DE PERTVRBATIONE MOTVS TERRAE AB ACTIONE VENERIS ORIVNDA.

Auctore

L. EVLERO.

1.

Quamquam orbita Veneris a plano eclipticae declinat, tamen in praesenti inuestigatione, ab hac declinatione mentem abstrahamus, quandoquidem perturbatio, qua terra a plano eclipticae dimoueretur quam minima esset futura, quamobrem Venerem in ipso plano eclipticae motum suum absoluere assumamus, sicque tota nostra inuestigatio ad binas coordinatas reducetur.

2. Reperiantur ergo certo quodam tempore centra Solis, Terrae et Veneris in punctis ☉, ♁ et ♀ in plano eclipticae, in quo recta ☉♈ dirigatur ad punctum aequinoctiale vernum, massae autem horum trium corporum iisdem signis ☉, ♁, ♀ indicentur. Pro loco terrae autem vocentur coordinatae ☉X $= x$ et X♁ $= y$, ipsa vero a Sole distantia dicatur ☉♁ $= u$, Veneris vero a Sole distantia ☉♀ $= v$, eiusque a terra distantia ♀♁ $= w$, tum vero vocentur anguli Longitudinem exhibentes

Tab. X. Fig. 1.

♈☉♁

♈ ⊙ ♁ $= \theta$ et ♈ ⊙ ♀ $= \Phi$, ac manifeſtum eſt fore $= u$ coſ. θ et $y = u$ ſin. θ, ideoque $uu = xx + yy$, tum vero erit $ww = uu + vv - 2uv$ coſ. $(\Phi - \theta)$.

3. Iam ob actionem Solis terra vrgetur in directione ♁ ⊙ vi $= \frac{\odot}{uu}$, ob actionem Veneris autem, terra vrgetur in directione ♁ ♀ vi $= \frac{♀}{ww}$, praeterea vero quia Solem in quiete ſpectamus, vires quibus ipſe Sol ſollicitatur contrario modo in terram transferri oportet, a terra autem Sol vrgetur in directione ⊙ ♁ vi $= \frac{♁}{uu}$, ex quo eadem vis ipſi terrae in directione ♁ ⊙ applicata eſt cenſenda. Deinde quia Sol a Venere vrgetur ſecundum ⊙ ♀ vi $= \frac{♀}{vv}$, ducta recta ♁ V ipſi ♀ ⊙ parallela, terra quoque cenſenda eſt vrgeri in directione ♁ V, vi $\frac{♀}{vv}$ quocirca terra omnino his tribus viribus ſollicitari eſt concipienda, *primo* in directione ♁ ⊙ adeſt vis $= \frac{\odot + ♁}{uu}$. *Secunda* vis agit in directione ♁ V eſt que $= \frac{♀}{vv}$. *Tertia* vero vis agit in directione ♁ ♀, quae eſt $= \frac{♀}{ww}$.

4. Quo iam has vires ſecundum directiones coordinatarum noſtrarum reſoluamus, demittatur ex ♀ in ⊙ ♈ perpendiculum ♀ P, ad quod ex ♁ rectae ⊙ ♈ parallela agatur ♁ Q, eritque ⊙ P $= v$. coſ. Φ et P ♀ $=$ V ſin. Φ, vnde colligitur ♁ Q $= x - v$ coſ. Φ, et ♀ Q $= v$ ſin. $\Phi - y$, atque hinc pro directione abſciſſae X ⊙ reſultant iſtae vires

$$= \frac{\odot + ♁}{uu} \cdot \frac{x}{u} + \frac{♀}{vv} \text{coſ.} \Phi + \frac{♀}{ww} \cdot \frac{x - v \,\text{coſ.} \Phi}{w} = \frac{(\odot + ♁)x}{u^3} + \frac{♀ \,\text{coſ.} \Phi}{vv} + \frac{♀(x - v \,\text{coſ.} \Phi)}{w^3}.$$

 Tum

Tum vero pro directione applicatae $♅ X$, colliguntur hae vires

$$\frac{\odot + ♅}{uu} \cdot \frac{y}{u} + \frac{♀}{vv} \text{ fin. } \Phi - \frac{♀ (v \text{ fin. } \Phi - y)}{w^3}.$$

Denotet nunc $d\tau$ elementum temporis pro constante habendum, sitque α quantitas constans naturae huius temporis conueniens ac principia motus nobis binas sequentes suppeditant aequationes:

$$\text{I. } \frac{ddx}{\alpha d\tau^2} + \frac{(\odot + ♅)x}{u^3} + \frac{♀ \text{ cof. } \Phi}{vv} + \frac{♀ (x - v \text{ cof. } \Phi)}{w^3} = 0$$

$$\text{II. } \frac{ddy}{\alpha d\tau^2} + \frac{(\odot + ♅)y}{u^3} + \frac{♀ \text{ fin. } \Phi}{vv} + \frac{♀ (y - v \text{ fin. } \Phi)}{w^3} = 0.$$

5. Calculum autem ab his coordinatis, ad alias traduci conuenit, quae ad longitudinem terrae mediam referantur. Ducatur ergo recta $\odot M$ longitudinem terrae mediam repraesentans, ad quam ex terra demittatur perpendicularis $♅ x$, vt habeantur nouae coordinatae, $\odot x = X$ et $x ♅ = Y$, tum vero vocetur angulus $♈ \odot M = t$, qui ipsam longitudinem terrae mediam exprimit cuius differentiale dt vtique illi elemento temporis $d\tau$ est proportionale, ideoque eius loco vsurpari poterit, vti mox videbimus, hinc autem coordinatae praecedentes ita determinantur, vt sit

$$x = X \text{ cof. } t - Y \text{ fin. } t, \quad y = Y \text{ cof. } t + X \text{ fin. } t,$$

manebit autem vt ante

$$uu = X^2 + Y^2, \text{ et } ww = uu + vv - 2uv \text{ cof. } (\Phi - t).$$

6.

6. Per differentiationem igitur elicimus:

$$dx = dX\,\mathrm{cof.}\,t - dY.\,\mathrm{fin.}\,t. - dt\,(X\,\mathrm{fin.}\,t + Y\,\mathrm{cof.}\,t)$$

$$= dX\,\mathrm{cof.}\,t - dY.\,\mathrm{fin.}\,t - y\,dt \quad \text{et}$$

$$dy = dY.\,\mathrm{cof.}\,t + dX\,\mathrm{fin.}\,t + dt\,(X\,\mathrm{cof.}\,t - Y\,\mathrm{fin.}\,t)$$

$$= dX\,\mathrm{fin.}\,t + dY.\,\mathrm{cof.}\,t + x\,dt.$$

Denuoque differentiando

$$ddx = ddX\,\mathrm{cof.}\,t - ddY\,\mathrm{fin.}\,t - dX.\,dt\,\mathrm{fin.}\,t - dY\,dt\,\mathrm{cof.}\,t - dy\,dt$$

$$= ddX.\,\mathrm{cof.}\,t - ddY\,\mathrm{fin.}\,t - 2dt(dX\,\mathrm{fin.}\,t + dY\,\mathrm{cof.}\,t) - dt^2(X\,\mathrm{cof.}\,t - Y\,\mathrm{fin.}\,t)$$

$$ddy = ddX\,\mathrm{fin.}\,t + ddY.\,\mathrm{cof.}\,t + 2dt(dX\,\mathrm{cof.}\,t - dY\,\mathrm{fin.}\,t) - dt^2(X\,\mathrm{fin.}\,t + \mathrm{cof.}\,t)$$

vnde aequationes noftrae ita erunt comparatae:

$$\text{I.}\quad \frac{ddX\,\mathrm{Cof.}\,t - ddY\,\mathrm{fin.}\,t - 2dt(dX\,\mathrm{fin.}\,t + dY\,\mathrm{Cof.}\,t) - dt^2(X\,\mathrm{Cof.}\,t - Y\,\mathrm{fin.}\,t)}{\alpha\,d\tau^2}$$

$$+ \frac{(\odot + \text{♁})(X\,\mathrm{Cof.}\,t - Y\,\mathrm{fin.}\,t)}{u^3} + \frac{\text{♀}\,\mathrm{cof.}\,\Phi}{vv} + \frac{\text{♀}\,(X\,\mathrm{Cof.}\,t - Y\,\mathrm{fin.}\,t - v\,\mathrm{cof.}\,\Phi)}{w^3} = 0$$

$$\text{II.}\quad \frac{ddX.\,\mathrm{fin.}\,t + ddY\,\mathrm{Cof.}\,t + 2dt(dX\,\mathrm{Cof.}\,t - dY\,\mathrm{fin.}\,t) - dt^2(X\,\mathrm{fin.}\,t + Y\,\mathrm{Cof.}\,t)}{\alpha\,d\tau^2}$$

$$+ \frac{(\odot + \text{♁})(X\,\mathrm{fin.}\,t + Y\,\mathrm{Cof.}\,t)}{u^3} + \frac{\text{♀}\,\mathrm{fin.}\,\Phi}{vv} + \frac{\text{♀}\,(X\,\mathrm{fin.}\,t + Y\,\mathrm{Cof.}\,t - v\,\mathrm{fin.}\,\Phi)}{w^3} = 0.$$

7. Quo autem hinc formas fimpliciores eruamus, duplici combinatione vtemur, prima fcilicet combinatio. I. cof. t + II. fin. t fuppeditat:

$$\frac{ddX - 2dt\,dY - X\,dt^2}{\alpha\,d\tau^2} + \frac{(\odot + \text{♁})X}{u^3} + \frac{\text{♀}\,\mathrm{cof.}\,(\Phi - t)}{vv} + \frac{\text{♀}\,(X - v.\,\mathrm{cof.}\,(\Phi - t))}{w^3} = 0$$

altera vero combinatio efto

I. − fin. t + II. cof. t quae praebet:

$$\frac{ddY + 2dt\,dX - Y\,dt^2}{\alpha\,d\tau^2} + \frac{(\odot + \text{♁})Y}{u^3} + \frac{\text{♀}\,\mathrm{fin.}\,(\Phi - t)}{vv} + \frac{\text{♀}\,(Y - v.\,\mathrm{fin.}\,(\Phi - t))}{w^3} = 0.$$

 Quo

Quo has aequationes adhuc tractabiliores reddamus, mentem primo abſtrahamus ab actione Veneris, ſeu ponamus $♀ = 0$, vt habeamus iſtas aequationes

$$\frac{ddX - 2dt\,dY - X\,dt^2}{\alpha\,d\tau^2} + \frac{(\odot + ♁)X}{u^3} = 0$$

$$\frac{ddY + 2dt.\,dX - Y\,dt^2}{\alpha\,d\tau^2} + \frac{(\odot + ♁)Y}{u^3} = 0$$

nunc autem terram ſecundum ipſum motum medium ferri aſſumamus, ita vt poſita diſtantia media terrae a Sole $= a$, pro hoc caſu habituri ſimus $X = a$; $Y = 0$ et $u = a$, qui valores in noſtris aequationibus producunt

$$-\frac{dt^2}{\alpha\,d\tau^2} + \frac{\odot + ♁}{a^3} = 0,$$

altera vero ſponte euaneſcit, vnde intelligimus loco elementi $d\tau$ differentiale motus medii dt introductum iri, ſi modo loco $\alpha\,d\tau^2$ ſcribatur $\frac{a^3\,dt^2}{\odot + ♁}$, hacque adeo ſubſtitutione in genere vti licebit, quo pacto non ſolum formula indefinita $\alpha\,d\tau^2$, ſed etiam notio maſſarum $\odot + ♁$ e calculo euaneſcit. Multiplicemus ſcilicet noſtras aequationes per $\frac{a^3}{\odot + ♁}$, tum vero ponatur fractio $\frac{♀}{\odot + ♁} = \lambda$, atque noſtrae expreſſiones ita ſatis concinnae expreſſae prodibunt:

$$\text{I. } \frac{ddX}{dt^2} - \frac{2dY}{dt} - X + \frac{a^3.X}{u^3} + \frac{\lambda a^3 \cos.(\Phi - t)}{vv} + \frac{\lambda a^3 (X - v\cos.(\Phi - t))}{w^3} = 0$$

$$\text{II. } \frac{ddY}{dt^2} + \frac{2dX}{dt} - Y + \frac{a^3.Y}{u^3} + \frac{\lambda a^3.\sin.(\Phi - t)}{vv} + \frac{\lambda a^3 (Y - v.\sin.(\Phi - t))}{w^3} = 0.$$

8. Poſtremam nunc transformationem adhibeamus inde deductam, quod ob excentricitatem terrae ſatis paruam locus ♁ a puncto x nunquam, admodum ſit diſcrepaturus; hunc in finem ſtatuamus

$X = a$

$X = a(1+x)$ et $Y = ay$, vnde fit $u = a\sqrt{((1+x)^2+yy)}$ et aequationes noſtrae per a diuiſae euadent

$$\text{I. } \frac{ddx}{dt^2} - \frac{2dy}{dt} - (1+x) + \frac{(1+x)}{((1+x)^2+yy)^{3:2}} + \frac{\lambda aa}{vv}\operatorname{cof.}(\Phi - t) + \frac{\lambda a^3(1+x-\frac{v}{a}\operatorname{cof.}\Phi - t)}{w^3} = 0$$

$$\text{II. } \frac{ddy}{dt^2} + \frac{2dx}{dt} - y + \frac{y}{((1+x)^2+yy)^{3:2}} + \frac{\lambda aa}{vv}\operatorname{fin.}(\Phi - t) + \frac{\lambda aa}{w^3}(ay - v\operatorname{fin.}(\Phi - t)) = 0$$

vbi notandum ambas has nouas coordinatas x et y prae vnitate ſemper fore ſatis exiguas, quam ob cauſſam formulam u^3 facile in ſeriem valde conuergentem euoluere licet. Eſt autem *B. Lector* monendus, ne has litteras x et y confundat cum ſuperioribus, quas iam penitus obliuiſci oportet.

Euolutio harum formularum remota actione Veneris.

9. Antequam Veneris rationem habeamus, vtique neceſſe eſt, pro binis noſtris incognitis x et y eos valores inueſtigari, qui ex ſola actione Solis et Terrae oriuntur, hancobrem nobis iſtae duae aequationes ſint propoſitae:

$$\text{I. } \frac{ddx}{dt^2} - \frac{2dy}{dt} - (1+x) + \frac{1+x}{((1+x)^2+yy)^{3:2}}$$

$$\text{II. } \frac{ddy}{dt^2} + \frac{2dx}{dt} \quad - y \quad + \frac{y}{((1+x)^2+yy)^{3:2}}$$

ex

ex quibus valores vtriusque incognitae x et y definiri oportet. Hunc in finem formulam irrationalem $((1+x)^2+yy)^{\frac{-3}{2}}$ in feriem conuergentem refoluamus, quae eft

$$\frac{1}{(1+x)^3}-\frac{3.\,yy}{2(1+x)^5}+\frac{15.\,y^4}{8(1+x)^7},$$

et quum porro fit

$$\frac{1}{(1+x)^n}=1-nx+\frac{n(n+1)}{1.\,2}x^2-\frac{n(n+1)(n+2)}{1.\,2.\,3}x^3$$
$$+\frac{n(n+1)(n+2)(n+3)}{1.\,2.\,3.\,4}x^4-\text{etc.}$$

facta euolutione binae noftrae aequationes fequentes induent formas:

$$\text{I. } \frac{ddx}{dt^2}-\frac{2dy}{dt}-3x,\ +3xx-\tfrac{3}{2}y^2,\ +6xy^2-4x^3,$$
$$+5x^4-15xxyy+\tfrac{15}{8}y^4=0$$
$$\text{II. } \frac{ddy}{dt^2}+\frac{2dx}{dt}\qquad,\ -3xy\qquad,\ +6xxy-\tfrac{3}{2}y^3,$$
$$-10.\,x^3y+\tfrac{15}{2}xy^3=0$$

quae feries quum x et y fint fractiones fatis paruae vehementer conuergunt, quod quo clarius appareat, has aequationes in membra difpefcuimus fecundum dimenfionum numerum, quas binae litterae x et y adimplent. Ac prima quidem membra vocabimus principalia, fequentia vero annexa.

10. Facile hic perfpicere licet quantitatem x per certam feriem cofinuum exprimi debere, alteram vero y per fimilem feriem finuum, hinc in fubfidium folutionis obferuaffe iuuabit, fi membra annexa prioris aequationis contineant $\mathfrak{M}$ cof. μt, pofterioris

vero

vero aequationis hunc terminum M fin. μt, tum in in ipfis feriebus ipfarum x et y, fimiles terminos occurrere debere. Pro x igitur occurrere. fumamus terminum $\mathfrak{N}$.cof. μt, pro y vero N fin. μt, qui valores in partes principales inducti praebent:

$$\mathfrak{M} = \mu^2 . \mathfrak{N} + 2\mu N + 3\mathfrak{N} = \mathfrak{N}(\mu^2 + 3) + 2\mu N$$
$$M = \mu^2 . N + 2\mu\mathfrak{N},$$

vnde colligimus

$$\mathfrak{N} = \frac{\mu\mathfrak{M} - 2M}{\mu(\mu\mu - 1)}; \quad N = \frac{(\mu^2 + 3)M - 2\mu\mathfrak{M}}{\mu\mu(\mu\mu - 1)}$$

fiue quatenus $\mathfrak{N}$ iam eft inuentum erit

$$N = \frac{M}{\mu\mu} - \frac{2\mathfrak{N}}{\mu}.$$

Hinc notari conuenit cafu quo $\mu = 0$, fore $\mathfrak{M} = 3\mathfrak{N}$; $\mathfrak{N} = \frac{1}{3}\mathfrak{M}$ fiquidem fuerit $M = 0$, id quod femper eueniet, tum vero cafu quo $\mu = 1$, neceffario fiet $\mathfrak{M} = 2M$, tum vero valor $\mathfrak{N}$ manet indefinitus, ex eoque prodibit $N = M - 2\mathfrak{N}$.

11. Quoniam iam vidimus pro motu terrae medio fiue circulari, fieri tam $x = 0$, quam $y = 0$, vnde eatenus tantum quantitates x et y certos fortientur valores, quatenus orbita terrae excentricitate eft praedita, denotet igitur K excentricitatem orbitae terreftris, et pro vtraque quantitate x et y fequentes ordines conftitui conueniet

$$x = K\mathfrak{P} + K^2\mathfrak{Q} + K^3\mathfrak{R} + K^4\mathfrak{S};$$
$$y = KP + K^2 . Q + K^3 . R + K^4 . S$$

quos fingulos ordines fucceffiue euolui neceffe eft.

Pro primo igitur ordine, qui excentricitate ſimplici K eſt affectus, habebimus has duas aequationes:

$$\left.\begin{array}{ll}\text{I.} & \frac{dd\mathfrak{P}}{dt^2}-\frac{2dP}{dt}-3\mathfrak{P}=0\\ \text{II.} & \frac{ddP}{dt^2}+\frac{2d\mathfrak{P}}{dt}\qquad\qquad=0\end{array}\right\}\text{I.}$$

Pro *ſecundo ordine* qui quadrato K K eſt affectus:

$$\left.\begin{array}{ll}\text{I.} & \frac{dd\mathfrak{Q}}{dt^2}-\frac{2dQ}{dt}-3\mathfrak{Q},+3\mathfrak{P}^2-\frac{3}{2}P^2=0\\ \text{II.} & \frac{ddQ}{dt^2}+\frac{2d\mathfrak{Q}}{dt}\qquad,-3\mathfrak{P}P\qquad=0\end{array}\right\}\text{II.}$$

Pro *tertio ordine* qui cubo K^3 eſt affectus, aequationes noſtrae erunt

$$\left.\begin{array}{ll}\text{I.} & \frac{dd\mathfrak{R}}{dt^2}-\frac{2dR}{dt}-3.\mathfrak{R},+6\mathfrak{P}\mathfrak{Q}-3.PQ;+6\mathfrak{P}P^2-4\mathfrak{P}^3=0\\ \text{II.} & \frac{ddR}{dt^2}+\frac{2d\mathfrak{R}}{dt}\qquad,-3\mathfrak{P}Q-3P\mathfrak{Q};+6\mathfrak{P}^2.P-\frac{3}{2}P^3=0\end{array}\right\}\text{III.}$$

Pro *quarto* denique *ordine* biquadrato K^4 affecto, aequationes erunt

$$\left.\begin{array}{ll}\text{I.} & \frac{dd\mathfrak{S}}{dt^2}-\frac{2dS}{dt}-3\mathfrak{S};+6\mathfrak{P}\mathfrak{R}+3.\mathfrak{Q}^2;+6\mathfrak{Q}P^2+12.\mathfrak{P}PQ\\ & \qquad -3PR-\frac{3}{2}.Q^2\qquad -12\mathfrak{P}^2.\mathfrak{Q};\\ & \qquad +5\mathfrak{P}^4-15\mathfrak{P}^2.P^2+\frac{15}{8}.P^4=0\\ \text{II.} & \frac{ddS}{dt^2}+\frac{2d\mathfrak{S}}{dt};-3\mathfrak{P}R-3P\mathfrak{R}-3\mathfrak{Q}Q;+6\mathfrak{P}^2Q+12P\mathfrak{P}\mathfrak{Q}\\ & \qquad -\frac{9}{2}P^2Q;\\ & \qquad -10\mathfrak{P}^3.P+\frac{15}{2}\mathfrak{P}P^3=0\end{array}\right\}\text{IV.}$$

12. Euolutio primi ordinis omnino eſt facillima, quia membra annexa deſunt, quare pro angulo quocunque μt ſemper erit $\mathfrak{M}=0$ et $M=0$, vnde etiam $\mathfrak{N}$ et N euaneſcerent, ſolo excepto caſu $\mu=1$, quo caſu littera $\mathfrak{N}$ manet indeterminata pro qua igitur vnitatem ſcribere licet, quia iam habet

habet indefinitum coefficientem K, inde autem colligitur $N = -2$. Quocirca pro primo ordine hanc habemus solutionem: $\mathfrak{P} = \cos. t$; $P = -2 \sin. t$. Hic euidens est angulum t simul anomaliam mediam terrae designare, quae vtique pari passu cum longitudine procederet, si terra a sola vi Solari sollicitaretur, quemadmodum hic assumimus, hac scilicet hypothesi aphelium quiesceret, quippe quod nonnisi ob perturbationes motum adipiscitur. Supra quidem hunc angulum t ab initio arietis computauimus et hanc ob rem, hic isti angulo t constantem adiici oportebat, quae quum facile subintelligatur, eam breuitatis caussa omisimus.

13. Progrediamur ergo ad ordinem secundum litteris $\mathfrak{Q}$ et Q contentum ac pro priore aequatione membrum annexum

$$3\mathfrak{P}^2 - \tfrac{3}{2}P^2 \text{ praebet } -\tfrac{3}{2} + \tfrac{9}{2}\cos. 2t$$

pro altera vero aequatione membrum annexum

$$-3\mathfrak{P}P \text{ praebet } + 3 \sin. 2t.$$

Hic ergo pro parte constante seu angulo $\mu t = 0$, est $\mathfrak{M} = -\frac{3}{2}$ et $M = 0$, vnde definitur $\mathfrak{N} = -\frac{1}{2}$, tum vero $N = 0$, pro angulo autem $2t$, vbi $\mu = 2$ habemus $\mathfrak{M} = +\frac{9}{2}$ et $M = +3$, vnde colligimus $\mathfrak{N} = \frac{1}{2}$; $N = \frac{1}{4}$, pro ordine ergo secundo nacti sumus

$$\mathfrak{Q} = -\tfrac{1}{2} + \tfrac{1}{2}\cos. 2t;\ Q = +\tfrac{1}{4}\sin. 2t.$$

14. Hinc pro ordine tertio membra annexa prioris aequationis praebent vt sequitur:

 pro

pro $\mathfrak{M}$

$$
\begin{aligned}
+6\,\mathfrak{P}\,\mathfrak{Q} &= -\tfrac{3}{2}\,\text{cof.}\,t + \tfrac{3}{2}\,\text{cof.}\,3\,t\\
-3\,P\,Q &= +\tfrac{3}{4}\,\text{cof.}\,t - \tfrac{3}{4}\,\text{cof.}\,3\,t\\
+6\,\mathfrak{P}\,P^{2} &= +6.\,\text{cof.}\,t - 6.\,\text{cof.}\,3\,t\\
-4\,\mathfrak{P}^{3} &= -3.\,\text{cof.}\,t - 1.\,\text{cof.}\,3\,t\\
\hline
& \tfrac{9}{4}\,\text{cof.}\,t - \tfrac{25}{4}\,\text{cof.}\,3\,t.
\end{aligned}
$$

Simili modo ex altera aequatione huius ordinis colligimus:

$$
\begin{aligned}
-3\,\mathfrak{P}\,Q &= -\tfrac{3}{8}\,\text{fin.}\,t - \tfrac{3}{8}\,\text{fin.}\,3\,t\\
-3\,P\,\mathfrak{Q} &= -\tfrac{9}{2}\,\text{fin.}\,t + \tfrac{3}{2}\,\text{fin.}\,3\,t\\
+6.\,\mathfrak{P}^{2}P &= -3.\,\text{fin.}\,t - 3\,\text{fin.}\,3\,t\\
-\tfrac{3}{2}.\,P^{3} &= +9.\,\text{fin.}\,t - 3\,\text{fin.}\,3\,t\\
\hline
& +\tfrac{9}{8}.\,\text{fin.}\,t - \tfrac{39}{8}.\,\text{fin.}\,3\,t.
\end{aligned}
$$
ergo pro M

Hinc igitur pro priore angulo t, eſt $\mu = 1$, $\mathfrak{M} = \frac{9}{4}$ et $M = \frac{9}{8}$ vnde fit $\mathfrak{N} = \dfrac{\frac{9}{4} - \frac{2\cdot 9}{8}}{0} = \frac{0}{0}$ ſicque hic valor foret indefinitus, quia autem iam ſupra termino $\text{cof.}\,t$ debitus coefficiens eſt tributus, hic poni oportet $\mathfrak{N} = 0$, ex quo fit $N = +\frac{9}{8}$. Pro altero autem angulo $3\,t$ ſeu $\mu = 3$, habemus $\mathfrak{M} = -\frac{25}{4}$ et $M = -\frac{39}{8}$ vnde deducimus $\mathfrak{N} = -\frac{3}{8}$ et $N = -\frac{7}{24}$ quocirca pro tertio ordine eruimus

$$\mathfrak{N} = +0.\,\text{cof.}\,t - \tfrac{3}{8}.\,\text{cof.}\,3\,t;\quad R = +\tfrac{9}{8}\,\text{fin.}\,t - \tfrac{7}{24}.\,\text{fin.}\,3\,t.$$

15. Ordini quarto hic non immoramur quoniam hanc inueſtigationem iam alibi fuſius docuimus atque ad praeſens inſtitutum ſufficit, motum regula-

rem

rem proxime tantum nouisse, cui quidem instituto solus primus ordo abunde sufficeret, collectis autem his tribus ordinibus, binae coordinatae nostrae x et y pro motu regulari ita sunt expressae,

$$x = -\tfrac{1}{2}K^2 + K\cos. t + \tfrac{1}{2}K^2.\cos. 2t - \tfrac{3}{8}K^3.\cos. 3t$$

$$y = \qquad -(2K - \tfrac{9}{8}K^3)\sin. 2t + \tfrac{1}{4}K^2.\sin. 2t - \tfrac{7}{24}K^3.\sin. 3t.$$

Euolutio nostrarum formularum accedente actione Veneris.

16. Propter actionem Veneris valores isti pro x et y inuenti exigua quaedam incrementa accipient coefficiente minimo λ affecta, quo igitur haec incrementa inueniamus, veros valores coordinatarum x et y sequenti modo repraesentemus,

$$x = X + \lambda X' \text{ et } y = Y + \lambda Y'$$

ipsae autem coordinatae pro loco terrae erunt

$$\odot x = a(1 + x) = a(1 + X + \lambda X') \text{ et}$$

$$\male x = ay = aY + a\lambda Y'$$

vbi X et Y denotant valores modo ante inuentos scilicet

$$X = -\tfrac{1}{2}K^2 + K\cos. t + \tfrac{1}{2}K^2.\cos. 2t - \tfrac{3}{8}K^3.\cos. 3t$$

$$Y = -(2K - \tfrac{9}{8}K^3)\sin. t + \tfrac{1}{4}K^2.\sin. 2t - \tfrac{7}{24}.K^3.\sin. 3t$$

partes autem annexae $\lambda X'$ et $\lambda Y'$ sunt eae ipsae quantitates, quas inuestigari oportet, et quas manifestum est fore quam minimas, ita vt prae X et Y quasi pro euanescentibus haberi queant. Ceterum hic

 monen-

monendum has litteras X et Y non esse confundendas cum iis quibus supra sumus vsi.

17. Binae autem aequationes ex quibus hae determinationes elici debent, ita sunt expressae

$$\text{I. } \frac{ddx}{dt^2}-\frac{2dy}{dt}-3x,+3xx-\tfrac{3}{2}yy,\ldots+\frac{\lambda aa}{vv}\operatorname{cos.}(\Phi-t)+\frac{\lambda a^3\left(1+x-\frac{v}{a}\operatorname{cos.}(\Phi-t)\right)}{w^3}=0$$

$$\text{II. } \frac{ddy}{dt^2}+\frac{2dx}{dt},\ -3xy\ldots+\frac{\lambda aa}{vv}.\operatorname{sin.}(\Phi-t)+\frac{\lambda aa}{w^3}(ay-v.\operatorname{sin.}(\Phi-t))=0$$

vbi vltimi termini littera λ affecti prae reliquis manifesto sunt quam minimi, deinde vero etiam membra quae supra annexa vocauimus prae membris principalibus etiam pro valde paruis sunt habenda, quandoquidem ipsae quantitatos x et y prae vnitate sunt satis exiguae, hanc ob caussam si hic velimus loco x et y valores modo indicatos $X+\lambda X'$ et $Y+\lambda Y'$ substituere, in membris annexis atque multo magis in postremis sufficiet tantum partes principales X et Y adhibere, vnde quum per hypothesin membra annexa destruantur a valoribus X et Y loco x et y substitutis aequationes quas adhuc resolui oportet diuisione per λ facta ita erunt comparatae:

$$\text{I. } \frac{ddX'}{dt^2}-\frac{2dY'}{dt}-3X',+\frac{aa}{vv}\operatorname{cos.}(\Phi-t)+\frac{aa}{w^3}(a(1+X)-v\operatorname{cos.}(\Phi-t))=0.$$

$$\text{II. } \frac{ddY'}{dt^2}+\frac{2dX'}{dt},\ \ +\frac{aa}{vv}\operatorname{sin.}(\Phi-t)+\frac{aa}{w^3}(aY-v\operatorname{sin.}(\Phi-t))=0$$

18. Quia hic termini postremi solas quantitates cognitas complectuntur, quas ad quoduis tempus facile assignare licet, statuamus breuitatis gratia

$$U=\frac{aa}{vv}\operatorname{cos.}(\Phi-t)+\frac{aa}{w^3}(a(1+X)-v\operatorname{cos.}(\Phi-t))$$

et

et

$$V = \frac{aa}{vv} \text{fin.} (\Phi - t) + \frac{aa}{v^3} (a\, Y - v \,\text{fin.} (\Phi - t))$$

et posterior aequatio ducta in dt et integrata statim praebet:

$$\frac{dY'}{dt} + 2\, X' + \int V\, dt = 0$$

vnde fit

$$\frac{dY'}{dt} = -2\, X' - \int V\, dt$$

qui valor in priori substitutus dat

$$\frac{ddX'}{dt^2} + X' + 2 \int V\, dt + U = 0$$

quam igitur integrari oportet.

19. Euadet autem haec aequatio integrabilis, multiplicando eam per dt cof. t, quippe integrale reperitur:

$$\frac{dX'}{dt} \text{cof.} t + X' \text{fin.} t + 2 \int dt. \text{cof.} t \int V\, dt + \int U\, dt. \text{cof.} t = 0$$

quae expressio facile reducitur ad hanc:

$$\frac{dX'}{dt}. \text{cof.} t + X'. \text{fin.} t + 2\, \text{fin.} t \int V\, dt - 2 \int V\, dt \cdot \text{fin.} t$$
$$+ \int U\, dt\, \text{cof.} t = 0$$

quae diuisa per cof. t^2 denuo redditur integrabilis. Sed multo facilius scopum attingemus si ipsam aequationem differentialem secundi gradus in dt fin. t ducamus, tum enim integrale deprehenditur fore

$$\frac{dX'}{dt} \text{fin.} t - X'. \text{cof.} t + 2 \int dt\, \text{fin.} t \int V\, dt + \int U\, dt. \text{fin.} t = 0$$

quae pari modo reducitur ad hanc formam:

$$\frac{dX'}{dt} \text{fin.} t - X'. \text{cof.} t - 2\, \text{cof.} t \int V\, dt + 2 \int V\, dt\, \text{cof.} t$$
$$+ \int U\, dt\, \text{fin.} t = 0$$

20.

20. Ex his duabus aequationibus differentialibus primi gradus elidamus differentiale dX', quod fit dum prima in fin. t altera vero in $-$ cof t ducitur, tum enim prodit

$$X' + 2\int V dt - 2 \text{fin.} t \int V dt . \text{fin.} t + \text{fin.} t \int U dt \text{ cof.} t \atop - 2 \text{cof.} t \int V dt . \text{cof.} t - \text{cof.} t \int U dt . \text{fin.} t = 0$$

ficque hinc impetramus

$$X' = -2\int V dt \begin{matrix} + 2 \text{fin.} t \int V dt . \text{fin.} t - \text{fin.} t \int U dt . \text{cof.} t \\ + 2 \text{cof.} t \int V dt . \text{cof.} t + \text{cof.} t \int U dt . \text{fin.} t \end{matrix}$$

Quo autem hinc facilius etiam alteram coordinatam y eruamus, vtamur formula iam fupra inuenta

$dY' = -2 X' dt - dt \int V dt$, hoc eft

$$dY' = +3 dt \int V dt - 4 . dt . \text{fin.} t \int V dt \text{ fin.} t + 2 dt \text{ fin.} t \int U dt . \text{cof.} t \atop - 4 . dt . \text{cof.} t \int V dt . \text{cof.} t - 2 dt . \text{cof.} t \int U dt . \text{fin.} t$$

quas fingulas partes fequenti modo integramus

$$\int dt \int V dt = t \int V dt - \int V t dt$$

quae reductio autem nihil iuuat:

$$\int dt \text{ fin.} t \int V dt \text{ fin.} t = - \text{cof.} t \int V dt . \text{fin.} t + \int V dt . \text{fin.} t \text{ cof.} t$$

$$\int dt \text{ cof.} t \int V dt \text{ cof.} t = + \text{fin.} t \int V dt . \text{cof.} t - \int V dt . \text{fin.} t . \text{cof.} t$$

$$\int dt \text{ fin.} t \int U dt \text{ cof.} t = - \text{cof.} t \int U dt \text{ cof.} t + \int U dt . \text{cof.} t^2$$

$$\int dt \text{ cof.} t \int U dt \text{ fin.} t = + \text{fin.} t \int U dt . \text{fin.} t - \int U dt . \text{fin.} t^2$$

vnde colligimus fequentem valorem:

$$Y' = +3 \int dt \int V dt + 4 \text{ cof.} t \int V dt . \text{fin.} t - 2 \text{ cof.} t \int U dt \text{ cof.} t + 2 \int U dt \atop - 4 \text{ fin.} t \int V dt . \text{cof.} t - 2 \text{ fin.} t \int U dt . \text{fin.} t .$$

21.

21. Ad quoduis ergo tempus valores vtriusque quantitatis X' et Y' elicuimus, quos concinnius adhuc ſequenti modo repraeſentare licet

$$X' = -2\int \mathrm{V}\,dt + \operatorname{coſ}.t\left(\int dt\,(2\,\mathrm{V}\operatorname{coſ}.t + \mathrm{U}\operatorname{ſin}.t)\right)$$
$$+ \operatorname{ſin}.t\left(\int dt\,(2\,\mathrm{V}\operatorname{ſin}.t \quad \mathrm{U}\operatorname{coſ}.t\right)$$
$$Y' = +3\int dt\int \mathrm{V}\,dt + 2\int \mathrm{U}\,dt + 2\operatorname{coſ}.t\left(\int dt\,(2\,\mathrm{V}\operatorname{ſin}.t - \mathrm{U}\operatorname{coſ}.t)\right)$$
$$-2\operatorname{ſin}.t\left(\int dt\,(2\,\mathrm{V}\operatorname{coſ}.t + \mathrm{U}\operatorname{ſin}.t)\right)$$

quemadmodum autem quouis caſu has formulas tractari conueniat, in ſequentibus fuſius ſumus explicaturi.

22. Quod autem ad conſtantes attinet, quae his integrationibus inuehuntur, eas his quatuor formis complecti licet:

I°. A; II°. B t; III^{tio}. C coſ. t et IV°. D ſin. t

quarum prima locum medium reſpicit, ſecunda ad ipſum motum medium pertinet, duae autem poſtremae formulae ad locum aphelii reducuntur. Quare ſi iam ſupra haec elementa rite conſtituta eſſe aſſumimus, has quoque conſtantes in ſequentibus praetermittere poterimus. Verum quo melius has formulas expedire queamus, imprimis attendi oportet, actionem Veneris duabus contineri partibus, quarum prior denominatorem habet $v\,v$, poſterior vero w^3, illam quoniam ad Solem refertur partem Solarem vocemus, alteram vero qua Venus immediate in terram agit partem terreſtrem atque omnino conueniet has duas partes a ſe inuicem diſtingui.

23. Denique circa formulas cof. t et fin. t, quae in haec integralia funt ingreffae obferuandum eft, angulum t eatenus tantum effe introductum, quatenus eius differentiale eft dt, neque idcirco terminum a quo iftum angulum computari oportet, effe praefcriptum, fiquidem eadem integralia prodire deberent, fi loco t fcriberetur $t + \alpha$, fcilicet ifta conftans α in ipfa euolutione iterum ex calculo elidetur, quemadmodum mox clarius apparebit. Hoc ideo monendum duximus, ne quis putet hunc angulum t perinde atque illum qui fupra eft introductus a loco Aphelii effe computandum.

De parte priore actionis Veneris Solari dicta.

24. Hic igitur littera U denotabit formulam =

$$\frac{aa}{vv} \operatorname{cof.} (\Phi - t)$$

pro priori aequatione, littera autem

$$V = \frac{aa}{vv} \operatorname{fin.} (\Phi - t)$$

pro aequatione pofteriori. Hasque formulas iterum fubdiuidere licet, quatenus vel ad folum motum medium fpectamus, vel etiam excentricitatis Veneris rationem habere velimus, quod pofterius fuperfluum videri poteft, quum orbita Veneris minimam habeat excentricitatem, vnde vix vllus effectus in perturbationem Terrae oriri poteft.

25. Denotet igitur b diftantiam mediam Veneris a Sole et quia excentricitatem negligimus, habebimus $v = b$, tum vero angulus Φ longitudinem

Vene-

Veneris mediam designabit, ideoque angulus $\Phi - t$ elongationem mediam Veneris a Terra e Sole spectatam. Designemus igitur hanc elongationem angulo $= p$, qui quum tempori sit proportionalis, ponamus $\frac{dp}{dt} = m$ hinc igitur erit

$$U = \frac{aa}{bb} \text{cof.}\, p \quad \text{et} \quad V = \frac{aa}{bb} \text{fin.}\, p.$$

Vnde colligimus

$$2V \text{cof.}\, t + U \text{fin.}\, t = \frac{aa}{bb}\left(\tfrac{1}{2}\text{fin.}(p-t) + \tfrac{3}{2}\text{fin.}(p+t)\right)$$

$$2V \text{fin.}\, t - U \text{cof.}\, t = \frac{aa}{bb}\left(+\tfrac{1}{2}\text{cof.}(p-t) - \tfrac{3}{2}\text{cof.}(p+t)\right).$$

26. His igitur notatis singulae formulae integrales, quae valores X' et Y' constituunt sequenti modo se habebunt:

$$\int V\, dt = \frac{aa}{bb}\int \text{fin.}\, p.\, dt = -\frac{aa}{bb}\cdot\frac{\text{cof.}\, p}{m}$$

$$\int dt(2V \text{cof.}\, t + U \text{fin.}\, t) = \frac{aa}{bb}\int dt\left(\tfrac{1}{2}\text{fin.}(p-t) + \tfrac{3}{2}\text{fin.}(p+t)\right)$$

$$= -\frac{aa}{bb}\left(\frac{\text{cof.}(p-t)}{2(m-1)} + \frac{3\,\text{cof.}(p+t)}{2(m+1)}\right)$$

$$\int dt(2V \text{fin.}\, t - U \text{cof.}\, t) = \frac{aa}{bb}\int dt\left(\tfrac{1}{2}\text{cof.}(p-t) - \tfrac{3}{2}\text{cof.}(p+t)\right)$$

$$= \frac{aa}{bb}\left(\frac{\text{fin.}(p-t)}{2(m-1)} - \frac{3\,\text{fin.}(p+t)}{2(m+1)}\right),$$

$$\int dt \int V\, dt = -\frac{aa}{bb}\cdot\frac{\text{fin.}\, p}{mm}; \quad \int U\, dt = \frac{aa}{bb}\cdot\frac{\text{fin.}\, p}{m}$$

vnde colligimus ipsas quantitates X' et Y'

$$X = \frac{aa}{bb}\left(\frac{2\,\text{cof.}\, p}{m} - \frac{\text{cof.}\, p}{2(m-1)} - \frac{3\,\text{cof.}\, p}{2(m+1)}\right) = \frac{aa}{bb}\cdot\frac{m - 2.\, \text{cof.}\, p}{m(mm-1)}$$

$$Y' = \frac{aa}{bb}\left(-\frac{3\,\text{fin.}\, p}{mm} + \frac{2\,\text{fin.}\, p}{m} + \frac{\text{fin.}\, p}{(m-1)} - \frac{3\,\text{fin.}\, p}{m+1}\right) = \frac{aa}{bb}\left(\frac{(m^2 - 2m + 3)\,\text{fin.}\, p}{mm(m^2-1)}\right).$$

27. Omnino hic insigne dubium occurrit, quod casu $m = 1$ vtraque haec quantitas abeat in infini-

 tum,

tum, quod vtique maxime esset absurdum, verum perpendendum est, hoc casu angulum p, abire in t et quia iam supra vidimus ob constantes integrationum, ingredi talem formam indefinitam $A + Bt + C \operatorname{cos.} t + D \operatorname{sin.} t$, manifestum est illud infinitum constantibus C vel D tolli posse. Ceterum hic ipse casus $m = 1$ peculiarem meretur resolutionem, tum enim erit angulus $p - t$ constans qui sit α, vnde integrando erit

$$\int dt (2V \operatorname{cos.} t + U \operatorname{sin.} t) = \frac{aa}{bb}(\tfrac{1}{2} t \operatorname{sin.} \alpha - \tfrac{3}{4} \operatorname{cos.} (p + t))$$

$$\int dt (2V \operatorname{sin.} t - U \operatorname{cos.} t) = \frac{aa}{bb}(\tfrac{1}{2} t \operatorname{cos.} \alpha - \tfrac{3}{4} \operatorname{sin.} (p + t))$$

quamobrem hoc casu consequimur

$$X' = \frac{aa}{bb}(2 \operatorname{cos} p + \tfrac{1}{2} t . \operatorname{sin.} (t + \alpha) - \tfrac{3}{4} \operatorname{cos.} p) = \frac{aa}{bb}(\tfrac{5}{4} \operatorname{cos.} p + \tfrac{1}{2} t . \operatorname{sin.} (t + \alpha))$$

$$Y' = \frac{aa}{bb}(- \operatorname{sin.} p + t \operatorname{cos.} (t + \alpha) - \tfrac{3}{2} \operatorname{sin.} p) = \frac{aa}{bb}(- \tfrac{5}{2} \operatorname{sin.} p + t \operatorname{cos.} (t + \alpha)).$$

Vbi notandum his formulis

$$\tfrac{1}{2} t \operatorname{sin.} (t + \alpha) \quad \text{et} \quad t \operatorname{cos.} (t + \alpha)$$

motum aphelii terrae innui.

28. Hactenus tantum ad motum medium Veneris respeximus neglecta excentricitate, interim tamen operae pretium est etiam huius rationem habere, sit igitur $\varkappa$ ista excentricitas et angulus q anomalia media Veneris fiatque $dq = n\,dt$, quo posito habebimus vti constat

$$v = b(1 + \varkappa \operatorname{cos.} q) \quad \text{et} \quad \Phi = \zeta - 2\varkappa \operatorname{sin.} q,$$

denotante ζ longitudinem mediam, ita vt sit $\zeta - t = p$, quum ergo iam sit

$$\Phi - t = p - 2\varkappa \operatorname{sin.} q$$

erit

erit

$$\sin.(\phi - t) = \sin.p - 2\varkappa \sin.q \cos.p \quad \text{et} \quad \cos.(\phi - t) = \cos.p + 2\varkappa \sin.p.\sin.q.$$

Ergo quia

$$\frac{1}{vv} = \frac{1}{bb}(1 - 2\varkappa \cos.q),$$

hinc colligemus:

$$U = \frac{aa}{bb}(\cos.p - 2\varkappa\cos.(p+q) \quad \text{et} \quad V = \frac{aa}{bb}(\sin.p - 2\varkappa\sin.(p+q)).$$

29. Quoniam autem ea quae a partibus potioribus oriuntur iam expidiuimus, ſumamus tantum

$$U = +\frac{\varkappa aa}{bb}(-2\cos.(p+q) \quad \text{et} \quad V = \frac{\varkappa aa}{bb}(-2\sin.(p+q)$$

hincque porro

$$2V\cos.t + U\sin.t = \frac{\varkappa aa}{bb}(-\sin.(p+q-t) - 3.\sin.(p+q+t))$$

$$2V\sin.t - U\cos.t = \frac{\varkappa aa}{bb}(-\cos.(p+q-t) + 3\cos.(p+q+t)$$

atque hinc deducimus integralia noſtra

$$\int V\,dt = \frac{\varkappa aa}{bb}\left(\frac{2\cos.(p+q)}{m+n}\right)$$

$$\int dt(2V\cos.t + U\sin.t) = \frac{\varkappa aa}{bb}\left(+\frac{\cos.+q-t)}{m+n-1} + \frac{3\cos.(p+q+t)}{m+n+1}\right)$$

$$\int dt(2V\sin.t - U\cos.t) = \frac{\varkappa aa}{bb}\left(-\frac{\sin.(p+q-t)}{m+n-1} + \frac{3\sin.(p+q+t)}{m+n+1}\right)$$

$$\int dt\int V\,dt = \frac{\varkappa aa}{bb}\left(\frac{2\sin.(p+q)}{(m+n)^2}\right); \quad \int U\,dt = \frac{\varkappa aa}{bb}\left(-\frac{2\sin.(p+q)}{m+n}\right).$$

30. Ex his itaque colligimus quaeſitos valores pro X' et Y' vti ſequuntur:

$$X' = \frac{\varkappa aa}{bb}\left(-\frac{4\cos.(p+q)}{m+n} + \frac{\cos.(p+q)}{m+n-1} + \frac{3\cos.(p+q)}{m+n+1}\right)$$

$$= \frac{\varkappa aa}{bb}\left(\frac{-2(m+n-2)}{(m+n)((m+n)^2-1)}\right).\cos.(p+q)$$

$$Y' = \frac{\varkappa aa}{bb}\left(+\frac{6.\sin.(p+q)}{(m+n)^2} - \frac{4\sin.(p+q)}{m+n} - \frac{2\sin.(p+q)}{m+n-1} + \frac{6.\sin.(p+q)}{m+n+1}\right)$$

$$= -\frac{2\varkappa aa}{bb}\,\frac{((m+n)^2 - 2(m+n)+3)}{(m+n)^2((m+n)^2-1)}\sin.(p+q).$$

 Omnino

Omnino igitur ex parte Solari actionis Veneris oriuntur sequentes valores pro nostris X' et Y'.

$$X' = \frac{aa}{bb} \cdot \frac{m-2}{m(m^2-1)} \operatorname{cos.} p - \frac{2\varkappa aa}{bb} \cdot \frac{m+n-2}{(m+n)((m+n)^2-1)} \operatorname{cos.} (p+q)$$

$$Y' = \frac{aa}{bb} \cdot \frac{(m^2-2m+3)}{mm(m^2-1)} \operatorname{fin.} p - \frac{2\varkappa aa}{bb} \left(\frac{(m+n)^2-2(m+n)+3}{(m+n)^2((m+n)^2-1)} \right) \operatorname{fin.} (p+q)$$

quas formulas quum in genere euoluere licuerit, manifestum est easdem etiam ad actionem, cuiusuis alius Planetae in terram agentis accommodari posse, quin etiam loco Terrae quemlibet alium Planetam primarium substituere licebit, ita vt ea quae hactenus sunt tradita ad motum cuiusuis Planetae ab alio quocunque perturbatum transferri queant.

De parte altera terrestri actionis Veneris.

31. Hic igitur habemus:

$$U = \frac{aa}{w^3}(a(1+X) - v \operatorname{cos.} (\Phi - t)) \text{ et } V = \frac{aa}{w^3}(aY - v \operatorname{fin.} (\Phi - t))$$

quas formulas gemino modo tractari conueniet, primo scilicet tam pro terra, quam pro Venere ad solum motum medium respiciemus, quae tractatio tamquam fundamentum constituet perturbationis totalis, quandoquidem perturbatio hinc enata, neque ab excentricitate terrae neque Veneris pendebit, sed pro omnibus reuolutionibus easdem inaequalitates exhibebit a sola elongatione Veneris a terra pendentes, dum contra si excentricitatis ratio habeatur, quaelibet reuolutio peculiarem euolutionem requireret, prouti scilicet rectae ☉♁ et ☉♀ respectu vtriusque lineae apsidum fuerint dispositae.

32.

32. Primum igitur ad motum medium tantum attendentes, habebimus 1°. $X = 0$ et $Y = 0$, tum vero erit $v = b$. et angulus $\phi - t = p$, cui respondet numerus $m = \frac{dp}{dt}$ hinc autem deducitur diſtantia

$$♀♁ = w = \sqrt{(aa - 2bb \operatorname{cof.} p + bb)}$$

atque hinc pro noſtris formulis integrandis, fiet

$$U = \frac{a^2 (a - b \operatorname{cof.} p)}{(aa - 2ab. \operatorname{cof.} p + bb)^{3:2}} \text{ et } V = -\frac{aa}{w^3} b. \operatorname{fin.} p$$

vnde porro deducimus

$$2V \operatorname{cof.} t + U \operatorname{fin.} t = \frac{aa}{w^3}(a \operatorname{fin.} t - \tfrac{1}{2} b. \operatorname{fin.}(p-t) - \tfrac{3}{2} b \operatorname{fin.}(p+t))$$

$$2V \operatorname{fin.} t - U \operatorname{cof.} t = \frac{aa}{w^3}(-a \operatorname{cof.} t - \tfrac{1}{2} b \operatorname{cof.}(p-t) + \tfrac{3}{2}. b \operatorname{cof.}(p+t));$$

vbi in ſubſidium calculi notaſſe iuuabit fore

$$w^2 = (a - b \operatorname{cof.} p)^2 + bb \operatorname{fin.} p^2.$$

33. Tabulas autem Aſtronomicas motuum terrae et Veneris conſulentes, ſumta diſtantia terrae media $= 1$, reperimus diſtantiam mediam Veneris $b = 0,72340$, deinde quum motus medius Solis per 30 dies $29^\circ. 34'. 10'' = 106450''$, Veneris autem per idem interuallum $48^\circ. 3'. 54'' = 173034$ erit motus Veneris a terra $= 18^\circ. 29'. 44'' = 66584$ hincque definitur numerus noſter $m = \frac{dp}{dt} = \frac{66584}{106450} = 0,62549$, ſicque angulo $t = 10^\circ$, reſpondet angulus $p = 6^\circ. 15'. 18''$ ideoque angulo $t = 5^\circ$, reſpondebit $p = 3^\circ. 7'. 39''$.

34. Hic autem ſtatim maxima difficultas occurrit in formula irrationali

$$w = \sqrt{(aa - 2ab \operatorname{cof.} p + bb)}$$

quam nullo modo in ſeriem conuergentem reſoluere licet, id quod integratio more praecedente inſtituenda requireret, quam ob cauſſam integralia noſtra, mechanice tantum definire cogimur, id quod ſequenti modo praeſtari poterit. Incipiamus ab eiusmodi ſitu, quo terra et Venus erant in coniunctione veluti terra in T et Venus in V, a quo ſitu motum vtrinque per exigua interualla vsque ad coniunctionem ſequentem proſequamur. Haec autem interualla aequalia faciamus, quibus angulus $p = 5°$ reſpondeat, ita vt integra reuolutio in 72 momenta diuidatur, pro ſingulis autem fiet angulus $t = \frac{1}{m} . 5° = 7° . 59' . 37''$ pro quo commode 8° vſurpare liceret.

Tab. X. Fig. 2.

Pro ipſo ergo hoc initio ob $a = 1$, et $b = 0,72340$ et $p = 0$ reperitur $w = a - b = 0,27660$, hincque

$$U = 13,07060 \text{; et } V = 0$$

hinc ob $t = 0$

$$2V \operatorname{cof.} t + U \operatorname{fin.} t = 0 \text{ et}$$

$$2V \operatorname{fin.} t - U \operatorname{cof.} t = -13,07060.$$

Pro ſequentibus momentis calculus ita commodiſſime inſtituetur: ſumtis valoribus $a - b \operatorname{cof.} p$ et $b \operatorname{fin.} p$, quaeratur angulus A vt ſit $\operatorname{tang.} A = \frac{b \operatorname{fin.} p}{a - b \operatorname{cof.} p}$ ſcilicet erit $A = \odot ♁ ♀$, vnde ſtatim deducitur

$$♀ ♁ = \frac{b \operatorname{fin.} p}{\operatorname{fin.} A} = (a - b \operatorname{cof.} p) \operatorname{fec.} A$$

Deinde

Deinde inuentis U et V, quaeratur angulus B vt ſit tang. $B = \frac{2V}{U}$, tum enim reperietur

$$2V \operatorname{coſ}. t + U \operatorname{ſin}. t = \frac{U. \operatorname{ſin}. (B + t)}{\operatorname{coſ}. B} = U \operatorname{ſin}. (B + t) \operatorname{Sec}. B$$

$$2V \operatorname{ſin}. t - U \operatorname{coſ}. t = - U. \operatorname{Sec}. B. \operatorname{coſ}. (B + t).$$

Quare ſecundum haec praecepta, ſingula illa momenta euoluantur.

35. Secundum haec praecepta computata eſt ſequens tabula quae pro ſingulis momentis exhibet valores quantitatum p, t, Log. w, Log. U, Log. V, L (2 V coſ. t + U ſin. t), Log. (2 V ſin. t − U coſ. t).

p	t.	Log. w	Log. U
0	0.	9,4418522	1,1162956
5	7°. 59′. 37″	9,4569345	1,0753452
10	15. 59.	9,4967064	0,9686546
15	23. 59.	9,5498767	0,8292970
20	31. 58.	9,6070652	0,6842664
25	39. 58.	9,6632535	0,5472774
30	47. 58.	9,7159907	0,4243418
35	55. 57.	9,7645091	0,3165150
40	63. 57.	9,8089831	0,2222297
45	71. 56.	9,8495749	0,1402221
50	79. 56.	9,8865906	0,0685901
55	87. 56.	9,9205273	0,0056333
60	95. 55.	9,9515346	9,9504210
65	103. 55.	9,9800156	9,9014878
70	111. 55.	0,0060488	9,8584063
75	119. 54.	0,0301129	9,8196290
80	127. 54.	0,0522237	9,7850291
85	135. 53.	0,0726427	9,7537930
90	143. 53.	0,0914012	9,7257964
95	151. 53.	0,1086588	9,7005732
100	159. 52.	0,1245604	9,6777106
105	167. 52.	0,1391401	9,6571146
110	175. 52.	0,1524578	9,6386393
115	183. 51.	0,1646910	9,6217770
120	191. 51.	0,1757526	9,6068236

Log. V.	L. P. I.	Log. P. II.
$-\infty$	$-\infty$	−1,1162956
0,4288710	+0,8434623	−1,0426259
0,6089295	1,0159197	−0,8264993
0,6227446	1,0174272	−0,4404008
0,5722346	0,9490722	−9,1687130
0,4955663	0,8490539	+0,1205327
0,4103764	0,7337769	+0,3099703
0,3244425	0,6107520	+0,3686832
0,2404967	0,4809425	+0,3789528
0,1602388	0,3444499	+0,3659349
0,0838607	0,1978678	+0,3392659
0,0111601	0,0359778	+0,3039294
9,9423053	9,8493453	+0,2632829
9,8766074	9,6144753	+0,2182427
9,8142179	9,2622612	+0,1699952
9,7539836	+7,8124342	+0,1182556
9,6960589	−9,1114274	+0,0638452
9,6397946	9,3644290	+0,0062803
9,5851749	9,4887634	+9,9460757
9,5317463	9,5604947	+9,8826492
9,4790488	9,6043823	+9,8159294
9,4269020	9,6305167	+9,7452747
9,3749909	9,6451575	+9,6704380
9,3225812	9,6507382	+9,5904115
9,2696513	−9,6505104	+9,5043127

p	t.	Log. w	Log. U
125	199. 50.	0, 1858533	9, 5931719
130	207. 50.	0, 1948645	9, 5812410
135	215. 50.	0, 2029675	9, 5705114
140	223. 49.	0, 2100971	9, 5612044
145	231. 49.	0, 2163564	9, 5530321
150	239. 49.	0, 2217224	9, 5460806
155	247. 48.	0, 2260475	9, 5408179
160	255. 48.	0, 2299155	9, 5355035
165	263. 47.	0, 2327592	9, 5318509
170	271. 47.	0, 2347756	9, 5292808
175	279. 47.	0, 2359843	9, 5277396
180	287. 46.	0, 2363861	9, 5272278

Log. V.	L. P. I.	Log. P. II.
9, 2151831	−9,6451785	+9,4104024
9, 1590389	−9,6366590	+9,3063203
9, 0999610	−9,6246553	+9,1915869
9, 0371547	−9,6120469	+9,0486173
8, 9689006	−9,5975991	+8,8725635
8, 8931813	−9,5827465	+8,6187154
8, 8071843	−9,5683258	+8,0966748
8, 7036837	−9,5532548	−8, 1410493
8, 5740971	−9,5395963	−8,5767402
8, 3948218	−9,5270811	−8,7792723
8, 0917220	−9,5158652	−8,9115587
$-\infty$	−9,5060048	−9,0117288

36. Repraesententur haec momenta more Geometrico, super axe A O per interualla aequalia A B, B C, C D etc. et in his singulis punctis applicatae erigantur A a, B b, C c etc. referentes eas quantitates U vel V, vel 2 V cos. t + U sin. t vel 2 V sin. t − U cos. t quas integrari oportet, ita vt areae huius lineae curuae exhibeant integralia quaesita, atque iam manifestum est, si inuenta fuerit area AakK, sequentem AalL facile inueniri si ad illam addatur trapezium KklL, cuius area proxime est $\frac{1}{2}$KL(Kk+Ll), verum quia hoc interuallum KL quasi esset $= dt$ est $= 7^\circ.\ 59'.\ 37''$ eius valor in partibus radii expressus erit 0, 1394835 cuius log. est 9, 1438047 singula autem integralia ita capiamus, vt in ipso initio A euanescant, hos igitur calculos sequens Tabula exhibebit: Tab. X. Fig. 3.

p	$\int U\,dt$	$\int V\,dt$
0	+ 0	+ 0
Incr.	1,7383	0,1869
5.	1,7383	0,1869
Incr.	1,4760	0,4698
10.	3,2143	0,6567
Incr.	1,1178	0,5750
15.	4,3321	1,2317
Incr.	0,8066	0,5522
20.	5,1387	1,7839
Incr.	0,5815	0,4780
25.	5,7202	2,2619
Incr.	0,4299	0,3970
30.	6,1501	2,6589
Incr.	0,3292	0,3260
35.	0,4793	2,9849
Incr.	0,2604	0,2680
40.	6,7397	3,2529
Incr.	0,2122	0,2218
45.	6,9519	3,4847
Incr.	0,1776	0,1852
50.	7,1295	3,6699
Incr.	0,1520	0,1559
55.	7,2815	3,8258
Incr.	0,1324	0,1324
60.	7,4139	3,9582
Incr.	0,1176	0,1134
65.	+ 7,5315	+ 4,0716

$\int dt \int V\,dt$	$\int dt(2V\cos t + U\sin t)$	$\int dt(2V\sin t - U\cos t)$
+0	+0	−0
0,0130	0,4855	−1,6782
0,0130	0,4855	−1,6782
0,0588	1,2077	−1,2351
0,0718	1,6932	−2,9133
0,1317	1,4470	−0,6589
0,2035	3,1402	−3,5722
0,2101	1,3440	−0,2022
0,4136	4,4842	−3,7744
0,2817	1,1110	+0,0816
0,6953	5,5952	−3,6928
0,3426	0,8690	+0,2340
1,0379	6,4642	−3,4588
0,3937	0,6612	+0,3048
1,4316	7,1254	−3,1540
0,4390	0,4948	+0,3293
1,8706	7,6202	−2,8247
0,4701	0,3646	+0,3283
2,3407	7,9848	−2,4964
0,4991	0,2637	+0,3137
2,8398	8,2485	−2,1827
0,5214	0,1854	+0,2922
3,3612	8,4339	−1,8905
0,5454	0,1249	+0,2678
3,9066	8,5588	−1,6227
0,5591	+0,0780	+0,2427
+4,4657	+8,6368	−1,3800

p

p	$\int V\,dt$	$\int V\,dt$
65.	+7,5315	+4,0716
Incr.	0,1058	0,0977
70.	7,6373	4,1693
Incr.	0,0963	0,0848
75.	7,7336	4,2541
Incr.	0,0884	0,0741
80.	7,8220	4,3282
Incr.	0,0819	0,0650
85.	7,9039	4,3932
Incr.	0,0765	0,0572
90.	7,9804	4,4504
Incr.	0,0719	0,0505
95.	8,0523	4,5009
Incr.	0,0680	0,0447
100.	8,1203	4,5456
Incr.	0,0647	0,0396
105.	8,1850	4,5852
Incr.	0,0619	0,0351
110.	8,2469	4,6203
Incr.	0,0595	0,0311
115.	8,3064	4,6514
Incr.	0,0574	0,0270
120.	8,3638	4,6784
Incr.	0,0556	0,0244
125.	8,4194	4,7028
Incr.	0,0541	0,0214
130.	+8,4635	+4,7242

$\int V\,dt$

$\int V\,dt$	$\int dt(2V\cos.t + U\sin.t)$	$\int dt(2V\sin.t - U\cos.t)$
+ 4,4657	+ 8,6368	− 1,3800
0,5738	+ 0,0415	+ 0,2181
5,0395	+ 8,6783	− 1,1619
0,5865	+ 0,0132	+ 0,1944
5,6260	+ 8,6915	− 0,9675
0,5976	− 0,0086	+ 0,1721
6,2236	+ 8,6829	− 0,7954
0,6073	− 0,0251	+ 0,1514
6,8309	+ 8,6578	− 0,6440
0,6158	− 0,0376	+ 0,1322
7,4467	+ 8,6202	− 0,5118
0,6232	− 0,0467	+ 0,1146
8,0699	+ 8,5735	− 0,3972
0,6299	− 0,0533	+ 0,0987
8,6998	+ 8,5202	− 0,2985
0,6357	− 0,0578	+ 0,0843
9,3355	+ 8,4624	− 0,2142
0,6409	− 0,0605	+ 0,0713
9,9764	+ 8,4019	− 0,1429
0,6456	− 0,0618	+ 0,0598
10,6220	+ 8,3401	− 0,0831
0,6497	− 0,0622	+ 0,0495
11,2717	+ 8,2779	− 0,0326
0,6531	− 0,0618	+ 0,0401
11,9248	+ 8,2161	+ 0,0075
0,6563	− 0,0608	+ 0,0320
+ 12,5811	+ 8,1553	+ 0,0395

p	$\int U\,dt$	$\int V\,dt$
130.	+ 8,4635	+ 4,7242
Incr.	0,0525	0,0188
135.	8,5160	4,7430
Incr.	0,0512	0,0163
140.	8,5672	4,7593
Incr.	0,0501	0,0140
145.	8,6173	4,7733
Incr.	0,0493	0,0119
150.	8,6666	4,7852
Incr.	0,0487	0,0099
155.	8,7153	4,7951
Incr.	0,0481	0,0080
160.	8,7634	4,8031
Incr.	0,0476	0,0061
165.	8,8110	4,8092
Incr.	0,0473	0,0043
170.	8,8583	4,8135
Incr.	0,0471	0,0026
175.	8,9054	4,8161
Incr.	0,0469	0,0009
180.	+ 8,9523	+ 4,8170

37. Hinc iam facile colligentur valores tam ipsius X′ quam Y′, ope formularum traditarum §. 21. quos in sequente Tabula exhibebimus, iis autem insuper adiungemus, perturbationis priores illas partes ex actione Solari oriundas, neglecta scilicet excen-

$\int dt \int V dt$	$\int dt(2V\cos.t + U\sin.t)$	$\int dt(2V\sin.t - U\cos.t)$
+12,5811	+8,1553	+0,0395
+0,6591	−0,0594	+0,0249
+13,2402	+8,0959	+0,0644
+0,6616	−0,0578	+0,0186
+13,9018	+8,0381	+0,0830
+0,6638	−0,0560	+0,0130
+14,5656	+7,9821	+0,0960
+0,6656	−0,0541	+0,0082
+15,2312	+7,9280	+0,1042
+0,6670	−0,0524	+0,0038
+15,8982	+7,8756	+0,1080
+0,6682	−0,0507	+0,0000
+16,5664	+7,8249	+0,1080
+0,6692	−0,0490	−0,0035
+17,2356	+7,7759	+0,1045
+0,6699	−0,0475	−0,0068
+17,9055	+7,7284	+0,0977
+0,6704	−0,0462	−0,0099
+18,5759	+7,6822	+0,0888
+0,6707	−0,0451	−0,0130
+19,2466	+7,6371	+0,0758

excentricitate Veneris, inuenimus autem ſupra iſtam partem priorem pro angulo p

$$X' = \frac{aa}{bb} \cdot \frac{m-2}{m(m^2-1)} \cos. p = +6,8980. \cos. p$$

coefficientis Logarithmo exiſtente +0,8387228

$$Y' = \frac{aa}{bb} \cdot \frac{m^2-2m+3}{mm(mm-1)} \sin. p = -17,17201. \sin. p$$

coefficientis Logarithmo exiſtente = −1,2348211.

Tabula pro X et Y

pro X

	pars I.	pars II.	totum
$p = 0$	$+6,8980$	0	$+6,8980$
5°	$+6,8717$	$-0,1266$	$+6,7451$
10	$+6,7932$	$-0,4879$	$+6,3053$
15	$+6,6629$	$-1,0463$	$+5,6166$
20	$+6,4820$	$-1,7619$	$+4,7201$
25	$+6,2517$	$-2,6075$	$+3,6442$
30	$+5,9738$	$-3,5586$	$+2,4152$
35	$+5,6505$	$-4,5935$	$+1,0570$
40	$+5,2841$	$-5,6971$	$-0,4130$
45	$+4,8776$	$-6,8664$	$-1,9888$
50	$+4,4339$	$-8,0471$	$-3,6132$
55	$+3,9565$	$-9,2368$	$-5,2803$
60	$+3,4490$	$-10,4127$	$-6,9637$
65	$+2,9152$	$-11,5599$	$-8,6447$
70	$+2,3593$	$-12,6624$	$-10,3031$
75	$+1,7853$	$-13,6795$	$-11,8942$
80	$+1,1978$	$-14,6178$	$-13,4200$
85	$+0,6012$	$-15,4503$	$-14,8491$
90	$+0,0000$	$-16,1661$	$-16,1661$
95	$-0,6012$	$-16,7507$	$-17,3519$
100	$-1,1978$	$-17,1934$	$-18,3912$
105	$-1,7853$	$-17,4888$	$-19,2741$
110	$-2,3593$	$-17,6309$	$-19,9902$
115	$-2,9152$	$-17,6185$	$-20,5337$
120	$-3,4490$	$-17,4516$	$-20,9006$
125	$-3,9565$	$-17,1369$	$-21,0934$

Tabula pro X et Y

pro Y

pars I.	pro II.	totum
°	°	°
− 1, 4966	+ 0, 0570	− 1, 4396
− 2, 9819	+ 0, 1104	− 2, 8715
− 4, 4444	+ 0, 1943	− 4, 2501
− 5, 8732	+ 0, 3650	− 5, 5082
− 7, 2571	+ 0, 6679	− 6, 5892
− 8, 5861	+ 1, 1795	− 7, 4066
− 9, 8495	+ 1, 1940	− 7, 9355
− 11, 0379	+ 2, 9180	− 8, 1199
− 12, 1424	+ 4, 1953	− 7, 9471
− 13, 1545	+ 5, 7724	− 7, 3821
− 14, 0665	+ 7, 6534	− 6, 4131
− 14, 8716	+ 9, 8556	− 5, 0160
− 15, 5631	+ 12, 3567	− 3, 2064
− 16, 1364	+ 15, 1637	− 0, 9727
− 16, 5869	+ 18, 2406	+ 1, 6537
− 16, 9111	+ 21, 4890	+ 4, 5779
− 17, 1067	+ 25, 1991	+ 8, 0924
− 17, 1720	+ 28, 9657	+ 11, 7937
− 17, 1067	+ 32, 9341	+ 15, 8274
− 16, 9111	+ 37, 0350	+ 20, 1239
− 16, 5869	+ 41, 2379	+ 24, 6510
− 16, 1364	+ 45, 4968	+ 29, 3604
− 15, 5631	+ 49, 7646	+ 34, 2015
− 14, 8716	+ 54, 0061	+ 39, 1345
− 14, 0665	+ 58, 1742	+ 44, 1077

Tabula pro X et Y

pro X

$p = 0$	pars I.	pars II.	totum
130	−4, 4339	−16, 6785	−21, 1124
135	−4, 8776	−15, 9972	−20, 8748
140	−5, 2841	−15, 3761	−20, 6602
145	−5, 6505	−14, 5564	−20, 2069
150	−5, 9738	−13, 6464	−19, 6202
155	−6, 2517	−12, 6658	−18, 9175
160	−6, 4820	−11, 6304	−18, 1124
165	−6, 6629	−10, 5642	−17, 2271
170	−6, 7932	− 9, 4742	−16, 2674
175	−6, 8717	− 8, 4145	−15, 2862
180	−6, 8980	− 7, 3758	−14, 2738

Tabula pro X et Y

pro Y

pars I.	pars II.	totum
− 13, 1545	+ 62, 2157	+ 49, 0612
− 12, 1424	+ 66, 1272	+ 53, 9848
− 11, 0379	+ 69, 8504	+ 58, 8125
− 9, 8495	+ 73, 3614	+ 63, 5119
− 8, 5861	+ 76, 6282	+ 68, 0421
− 7, 2571	+ 69, 6272	+ 72, 3701
− 5, 8732	+ 82, 3446	+ 76, 4714
− 4, 4444	+ 84, 7664	+ 80, 3220
− 2, 9819	+ 86, 8883	+ 83, 9064
− 1, 4966	+ 88, 7097	+ 87, 2131
− 0	+ 90, 2364	+ 90, 2364

38. Quo autem facilius has determinationes ad vſum Aſtronomicum accommodemus, notandum eſt totum negotium ad valorem litterae Y' redire, ex quo facile ſtatim effectum perturbationis cognoſcere licet, quippe qui angulo perexiguo conſtat quem ad locum terrae ex Tabulis ſolitis deſumtum ſiue addere ſiue inde auferre oportet. Quum enim definiri debeat angulus $M \odot \female$ ad longitudinem terrae mediam addendus eius tangens eſt $\frac{y}{1+x} = \frac{Y + \lambda Y'}{1 + X + \lambda X'}$ quem ergo angulum hic in duas partes diſtribui convenit, quarum altera principalis ex ſolo motu regulari eſt petenda, cuius ergo tangens eſt $X \frac{Y}{1+X}$, quem angulum Tabulae Solares conſuetae exhibent, quem littera η indicemus, altera vero pars, ipſam perturbationem continens notetur littera ω, ita vt ad locum Terrae medium addi oporteat angulum $\eta + \omega$.

39. Quum igitur habeamus Tang. $(\eta + \omega) = \frac{Y + \lambda Y'}{1 + X + \lambda X'} = \text{Tang.}\,\eta + \frac{\omega}{\mathit{cof.}\,\eta^2}$; quoniam angulus ω prae η eſt vehementer paruus; tum vero fit Tang. $\eta = \frac{Y}{1+X}$, hinc colligimus $\frac{\omega}{\mathit{cof.}\,\eta^2} = \frac{\lambda Y'}{(1+X)^2}$ propterea quod quantitates $\lambda X'$ et $\lambda X'^2$ ſunt quaſi euaneſcentes. Praeterea vero quia angulus η eſt valde paruus eiusque coſinus vix ab vnitate differt tum vero etiam loco $1 + X$ tuto ſcribere licet 1, habebimus ſimpliciter $\omega = \lambda Y'$. Interim ſi etiam harum poſtremarum conditionum rationem habere velimus, reperitur $\omega = \frac{\lambda Y'}{(1+X)^2 + YY'}$,

quae

quae expreſſio a praecedente vix particula ſenſibili diſcrepabit.

40. Quoniam igitur tota perturbatio ω, quam hic quaerimus per ſolam quantitatem Y' determinatur, cuius valores modo ante vsque ad $180°$ exhibuimus, nunc accuratius nobis oſtendere incumbit, quemadmodum hi valores ad praxin accommodari queant. Supra autem iam innuimus, conſtantes per integrationes ingreſſas, ita capi debere vt poſt ſingulas reuolutiones, hae perturbationes iterum euaneſcant, quod etiam de dimidiis reuolutionibus eſt tenendum. Quare quum pro $180°$ valor ipſius Y' prodierit $+ 90,0264$, ob conſtantes illas memoratas a quolibet valore ipſius Y' pro angulo ♀⊙♁ ſubtrahi debet numerus proportionalis, ſcilicet

$$\frac{♀⊙♁}{180} \cdot 90,0264 = ♀⊙♁ \cdot 0,50014$$

vnde facile erit has correctiones in Tabulam ſuperiorem introducere.

41. Denique ſupereſt, vt valorem litterae λ conſideremus qui a ratione maſſae Veneris ad maſſam ſolarem pendet quae autem ratio adhuc plane eſt ignota. Satis probabile autem videtur Veneris maſſam vix a maſſa terrae diſcrepare propterea quod eius magnitudo non multum a terrae deficit, denſitas vero aliquanto maior cenſetur, quandoquidem planetae quo Soli ſunt propiores eo etiam denſiores aeſtimantur. Statuamus ergo Veneris maſſam ipſi maſſae terrae aequalem quam ex nouiſſimis paral-

laxis Solis obſeruationibus, colligimus $= \frac{1}{1000000}$ dum maſſa Solis vnitate referatur, hinc igitur habebimus $\lambda = \frac{1}{1000000}$ atque ex hoc valore ſingulos angulos ω, qui perturbationem continent ſupputemus: Si forte maſſa Veneris aliquanto maior vel minor eſſet, quantitates in ſequenti Tabula occurrentes in eadem ratione ſunt augendae vel minuendae. Praeterea notandum eſt argumentum p aequari angulo $\Phi - t$ ſeu deſignare elongationem mediam Veneris a Terra e Sole ſpectatam.

Tabula

Tabula oftendens perturbationes motus terrae, ab actione Veneris ortas:

p	Sig. —	I. Sig. —	II. Sig. —	III. Sig. —	IV. Sig. —	V. Sig. —	p
0	0, 0	13, 8	21, 6	20, 6	13, 0	4, 4	30
1	0, 5	14, 2	21, 7	20, 4	12, 7	4, 1	29
2	1, 0	14, 6	21, 8	20, 2	12, 4	3, 9	28
3	1, 5	15, 0	21, 9	20, 0	12, 1	3, 7	27
4	2, 0	15, 4	22, 0	19, 8	11, 8	3, 5	26
5	2, 4	15, 7	22, 1	19, 6	11, 5	3, 3	25
6	2, 9	16, 1	22, 2	19, 4	11, 2	3, 1	24
7	3, 4	16, 5	22, 2	19, 2	10, 9	2, 9	23
8	3, 9	16. 8	22, 3	19, 0	10, 6	2, 7	22
9	4, 4	17, 1	22, 3	18, 8	10, 3	2, 5	21
10	4, 8	17, 4	22, 3	18, 5	10, 0	2, 3	20
11	5, 3	17, 8	22, 3	18, 3	9, 7	2, 1	19
12	5, 8	18, 1	22, 3	18, 1	9, 4	1, 9	18
13	6, 3	18, 4	22, 3	17, 9	9, 1	1, 7	17
14	6, 8	18, 7	22, 2	17, 6	8, 8	1, 6	16
15	7, 3	18, 9	22, 2	17, 3	8, 5	1, 5	15
16	7, 8	19, 2	22, 2	17, 1	8, 2	1, 3	14
17	8, 3	19, 5	22, 1	16, 8	7, 9	1, 1	13
18	8, 8	19, 7	22, 1	16, 5	7, 6	1, 0	12
19	9, 2	19, 9	22, 0	16, 2	7, 3	0, 9	11
20	9, 6	20, 1	21, 9	15, 9	7, 0	0, 8	10
21	10, 1	20, 3	21, 8	15, 7	6, 7	0, 6	9
22	10, 6	20, 5	21, 7	15, 4	6, 4	0, 5	8
23	11, 0	20, 7	21, 6	15, 1	6, 1	0, 4	7
24	11, 4	20, 9	21, 5	14, 8	5, 9	0, 3	6
25	11, 8	21, 0	21, 3	14, 5	5, 7	0, 2	5
26	12, 2	21, 2	21, 2	14, 2	5, 4	0, 2	4
27	12, 6	21, 3	21, 1	13, 9	5, 1	0, 1	3
28	13, 0	21, 4	21, 0	13, 6	4, 8	0, 1	2
29	13, 4	21, 5	20, 8	13, 3	4, 6	0, 0	1
30	13, 8	21, 6	20, 6	13, 0	4. 4	0, 0	0
	+	+	+	+	+	+	
p	XI. Sig.	X. Sig.	IX. Sig.	VIII. Sig.	VII. Sig.	VI. Sig.	p