtn)$$

quae

quae duo elementa nosse sufficit, cum inde tertium, nempe spatiolum S s facile concludatur.

Vt iam inuestigemus quomodo vtramque scalam vltra A continuari oporteat, sumamus punctum S in ipso termino A, vt sit $s = 0$, et abscissis vtrinque A T$'$ = A t' = $c\,t$, quoniam areas curvarum a puncto A dextrorsum computauimus, quae nunc sinistrorsum cadunt, negatiue capi debent, eritque pro hoc puncto A

$$q = Q\left(1 + \tfrac{1}{2a}(T'M' + t'm' - 2AC) - \tfrac{1}{2a}(T'N' - t'n')\right)$$

$$\upsilon = \tfrac{c}{2a}(t'm' - T'M') + \tfrac{c}{2a}(T'N' + t'n') + \tfrac{c}{2aa}(ACT'M' + ACt'm')$$

$$- \tfrac{c}{2a^2}(ADT'N' - ADt'n').$$

Nunc duo casus sunt expediendi prout tubus in termino A fuerit apertus vel clausus:

I. Sit tubus in A a apertus et quia ibi densitas perpetuo manet naturalis, vt sit $Q = B$ et $q = B$, erit $AC = 0$; fierique necesse est

$$T'M' + t'm' = 0 \quad \text{et} \quad T'N' = t'n'$$

hoc ergo casu vtraque scala prorsus vt in tubis cylindricis continuatur; scala scilicet densitatum C Q c ad alteram axis partem, celeritatum vero D V d ad eandem describitur.

II. Sit tubus in A a clausus, et quia ibi celeritas perpetuo manet nulla, pro scala densitatum primo esse debet:

$$a(t'm' - T'M') + ACT'M' + ACt'm' = 0.$$

 statua-

statuamus

$$AT' = At' = x,\quad T'M' = y \text{ et } t'm' = z,$$

vt habeamus:

$$a(z-y)+\int(y+z)dx = 0 \text{ seu } adz+zdx-ady+ydx = 0$$

quae integrata dat:

$$e^{\frac{x}{a}}az = \int e^{\frac{x}{a}}(ady-ydx) = e^{\frac{x}{a}}ay - 2\int e^{\frac{x}{a}}ydx = -e^{\frac{x}{a}}ay + 2a\int e^{\frac{x}{a}}dy$$

hincque

$$z = t'm' = y - \tfrac{2}{a}e^{-\frac{x}{a}}\int e^{\frac{x}{a}}ydx = -y + 2e^{-\frac{x}{a}}\int e^{\frac{x}{a}}dy.$$

Deinde pro scala celeritatum oportet sit:

$$a(T'N'+t'n') - ADT'N' + ADt'n' = 0$$

faciamus

$$AT' = At' = x,\quad T'N' = y,\quad t'n' = z,$$

et aequatio

$$a(y+z)-\int ydx+\int zdx = 0 \text{ seu } adz+zdx+ady-ydx = 0$$

integrata praebet:

$$e^{\frac{x}{a}}az = \int e^{\frac{x}{a}}(ydx-ady) = -e^{\frac{x}{a}}ay + 2\int e^{\frac{x}{a}}ydx = e^{\frac{x}{a}}ay - 2a\int e^{\frac{x}{a}}dy$$

hincque

$$z = t'n' = -y + \tfrac{2}{a}e^{-\frac{x}{a}}\int e^{\frac{x}{a}}ydx = y - 2e^{-\frac{x}{a}}\int e^{\frac{x}{a}}dy$$

vtroque casu integralia ita definiri debent, vt posito $x = 0$ fiat $z = y$.

Coroll. 1.

143. Cum casu, quo tubus in Aa est apertus

tus ambae ſcalae eodem modo continuentur quo in tubis cylindricis, eadem quoque lex continuationis locum habebit ſi tubus conicus etiam in altero termino B b fuerit apertus, vnde ſequitur in tubo conico vtrinque aperto agitationes aeris prorſus conuenire cum iis, quas ſupra pro tubis cylindricis definiuimus.

Coroll. 2.

144. Quae igitur ſupra de ſonis, quos tibiae apertae edunt annotauimus, eadem quoque locum habent ſi tibiis figura conica tribuatur. Neque tamen figura nimis diuergens admitti poteſt, quia tum agitationes per vnamquamque ſectionem non amplius forent aequabiles, vti Theoria noſtra poſtulat.

Coroll. 3.

145. Ex hoc problemate theoria tuborum cylindricorum deducitur ſi diſtantia I A $= a$ ſtatuatur infinita. Tum autem pro caſu quo tubus in A a eſt clauſus eadem lex pro vtriusque ſcalae continuatione colligitur, quia ob $a = \infty$ erit $t' m' = T' M'$ et $t' n' = - T' N'$.

Scholion 1.

146. Nunc dubia quae ſupra circa propagationem pulſus in ipſo vertice coni excitati ſupererant, perfecte tolluntur, cum enim totum negotium eo redeat, quomodo vtramque ſcalam vltra verticem coni oporteat continuari, ad hunc caſum noſtrum

ſtrum problema accommodabimus, ſi diſtantiam $IA = a$ euaneſcentem ſimulque tubum in Aa clauſum ſtatuamus quoniam enim conus in vertice per ſe clauditur ibi certe aer nullum motum concipere poteſt. Ex quo primum pro continuatione ſcalae denſitatum quaeſtio huc redit, vt ex hac aequatione

$$z = -y + 2\, e^{\frac{-x}{a}} \int e^{\frac{x}{a}}\, dy$$

valor ipſius z definiatur caſu quo $a = 0$, quod cum ob exponentem $\frac{x}{a}$ infinitum minus pateat ad aequationem differentialem

$$a\,dz + z\,dx - a\,dy + y\,dx = 0$$

confugiamus, vnde ob $a = 0$ manifeſto ſequitur $z = -y$, ita vt ſcala denſitatum axi ad partem contrariam applicari debeat, ſecus ac ſupra per euaneſcentiam interualli $S = 0$ decepti fecimus. Deinde vero pro ſcala celeritatum ex aequatione differentiali

$$a\,dz + z\,dx + a\,dy - y\,dx = 0 \text{ ob } a = 0$$

deducimus $z = y$, ita vt haec ſcala ad eandem axis partem conſtitui debeat vtraque ſcilicet continuatio eadem lege peragitur ac ſi tubus eſſet apertus. Hinc
Tab. IX. Fig. 105. ergo poſtrema euolutio in probl. 94 ita emendabitur vt cum ſcalae in contrarias plagas cadant atque in figura ſunt repraeſentatae elapſo tempore $t = \frac{S\,d}{c}$, fiat $t'm' = 0$, $t'n' = +DF$ area $At'm' = +Amd = AMD$ quia hic duplex ſigni mutatio fieri debet altera quatenus haec area vltra A, altera quatenus

infra

infra axem cadit. Denique vero fit area $A t' n' = - A \delta f = - A D. D F$; vnde consequimur:

$T'M' + t'm' = o$; $T'N' - t'n' = o$; $A t'm' - A T'M' = o$

$t'm' - T'M' = o$; $T'N' + t'n' = 2DF$; $AT'N' + At'n' = 2S.DF$

hincque porro

densitatem $q = B$ et celeritatem

$$\vartheta = \tfrac{c}{2S}.o + \tfrac{c}{2S}.2DF - \tfrac{c}{2SS}.o - \tfrac{c}{2SS}.2S.DF \text{ seu}$$

$$\vartheta = \tfrac{c}{S}.DF - \tfrac{c}{S}.DF = o$$

ex quo manifestum est elapso tempore $t = \frac{Sd}{c}$ aerem in S penitus in aequilibrium restitui, sicque veritatem egregie saluari.

Scholion 2.

147. Quoniam vidimus tibias conicas pro longitudine eosdem edere sonos atque cylindricos siquidem sint apertae, probe notandum est hanc conuenientiam neutiquam in tibiis clausis locum habere. Cum enim si tubus conicus ad $A a$ fuerit clausus, ambae scalae in lineas ipsis maxime dissimiles continuari debeant, hinc nullae oscillationes regulares oriri poterunt; sed sonus iis editus admodum erit rudis et inconditus minimeque ad harmoniam efficiendam accommodatus.