NOVA METHODVS
MOTVS PLANETARVM PRINCIPALIVM AD TABVLAS ASTRONOMICAS REDVCENDI.

Auctore

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1.

Tab. III. Fig. 16. Repraesentet Tabula planum eclipticae, in quo punctum S sit centrum Solis, et recta SA ad fixa et ad punctum aequinoctiale vernum directa. Versetur iam Planeta in loco quocunque Z extra eclipticam, vnde eo demittatur perpendiculum ZY, et ex Y ad rectam SA agatur normalis YX, ita vt locus planetae determinetur his tribus coordinatis

$$SX = x, \quad XY = y \quad \text{et} \quad YZ = z;$$

tum vero ponatur breuitatis gratia distantia a Sole

$$= \sqrt{(xx + yy + zz)} = v.$$

2. Quodsi iam massa Solis sit $= S$, et elementum temporis dt constans assumatur, quoniam vis, qua Planeta ad Solem vrgetur, est $= \frac{S}{vv}$, pro motu Planetae habebimus tres sequentes aequationes:

I. $\frac{ddx}{dt^2} + \frac{Sx}{v^3} = 0$; II. $\frac{ddy}{dt^2} + \frac{Sy}{v^3} = 0$; III. $\frac{ddz}{dt^2} + \frac{Sz}{v^3} = 0$

vbi

vbi notasse iuuabit, si tempus t per motum anomaliae mediae Solis exprimatur, et distantia media terrae a Sole vnitate designetur, tum fore $S = 1$.

3. Ad tempus propositum exhibeat recta SB longitudinem mediam Planetae, ac ponatur angulus $ASB = p$, qui quum sit tempori proportionalis, statuatur $dp = m\,dt$, ducta ex Y ad hanc directionem SB normali Yx vocentur nouae coordinatae

$$Sx = X, \quad Yx = Y \quad \text{et} \quad Yz = Z$$

eritque itidem

$$vv = XX + YY + ZZ,$$

tum vero erit

$$X = x\cos.p + y\sin.p; \quad \text{et} \quad Y = y\cos.p - x\sin.p \quad \text{et} \quad Z = z$$

hinc erit differentiando

$$dX = dx\cos.p + dy\sin.p + m\,dt(y\cos.p - x\sin.p) = dx\cos p + dy\sin.p + mY\,dt$$

$$\text{et } dY = dy\cos.p - dx\sin.p - mX\,dt,$$

ita vt sit

$$dx\cos.p + dy\sin.p = dX - mY\,dt; \quad dy\cos.p - dx\sin.p = dY + mX\,dt$$

quae formae denuo differentiatae praebent

$$ddx\cos.p + ddy\sin.p + mdt(dY + mX\,dt) = ddX - mdY\,dt$$

$$ddy\cos.p - ddx\sin.p - mdt(dX - mY\,dt) = ddY + mdX\,dt.$$

4. Quum nunc ex superioribus formulis posito $S = 1$, sit

$$ddx = -\frac{x\,dt^2}{v^3} \quad \text{et} \quad ddy = -\frac{y\,dt^2}{v^3}$$

si hi valores ibi substituantur ob

$$x^2 + y^2 = XX + YY$$

Y y z habe-

habebimus has aequationes

$$-\frac{X\,dt^2}{v^3} + m^2 X\,dt^2 = ddX - 2m\,dY\,dt$$

$$-\frac{Y\,dt^2}{v^3} + m^2 Y\,dt^2 = ddY + 2m\,dX\,dt$$

quae redigantur ad sequentes formas

$$\frac{ddX}{dt^2} - \frac{2m\,dY}{dt} - m^2 X + \frac{X}{v^3} = 0$$

$$\frac{ddY}{dt^2} + \frac{2m\,dX}{dt} - m^2 Y + \frac{Y}{v^3} = 0.$$

Vbi natura motus medii postulat, vt Y nunquam vltra certum terminum excrescat, atque etiam ipsa quantitas x intra certos limites contineatur pro magnitudine excentricitatis, tertia vero aequatio manet vt ante

$$\frac{ddZ}{dt^2} + \frac{Z}{v^3} = 0.$$

5. Designet a distantiam mediam Planetae a Sole et quia X non multum ab ea discrepat, statuamus

$$X = a(1+x) \quad \text{et} \quad Y = ay,$$

similique modo $Z = az$, hic scilicet assumo harum litterarum x, y, z valores superiores iam obliuioni esse traditos, atque hinc statim habemus

$$v = a\sqrt{((1+x)^2 + yy + zz)},$$

ex quo tres nostrae aequationes ita se habebunt

$$\text{I.}\quad \frac{ddx}{dt^2} - \frac{2m\,dy}{dt} - m^2(1+x) + \frac{(1+x)}{a^3((1+x)^2+yy+zz)^{\frac{3}{2}}} = 0$$

$$\text{II.}\quad \frac{ddy}{dt^2} + \frac{2m\,dx}{dt} - m^2 y + \frac{y}{a^3((1+x)^2+yy+zz)^{\frac{3}{2}}} = 0$$

$$\text{III.}\quad \frac{ddz}{dt^2} + \frac{z}{a^3((1+x)^2+yy+zz)^{\frac{3}{2}}} = 0.$$

6. Quo-

6. Quoniam quantitates x, y et z ſpectantur vt valde paruae prae vnitate; formula irrationalis

$$\frac{1}{((1+x)^2+yy+zz)\, 3:2}$$

commode reſoluitur in hanc ſeriem conuergentem:

$$\frac{1}{(1+x)^3}-\frac{3(yy+zz)}{2(1+x)^5}+\frac{15(yy+zz)^2}{8.(1+x)^7}\text{ etc.}$$

et euolutis poteſtatibus negatiuis $1+x$, terminisque ſecundum dimenſiones diſpoſitis habebimus

$$1,-3x,+6x^2-\tfrac{3}{2}yy-\tfrac{3}{2}zz,+10x^3+\tfrac{15}{2}xyy+\tfrac{15}{2}xzz,$$
$$+15x^4-\tfrac{45}{2}xxyy-\tfrac{45}{2}xxzz+\tfrac{15}{8}y^4+\tfrac{15}{4}yyzz+\tfrac{15}{8}z^4,\text{ etc.}$$

pro qua expreſſione breuitatis gratia ſcribamus $1-3x+W$ ita vt ſit

$$W=+6xx-\tfrac{3}{2}yy-\tfrac{3}{2}zz-10x^3+\tfrac{15}{2}xyy+\tfrac{15}{2}xzz+15x^4$$
$$-\tfrac{45}{2}xxyy-\tfrac{45}{2}xxzz+\tfrac{15}{8}y^4+\tfrac{15}{4}yyzz+\tfrac{15}{8}z^4\text{ etc.}$$

ſicque noſtrae aequationes erunt

$$\text{I. } \frac{ddx}{dt^2}-\frac{2mdy}{dt}-m^2(1+x)+\frac{1}{a^3}(1+x)-\frac{3x}{a^3}(1+x)+\frac{1+x.W}{a^3}=0$$

$$\text{II. } \frac{ddy}{dt^2}-\frac{2mdx}{dt}-m^2y+\frac{1}{a^3}y-\frac{3xy}{a^3}+\frac{yW}{a^3}=0$$

$$\text{III. } \frac{ddy}{dt^2}+\frac{1}{a^3}z-\frac{3}{a^3}.xz+\frac{1}{a^3}.zW=0.$$

7. Hactenus quantitas a aliter non eſt definita, niſi quod valorem quendam medium inter omnes abſciſſas X deſignet, ita vt quantitas x valores modo poſitiuos modo negatiuos ſortiretur, id quod infinitis modis fieri poſſet, dummodo a non multum a diſtantia media diſcreparet. Nunc autem conueniet ſtatui $\frac{1}{a^3}=mm$, quandoquidem hoc modo termini maiores in noſtris aequationibus deſtruuntur, calculusque ad noſtram hypotheſin, qua x ſumitur quantitas

valde parua modo positiua modo negatiua reuocatur, hoc modo nostrae aequationes in formas sequentes contrahentur

I. $\frac{ddx}{dt^2} - \frac{2m\,dy}{dt} - 3m^2x(1+x) + mm(1+x)W = 0$

II. $\frac{ddy}{dt^2} + \frac{2m\,dx}{dt} - 3m^2xy \quad + mmyW = 0$

III. $\frac{ddz}{dt^2} + mmz - 3m^2xz + mmzW = 0.$

8. Quoniam quantitates x, y et z per hypothesin prae vnitate sunt valde paruae, reiiciamus primo omnes terminos duas pluresue dimensiones harum quantitatum continentes sicque consequemur tres sequentes aequationes

I. $\frac{ddx}{dt^2} - \frac{2m\,dy}{dt} - 3m^2x = 0$

II. $\frac{ddy}{dt^2} + \frac{2m\,dx}{dt} \quad = 0$

III. $\frac{ddz}{dt^2} + mmz \quad = 0$

quarum duas priores, quoniam non amplius continent z, seorsim tractare licet, atque adeo commode de vsu venit, vt secunda per se sit integrabilis, dum eius integratio praebet $\frac{dy}{dt} + 2mx =$ Const., quam constantem quemadmodum comparatam esse oportet, hic accuratius est inuestigandum. Maneat ea primo indeterminata, vt sit

$\frac{dy}{dt} = C - 2mx$ qui valor in prima substitutus praebet

$\frac{ddx}{dt^2} + mmx - 2mC = 0$, cuius integrale completum est

$x = \frac{2C}{m} + \alpha\cos q$ existente $dq = m\,dt$,

iam hic valor in altera surrogatus praebebit

$\frac{dy}{dt} = -3C - 2m\alpha\cos q$ et

et integrando
$y = -3Ct$ ...
vbi statim liquet
quia alioquin qua
ra, quod indoli
sequitur, si positio
necessario esse opo
spicuum est, et
posse $= 0$, quippe
pariter inuoluitur
$x = \alpha\cos q$

9. Euoluto
pluriumue dimensi
loco x et y hos
$x = \alpha\cos q$
existente $dq = m$
et Cosinuum ad si
mus, in prima a
deducent ad serie
autem aequatione
nuum, propter q
Sinus Cosinusue co
facile definiri quea

10. Ponamu
iam pro x et y,
modi valores esse
noribus, seu iis qu
siones, tam prima

et integrando

$$y = -3\,C\,t - 2\,\alpha\,\text{ſin.}\,q + D,$$

vbi ſtatim liquet conſtantem C euaneſcere debere, quia alioquin quantitas y continuo maior eſſet euaſura, quod indoli motus medii aduerſaretur, ex quo ſequitur, ſi poſitio rectae S B rite fuerit conſtituta, neceſſario eſſe oportere $C = o$, deinde etiam perſpicuum eſt, et alteram conſtantem D tuto ſumi poſſe $= o$, quippe quod in idea longitudinis mediae pariter inuoluitur, ſicque pro hoc caſu habebimus

$$x = \alpha\,\text{coſ.}\,q \quad \text{et} \quad y = -2\,\alpha\,\text{ſin.}\,q.$$

9. Euoluto hoc caſu, quo termini duarum, pluriumue dimenſionum negliguntur, ſi iis admiſſis, loco x et y hos inuentos valores

$$x = \alpha\,\text{coſ.}\,q \quad \text{et} \quad y = -2\,\alpha\,\text{ſin.}\,q$$

exiſtente $dq = m\,dt$ ſubſtituamus et producta Sinuum et Coſinuum ad ſimplices Sinus vel Coſinus reuocemus, in prima aequatione termini hactenus neglecti deducent ad ſeriem certorum Coſinuum, in altera autem aequatione ad ſimilem ſeriem certorum Sinuum, propter quos deinde tam x, quam y ſimiles Sinus Coſinusue complecti debebunt, qui quomodo facile definiri queant, generatim oſtendiſſe iuuabit.

10. Ponamus igitur per approximationem iam pro x et y, quin etiam pro z certos huiusmodi valores eſſe inuentos, eosque in terminis minoribus, ſeu iis qui duas pluresue continent dimenſiones, tam primae quam ſecundae aequationis ſubſtitui,

stitui, atque in priore quidem aequatione occurrere terminum $+ M \cos. \omega$, in altera vero talem $N \sin. \omega$ existente $d\omega = \mu\, dt$ et quoniam quicquid de his terminis trademus simul quoque ad quotcunque similes extendi potest, consideremus primo secundam aequationem:

$$\frac{ddy}{dt^2} + \frac{2m\,dx}{dt} + N \sin. \omega = 0$$

quae integrata dat

$$\frac{dy}{dt} + 2mx - \frac{N}{\mu}\cos. \omega = 0 \quad \text{siue} \quad \frac{dy}{dt} = -2mx + \frac{N}{\mu}\cos. \omega$$

qui valor in prima substitutus praebet

$$\frac{ddx}{dt^2} + mmx - \frac{2mN}{\mu}\cos. \omega + M \cos. \omega = 0$$

vnde colligitur

$$x = \frac{M - \frac{2mN}{\mu}}{\mu\mu - mm}\cos. \omega,$$

quo inuento erit

$$\frac{dy}{dt} = -2mL\cos. \omega + \frac{N}{\mu}\cos. \omega$$

existente

$$L = \frac{M - \frac{2mN}{\mu}}{\mu\mu - mm}$$

vnde fit

$$y = \left(-\frac{2mL}{\mu} + \frac{N}{\mu\mu}\right)\sin. \omega$$

hos scilicet terminos, praeter iam inuentos ad x et y adiici oportet.

11. Vt argumentum hoc ordine pertractemus, quoniam in terminis secundae dimensionis iam occurrit zz, cuius valorem demum ex tertia aequatione

tione elici oportet
modo tertia aequa
seu quod eodem
ipso plano Eclipti

Casus prior
Ecl

12. Quum
tae his duabus aeq

I. $\frac{ddx}{dt^2} - \frac{2m\,dy}{dt} - 3$
$-4mmx^3 + 6m$

II. $\frac{ddy}{dt^2} + \frac{2m\,dx}{dt}; -$

quibus aequationibu
sumendo

$x = k \cos. q$

existente $dq = m$
euenire posset, vt
piam numero ita c
mini formae cos. q
oriundi destruerent
tus iam constat
motum anomaliae
medio esse aequaler
primit anomaliam
tricitatem orbitae.

13. Hic aute
satis esse exiguam,

tione elici oportet, primo statuamus $z = 0$, vt hoc modo tertia aequatio prorsus ex calculo eliminetur, seu quod eodem redit, primo casum quo Planeta in ipso plano Eclipticae mouetur euoluamus.

Casus prior quo Planeta in ipso plano Eclipticae mouetur.

12. Quum igitur hic sit $z = 0$ motus Planetae his duabus aequationibus euolutis, continebitur

$$\text{I. } \frac{ddx}{dt^2} - \frac{2mdy}{dt} - 3mmx, + 3mmxx - \tfrac{3}{2}mmyy$$
$$- 4mmx^3 + 6mmxyy, + 5mmx^4 - \tfrac{15}{2}mmxxyy + \tfrac{35}{8}mmy^4$$
$$\text{II. } \frac{ddy}{dt^2} + \frac{2mdx}{dt}; - 3m^2yx, + 6m^2xxy - \tfrac{3}{2}m^2y^3, - 10m^2x^3y + \tfrac{15}{2}mmy^4$$

quibus aequationibus iam proxime satisfieri vidimus, sumendo

$$x = k\cos.q \quad \text{et} \quad y = -2k\sin.q,$$

existente $dq = m\,dt$; ob sequentes vero terminos euenire posset, vt $\frac{dq}{dt}$ non foret $= m$ sed alii cuipiam numero ita comparato, vt omnes plane termini formae $\cos.q$, etiam ex minoribus membris oriundi destruerentur. Verum ex natura huius motus iam constat lineam apsidum quiescere ideoque motum anomaliae mediae quae est q ipsi motui medio esse aequalem. De caetero quia hic q exprimit anomaliam mediam, littera k exhibet excentricitatem orbitae.

13. Hic autem assumimus excentricitatem k satis esse exiguam, quia alioquin coordinatae x et y, nimis

nimis augeri possent. Admittamus iam etiam terminos secundae dimensionis et pro prima aequatione habebimus

$$1 + 3mmxx = 3mmk^2 \text{cos.}\, q = \tfrac{3}{2}mmkk + \tfrac{3}{2}mmkk \text{cos.}\, 2q$$

$$-\tfrac{3}{2}mmyy = -3mmkk + 3mmkk \text{cos.}\, 2q$$ ideoque coniunctim

$$-\tfrac{3}{2}mmkk + \tfrac{9}{2}mmkk \text{cos.}\, 2q.$$

Pro secunda aequatione autem

$$-3m^2 xy = +3mmkk \text{ sin. } 2q.$$

Hic primo notandum terminum illum constantem $-\tfrac{3}{2}mmkk$ per principalem $-3mmx$ tolli debere, vnde hinc fit $x = -\tfrac{1}{2}kk$, pro angulo autem $2q$, comparatione cum superiori regula instituta erit

$$\omega = 2q, \quad M = +\tfrac{9}{2}mmkk,$$

$N = +3mmkk$, atque $\mu = 2m$, vnde colligimus $x = \frac{kk}{2}\text{cos.}\, 2q$ ideoque $L = +\frac{kk}{2}$ ex quo denique fit

$$y = +\tfrac{1}{4}kk \text{ sin. } 2q.$$

14. Vocemus has partes, quas modo tam pro x. quam pro y inuenimus, secundi ordinis; siquidem quadratum excentricitatis k inuoluunt, ita vt nunc coniunctim habeamus

$$x = k \text{ cos. } q, \; -\tfrac{1}{2}kk + \tfrac{1}{2}kk \text{ cos. } 2q \text{ et}$$
$$y = -2k \text{ sin. } q, \; +\tfrac{1}{4}kk \text{ sin. } 2q.$$

Quod si iam simili modo sequentes partes indagare velimus, quo eas a se inuicem clarius distinguamus, statuamus in genere

$$x = k.\,\mathfrak{P} + kk\,\mathfrak{Q} + k^3.\,\mathfrak{R} + k^4\,\mathfrak{S} \text{ etc.}$$
$$y = k.\,\mathrm{P} + kk\,\mathrm{Q} + k^3.\,\mathrm{R} + k^4\,\mathrm{S} \text{ etc.}$$

vbi

vbi quidem iam conftat effe

$$\mathfrak{P} = \text{cof.}\, q; \quad \mathfrak{Q} = -\tfrac{1}{2} + \tfrac{1}{2}\,\text{cof.}\, 2q; \quad P = -2\,\text{fin}\, q;$$
$$Q = +\tfrac{1}{4}\,\text{fin.}\, 2q.$$

15. Subftituamus autem valores iftos affumtos in vtraque aequatione et membris fecundum dimenfionem ipfius k a fe inuicem disiunctis nancifcemur fequentes aequationes.

I. Ordo

$$k\begin{cases}\dfrac{dd\mathfrak{P}}{mm\,dt^2} - \dfrac{2\,dP}{m\,dt} - 3\mathfrak{P} = 0\\[2mm] \dfrac{ddP}{mm\,dt^2} + \dfrac{2\,d\mathfrak{P}}{m\,dt} = 0\end{cases}$$

II. Ordo

$$kk\begin{cases}\dfrac{dd\mathfrak{Q}}{mm\,dt^2} - \dfrac{2\,dQ}{m\,dt} - 3\mathfrak{Q} + 3\mathfrak{P}^2 - \tfrac{3}{4}P^2 = 0\\[2mm] \dfrac{ddQ}{mm\,dt^2} + \dfrac{2\,d\mathfrak{Q}}{m\,dt} - 3\mathfrak{P}P = 0\end{cases}$$

III. Ordo

$$k^3\begin{cases}\dfrac{dd\mathfrak{R}}{mm\,dt} - \dfrac{2\,dR}{m\,dt} - 3\mathfrak{R} + 6\mathfrak{P}\mathfrak{Q} - 3PQ - 4\mathfrak{P}^3 + 6\mathfrak{P}P^2 = 0\\[2mm] \dfrac{ddR}{mm\,dt^2} + \dfrac{2\,d\mathfrak{R}}{m\,dt} - 3\mathfrak{P}Q - 3P\mathfrak{Q} + 6\mathfrak{P}^2P - \tfrac{3}{2}P^3 = 0\end{cases}$$

IV. Ordo

$$k^4\begin{cases}\dfrac{dd\mathfrak{S}}{mm\,dt^2} - \dfrac{2\,dS}{m\,dt} - 3\mathfrak{S} + 6\mathfrak{P}\mathfrak{R} + 3\mathfrak{Q}^2 - 3PR - \tfrac{3}{2}Q^2 - 12\mathfrak{P}^2\mathfrak{Q} + 12\mathfrak{P}PQ\\ \qquad + 6\mathfrak{Q}P^2 + 5\mathfrak{P}^4 - 15\mathfrak{P}^2.P^2 + \tfrac{15}{8}P^4 = 0\\[2mm] \dfrac{ddS}{mm\,dt^2} + \dfrac{2\,d\mathfrak{S}}{m\,dt} - 3\mathfrak{P}K - 3\mathfrak{Q}Q - 3P\mathfrak{R} + 12\mathfrak{P}P\mathfrak{Q} + 6\mathfrak{P}^2Q - \tfrac{3}{2}P^2.Q\\ \qquad - 10\mathfrak{P}^3.P + \tfrac{15}{2}P^3\mathfrak{P} = 0.\end{cases}$$

16. Quoniam aequationes primi et fecundi ordinis iam expediuimus, aggrediamur binas aequationes tertii ordinis, et quia iam habemus

$$\mathfrak{P} = \text{cof.}\, q; \quad \mathfrak{Q} = -\tfrac{1}{2} + \tfrac{1}{2}\,\text{cof.}\, 2q$$
$$P = -2\,\text{fin.}\, q; \quad Q = \quad +\tfrac{1}{4}\,\text{fin.}\, 2q$$

 pro

pro priore aequatione inueniemus

$\mathfrak{P}\mathfrak{Q} = -\frac{1}{4}\cos. q + \frac{5}{4}\cos. 3q$

$PQ = -\frac{7}{4}\cos. q + \frac{1}{4}\cos. 3q;$

$\mathfrak{P}^3 = +\frac{3}{4}\cos. q + \frac{1}{4}\cos. 3q; \quad \mathfrak{P}P^2 = \cos. q - \cos. 3q.$

hinc iunctim $\frac{9}{4}\cos. q - \frac{25}{4}\cos. 3q$

Simili modo pro posteriori aequatione

$\mathfrak{P}Q = +\frac{1}{8}\sin. q + \frac{1}{8}\sin. 3q; \quad P\mathfrak{Q} = +\frac{3}{2}\sin. q - \frac{1}{2}\sin. 3q$

$\mathfrak{P}^2 P = -\frac{1}{2}\sin. q - \frac{1}{2}\sin. 3q; \quad P^3 = -6\sin. q + 2\sin. 3q$

iunctim $\frac{9}{8}\sin. q. - \frac{39}{8}\sin. 3q.$

17. Consideremus hic primo angulum q, vt sit $\omega = q$ et $\mu = m$, tum vero

$M = +\frac{9}{4}mm$ et $N = +\frac{9}{8}mm$,

vnde quia quod ante erat x vel y; hic est vel $\mathfrak{R}$ vel R, elicimus

$$\mathfrak{R} = \frac{\frac{9}{4}mm - \frac{9}{4}mm}{mm - mm}\cos. q = \frac{0}{0}\cos. q$$

sicque coefficiens huius termini est arbitrarius in se; quando ergo k designat totam excentricitatem hunc coefficientem $= 0$ statui oportet, ita vt sit $L = 0$ hinc autem fiet $R = +\frac{9}{8}\sin. q$. At vero pro altero angulo $3q$ vbi

$\mu = 3m$; $M = -\frac{25}{4}mm$ et $N = -\frac{39}{8}mm$

fiet $\mathfrak{R} = -\frac{3}{8}\cos. 3q$; ita vt sit $L = -\frac{3}{8}m$

hincque porro

$R = -\frac{7}{24}\sin. 3q$,

quocir-

quocirca pro hoc ordine omnino habemus

$$\mathfrak{R} = \quad - \tfrac{3}{8} \operatorname{cof.} 3q$$
$$R = + \tfrac{9}{8} \operatorname{fin.} q - \tfrac{7}{24} \operatorname{fin.} 3q.$$

18. Progrediamur eodem modo ad binas aequationes ordinis quarti et pro prima quidem aequatione colligimus

				multipl. per
$\mathfrak{P}\mathfrak{R}$	$=$	$-\frac{5}{16}\operatorname{cof.} 2q$	$-\frac{5}{16}\operatorname{cof.} 4q$	$+6$
$\mathfrak{Q}^2$	$= +\frac{3}{8}$	$-\frac{1}{2}$	$+\frac{1}{8}$	$+3$
PR	$= -\frac{9}{8}$	$+\frac{17}{12}$	$-\frac{7}{24}$	-3
Q^2	$= +\frac{1}{32}$		$-\frac{1}{32}$	$-\frac{3}{2}$
$\mathfrak{P}^2\mathfrak{Q}$	$= -\frac{1}{8}$		$-\frac{1}{8}$	-12
$\mathfrak{P}PQ$	$= +\frac{1}{8}$		$+\frac{3}{8}$	$+12$
$\mathfrak{Q}P^2$	$= -\frac{3}{2}$	$+2$	$-\frac{1}{2}$	$+6$
$\mathfrak{P}^4$	$= +\frac{3}{8}$	$+\frac{1}{2}$	$+\frac{1}{8}$	$+5$
$\mathfrak{P}^2 . P^2$	$= +\frac{1}{2}$		$-\frac{1}{2}$	-15
P^4	$= +6$	-8	$+2$	$+\frac{15}{8}$

iunctim $+\frac{69}{64} - \frac{59}{8} \operatorname{cof.} 2q + \frac{579}{64} \operatorname{cof.} 4q.$

At pro altera aequatione habebimus

			multipl. per
$\mathfrak{P}R$	$= +\frac{5}{12}\operatorname{fin.} 2q$	$-\frac{7}{48}\operatorname{fin.} 4q$	-3
$\mathfrak{Q}Q$	$= -\frac{1}{8}$	$+\frac{1}{16}$	-3
$P\mathfrak{R}$	$= -\frac{3}{8}$	$+\frac{3}{8}$	-3
$\mathfrak{P}\mathfrak{Q}P$	$= +\frac{1}{2}$	$-\frac{1}{4}$	$+12$
$\mathfrak{P}^2 Q$	$= +\frac{1}{8}$	$+\frac{6}{16}$	$+6$
$P^2 Q$	$= +\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{9}{2}$
$\mathfrak{P}^3 P$	$= -\frac{1}{2}$	$-\frac{1}{4}$	-10
$\mathfrak{P}P^3$	$= -2$	$+1$	$+\frac{15}{2}$

iunctim $-\frac{21}{4} \operatorname{fin.} 2q + \frac{61}{8} \operatorname{fin.} 4q.$

 19.

19. Quod hic primum ad terminum constantem attinet in aequatione priori, quoniam y per terminum principalem $-3\mathfrak{S}$ tolli debet, erit $\mathfrak{S} = +\frac{23}{64}$, deinde pro angulo $2q$ habemus

$$\mu = 2m;\ M = -\frac{59}{8} mm \text{ et } N = -\frac{21}{4} mm$$

atque hinc

$$\mathfrak{S} = \frac{17}{24} \cos. 2q,$$

ita vt sit

$$L = -\frac{17}{24} mm$$

ex quo denique colligitur

$$S = -\frac{29}{48} \sin. 2q.$$

Eodem modo pro angulo $4q$ ob

$$\mu = 4m;\ M = +\frac{579}{64} mm \text{ et } N = +\frac{61}{8} mm$$

fiet $\mathfrak{S} = +\frac{67}{192} \cos. 4q$,

ita vt sit

$$L = +\frac{67}{192} mm,$$

vnde tandem concluditur

$$S = +\frac{29}{96} \sin. 4q,$$

consequenter valores quarti ordinis $\mathfrak{S}$ et S sequenti modo determinantur

$$\mathfrak{S} = +\frac{23}{64} - \frac{17}{24} \cos. 2q + \frac{67}{192} \cos. 4q$$
$$S = \quad - \frac{29}{48} \sin. 2q + \frac{29}{96} \sin. 4q.$$

20. His igitur coniunctis valores binarum coordinatarum x et y sequenti modo se habebunt:

$$x = \left.\begin{matrix} -\frac{1}{2}kk \\ -\frac{23}{64}k^4 \end{matrix}\right. + \left.\begin{matrix} k \\ -\frac{17}{24}k^4 \end{matrix}\right\} \cos. q + \frac{1}{2}kk \cos. 2q - \frac{3}{8}k^3 \cos. 3q + \frac{67}{192}k^4 \cos. 4q$$

$$y = \left.\begin{matrix} -2k \\ +\frac{9}{8}k^3 \end{matrix}\right\} \sin. q \left.\begin{matrix} +\frac{1}{4}kk \\ -\frac{29}{48}k^4 \end{matrix}\right\} \sin. 2q - \frac{7}{24}k^3 \sin. 3q + \frac{29}{96}k^4 \sin. 4q$$

Harum

Harum igitur formularum ope facile erit, tabulas conſtruere, quae ad quamuis anomaliam mediam Planetae q, exhibeant valores vtriusque quantitatis x et y dummodo excentricitas k orbitae accurate fuerit cognita, inuentis autem ad quoduis tempus propoſitum numeris x et y, fractio $\frac{y}{1+x}$ dabit tangentem aequationis centri Planetae ad longitudinem mediam addendae vel ſubtrahendae, prout iſta fractio $\frac{y}{1+x}$, fuerit vel poſitiua vel negatiua, tum vero ſi haec aequatio centri dicatur E ob $\frac{y}{1+x} = \text{Tang. E}$ diſtantia Planetae a Sole erit

$$a\sqrt{((1+x)^2 yy)} = \frac{a(1+x)}{\text{coſ. E}},$$

vbi a denotat diſtantiam mediam planetae a Sole; ita vt ſit $a = \sqrt[3]{\frac{1}{mm}}$ ſi ſcilicet diſtantia media Solis a terra vnitate referatur, id quod omnino conſentaneum eſt regulae Kepleri, qua cubi diſtantiarum mediarum quadratis temporum periodicorum ſunt proportionales.

21. Quanquam hic ſumſimus $z = 0$, tamen haec ſolutio ad omnes plane planetas primarios patet, quatenus ſcilicet a perturbatione eorum mutua animum abſtrahimus; quoniam enim nulla neceſſitate vrgente planum eclipticae in plano Tabulae conſtituimus, id tantum opus eſt, vt cuiusque planetae motus ad id ipſum planum, in quo mouetur, referatur, quippe quo caſu etiam ſemper habebitur $z = 0$, interim tamen vt pateat, quomodo calculum tractari conueniat, ſi ſtatim Planetarum motus ad

ad planum Eclipticae referre velimus, etiam hunc casum seorsim et omni cura euoluamus.

Casus posterior quo orbita Planetae ad Eclipticam parumper inclinatur.

22. Seruatis valoribus, quos casu praecedente pro x et y inuenimus, quandoquidem illi parum ob zz immutantur, propterea quod z, vt valde paruum spectatur, incipiamus a tertia aequatione, vbi primo terminos duarum pluriumue dimensionum negligamus, ita vt habeamus hanc aequationem:

$$\frac{ddz}{dt^2} + mmz = 0$$

cuius integratio statim praebet $z = C$ sin. r existente $dr = mdt$, vbi manifesto, angulus r exprimit argumentum latitudinis Planetae medium, quod reperitur, si a loco medio Planetae in orbita, locus nodi ascendentis subtrahatur. Ex quo quum linea nodorum etiam quiescat, manifestum est vtique esse debere $\frac{dr}{dt} = m$. Porro autem coefficiens constans conuenit cum inclinatione orbitae Planetae ad Eclipticam, seu potius eius tangente quae si ponatur $= i$, habebimus pro hac prima approximatione $z = i$ sin. r.

23. Quodsi iam in terminis sequentibus minoribus loco z iste valor substituatur, simulque pro x et y valores inuenti scribantur, ex his terminis orietur series certorum sinuum, quorum si quilibet fuerit $=$ K sin. ω existente $d\omega = \mu\, dt$ inde redundabit similis terminus in valorem z, qui si statuatur

tur $= \lambda$ fin. ω, erit $-\lambda\mu^2 + \lambda mm + K = 0$ vnde fequitur $\lambda = \frac{K}{\mu\mu - mm}$, hoc praemiffo, confideremus terminum tantum fecundae dimenfionis $-3mmxz$, vbi etiam loco x tantum eius valorem principalem k cof. q ftatuamus, ficque prodibit haec aequatio

$$\frac{ddz}{dt^2} + mmz - \tfrac{3}{2}mmki\,\text{fin.}(r-q) - \tfrac{3}{2}mmki\,\text{fin.}(r+q) = 0$$

ex priore angulo $r-q$ nafcetur pro z terminus

$$+\tfrac{3}{2}ki\,\text{fin.}(r-q)$$

vbi manifefto angulus $r-q$ eft conftans, exhibens diftantiam inter lineam apfidum et lineam nodorum, qui angulus fi dicatur $\varkappa$ habebimus hinc

$$z = \ldots\ldots + \tfrac{3}{2}ik\,\text{fin.}\,\varkappa$$

ex altero autem angulo $r+q$ refultat

$$z = \ldots\ldots - \tfrac{1}{2}ik\,\text{fin.}(r+q)$$

hactenus ergo peruenimus ad hunc valorem

$$z = i\,\text{fin.}\,r + \tfrac{3}{2}ik\,\text{fin.}\,\varkappa - \tfrac{1}{2}ik\,\text{fin.}(r+q).$$

24. Admittamus nunc etiam terminos trium dimenfionum ac ponamus breuitatis gratia vt fupra

$$x = k\mathfrak{P} + kk\mathfrak{Q} + k^3.\mathfrak{R} + k^4\mathfrak{S} \text{ etc.}$$

$$y = kP + kkQ + k^3.R + k^4S \text{ etc.}$$

Simili autem modo ftatuamus

$$z = i\,\text{fin.}\,r + ikX + ik^2Y + ik^3Z \text{ etc.}$$

atque noftra aequatio refoluetur in fequentes ordines

I. Ordo

$$ik\left(\frac{ddX}{m^2dt^2}+X-3\mathfrak{P}\,\text{fin.}\,r\right)=0$$

II. Ordo

$$ikk\left(\frac{ddY}{m^2dt^2}+Y-3\mathfrak{Q}\,\text{fin.}\,r-3\mathfrak{P}X+6\mathfrak{P}^2.\,\text{fin.}\,r-\tfrac{3}{2}P^2.\,\text{fin.}\,r\right)=0$$

III. Ordo

$$ik^3\left(\frac{ddZ}{mmdt^2}+Z-3\mathfrak{R}\,\text{fin.}\,r-3\mathfrak{Q}X-3\mathfrak{P}Y+12\mathfrak{P}\mathfrak{Q}\,\text{fin.}\,r\right.$$
$$\left.+6\mathfrak{P}^2.X-3PQ\,\text{fin.}\,r-\tfrac{3}{2}P^2.X-10\mathfrak{P}^3\,\text{fin.}\,r+\tfrac{15}{2}\mathfrak{P}P^2\,\text{fin.}\,r\right)=0.$$

25. Harum aequationum primam iam expediuimus; atque inuenimus

$$X=+\tfrac{3}{2}\,\text{fin.}\,\varkappa-\tfrac{1}{2}\,\text{fin.}\,(r+q),$$

vbi angulus conftans $\varkappa=r-q$, hoc ergo valore fubftituto, pro fecundo ordine habemus

$-3)\ \mathfrak{Q}\,\text{fin.}\,r=$	$-\tfrac{1}{2}\,\text{fin.}\,r$	$-\tfrac{1}{4}\,\text{fin.}\,(2q-r)$	$+\tfrac{1}{4}\,\text{fin.}\,(2q+r)$
$-3)\ \mathfrak{P}X=$	$+\tfrac{1}{2}\,\text{fin.}\,r$	$-\tfrac{3}{4}\,\text{fin.}\,(2q-r)$	$-\tfrac{1}{4}\,\text{fin.}\,(2q+r)$
$+6)\ \mathfrak{P}^2\,\text{fin.}\,r=$	$+\tfrac{1}{2}$	$-\tfrac{1}{4}$	$+\tfrac{1}{4}$
$-\tfrac{3}{2})\ P^2\,\text{fin.}\,r=$	$+2$	$+1$	-1

iunctim $K=3\,\text{fin.}\,(2q+r)$ atque ob $\mu=3m$, fit

$$Y=\tfrac{3}{8}\,\text{fin.}\,(2q+r)$$

26. Hinc eodem modo determinemus ordinem Z

$3)\ \mathfrak{R}\,\text{fin.}\,r=$			$+\tfrac{3}{16}\,\text{fin.}\,(3q-r)$	$-\tfrac{3}{16}\,\text{fin.}\,(3q+r)$
$-3)\ \mathfrak{Q}X=$	$\tfrac{7}{8}\,\text{fin.}\,(q-r)$	$+\tfrac{5}{8}\,\text{fin.}\,(q+r)$	$-\tfrac{3}{8}$	$-\tfrac{1}{8}$
$-3)\ \mathfrak{P}Y=$		$+\tfrac{3}{16}$		$+\tfrac{3}{16}$
$+12)\ \mathfrak{P}\mathfrak{Q}\,\text{fin.}\,r=$	$+\tfrac{1}{8}$	$-\tfrac{1}{8}$	$-\tfrac{1}{8}$	$+\tfrac{1}{8}$
$+6)\ \mathfrak{P}^2X=$	$-\tfrac{5}{8}$	$+\tfrac{1}{8}$	$-\tfrac{3}{8}$	$-\tfrac{1}{8}$
$-3)\ PQ\,\text{fin.}\,r=$	$+\tfrac{1}{8}$	$-\tfrac{1}{8}$	$-\tfrac{1}{8}$	$+\tfrac{1}{8}$
$-\tfrac{3}{2})\ P^2X=$	$-\tfrac{7}{2}$	$-\tfrac{5}{2}$	$+\tfrac{3}{2}$	$+\tfrac{1}{2}$
$-10)\ \mathfrak{P}^3\,\text{fin.}\,r=$	$-\tfrac{3}{8}$	$+\tfrac{3}{8}$	$-\tfrac{1}{8}$	$+\tfrac{1}{8}$
$+\tfrac{15}{2})\ \mathfrak{P}P^2\,\text{fin.}\,r=$	$-\tfrac{1}{2}$	$+\tfrac{1}{2}$	$+\tfrac{1}{2}$	$-\tfrac{1}{2}$

iunctim

iunctim $K=+0\,\text{fin.}\,(q$

et $\mu=0$

confequenter

$Z=+\tfrac{5}{16}\,\text{fin.}\,(q-$

Hos ordines euolui

perimus

$z=i\,\text{fin.}\,r-\tfrac{3}{2}ik\,\text{fin.}$

$-\tfrac{1}{48}ik^3\,\text{fi}$

27. Inuento

correctiones, quae l

dant, inueftigari o

primo quadratum z

cem $iikk$ extenda

reperietur expreffun

$zz=ii\,\text{fin.}\,r^2+$

quae forma euoluit

$zz=ii(\tfrac{1}{2}-\tfrac{1}{2}\,\text{cof}\,2r)+i$

$+iikk(+\tfrac{5}{4}-\tfrac{3}{8}\,\text{cof.}\,2q$

cuius loco fcribamus

$zz=ii\,A+i$

ita vt fit

$A=\tfrac{1}{2}-\tfrac{1}{2}\,\text{cof.}\,2r;\ B=$

$C=+\tfrac{5}{4}-\tfrac{3}{8}\,\text{cof.}\,2q+$

Deinde pro quaefitis

ftatuamus

iunctim $K = +0 \sin.(q-r) + \frac{15}{16}\sin.(q+r) - \frac{1}{16}\sin.(3q-r) - 5\sin.(3q+r)$
et $\mu = 0 \qquad 2m \qquad 2m \qquad 4m$

consequenter

$$Z = +\tfrac{5}{16}\sin.(q+r) - \tfrac{1}{48}\sin.(3q-r) - \tfrac{1}{3}\sin.(3q+r).$$

Hos ordines euoluisse sufficiat, quibus collectis reperimus

$$z = i\sin.r - \tfrac{3}{2}ik\sin.(q-r) - \tfrac{1}{2}ik\sin.(q+r) + \tfrac{3}{8}ikk\sin.(2q+r) + \tfrac{5}{16}ik^2$$

$$-\tfrac{1}{48}ik^3\sin.(3q-r) - \tfrac{1}{3}ik^3\sin.(3q+r).$$

27. Inuento hoc valore ipsius z, nunc etiam correctiones, quae hinc in quantitates x et y redundant, inuestigari oportet, quem in finem definiamus primo quadratum zz, quod autem non vltra indicem $iikk$ extendamus, quod ergo sequenti modo reperietur expressum:

$$zz = ii\sin.r^2 + 2iik.X.\sin.r + 2iikkY\sin.r + iikk.XX$$

quae forma euoluitur in hanc

$$zz = ii(\tfrac{1}{2} - \tfrac{1}{2}\cos 2r) + iik(+\cos q - \tfrac{3}{2}\cos.(q-2r) + \tfrac{1}{2}\cos.(q+2r)) + iikk(+\tfrac{5}{4} - \tfrac{3}{8}\cos.2q + \tfrac{3}{4}\cos.2r - \tfrac{9}{8}\cos.(2q-2r) - \tfrac{1}{2}\cos.(2q+2r))$$

cuius loco scribamus

$$zz = ii\,A + iik.\,B + iikk\,C,$$

ita vt sit

$$A = \tfrac{1}{2} - \tfrac{1}{2}\cos.2r;\quad B = \cos q - \tfrac{3}{2}\cos.(q-2r) + \tfrac{1}{2}\cos.(q+2r);$$
$$C = +\tfrac{5}{4} - \tfrac{3}{8}\cos.2q + \tfrac{3}{4}\cos 2r - \tfrac{9}{8}\cos.(2q-2r) - \tfrac{1}{2}\cos.(2q+2r).$$

Deinde pro quaesitis correctionibus ipsarum x et y, statuamus

$$x = k\mathfrak{P} + kk\mathfrak{Q} + k^3\mathfrak{R} + k^4\mathfrak{S} + ii\mathfrak{T} + iik\mathfrak{U} + iikk\mathfrak{V}$$
$$y = kP + kkQ + k^3R + k^4S + iiT + iikU + iikkV$$

quibus valoribus in prima et fecunda aequatione fubftitutis, pro nouis ordinibus, fignis ii; iik et $iikk$ notatis obtinebimus fequentes aequationes:

Ordo ii
$$\begin{cases} \frac{dd\mathfrak{T}}{mm\,dt^2} - \frac{2\,dT}{m\,dt} - 3\mathfrak{T} - \frac{3}{2}A = 0 \\ \frac{ddT}{mm\,dt^2} + \frac{2\,d\mathfrak{T}}{m\,dt} = 0 \end{cases}$$

iik
$$\begin{cases} \frac{dd\mathfrak{U}}{mm\,dt^2} - \frac{2\,dU}{m\,dt} - 3\mathfrak{U} + 6\mathfrak{P}\mathfrak{T} - 3PT - \frac{3}{2}B + 6\mathfrak{P}A = 0 \\ \frac{ddU}{mm\,dt^2} + \frac{2\,d\mathfrak{U}}{m\,dt} - 3\mathfrak{P}T - 3P\mathfrak{T} - \frac{3}{2}AP = 0 \end{cases}$$

$iikk$
$$\begin{cases} \frac{dd\mathfrak{V}}{mm\,dt^2} - \frac{2\,dV}{m\,dt} - 3\mathfrak{V} + 6(\mathfrak{P}\mathfrak{U} + \mathfrak{Q}\mathfrak{T}) - 3(PU + QT) - 12\mathfrak{P}^2\mathfrak{T} \\ \qquad + 6P^2\mathfrak{T} + 12\mathfrak{P}PT - \frac{3}{2}C + 6\mathfrak{P}B + 6A\mathfrak{Q} = 0 \\ \frac{ddV}{mm\,dt^2} + \frac{2\,d\mathfrak{V}}{m\,dt} - 3\mathfrak{P}U - 3P\mathfrak{U} - 3\mathfrak{Q}T - 3Q\mathfrak{T} \\ \qquad + 6\mathfrak{P}^2T + 12\mathfrak{P}P\mathfrak{T} - \frac{9}{2}P^2T - \frac{3}{2}BP - \frac{3}{2}AQ = 0 \end{cases}$$

28. Euoluamus nunc terminos minores pro his ordinibus, ac pro primo ordine

$-\frac{3}{2}A = \frac{3}{2} \qquad - \frac{3}{2}\,\text{cof.}\,2r$
ergo $M = -\frac{3}{4} \qquad + \frac{3}{4}\,\text{cof.}\,2r$ at vero N erit $= 0$

vnde quia terminus conftans per terminum principalem $-3\mathfrak{T}$ tolli debet, hinc fit $\mathfrak{T} = -\frac{1}{4}$; deinde pro angulo $2r$, habemus $\mu = 2m$, tum vero $M = +\frac{3}{4}$ et $N = 0$ vnde $\mathfrak{T} = +\frac{1}{4}\,\text{cof.}\,2r$, ideoque $L = +\frac{1}{4}mm$ vnde porro fit $T = -\frac{1}{4}\,\text{fin.}\,2r$ quocirca pro hoc ordine nacti fumus

$$\mathfrak{T} = -\tfrac{1}{4} + \tfrac{1}{4}\,\text{cof.}\,2r; \quad T = -\tfrac{1}{4}\,\text{fin.}\,2r.$$

29.

29. Pergamus ad fecundum ordinem et euoluamus terminos minores vtriusque aequationis:

	$\cos q$	$\cos(q-2r)$	$\cos(q+2r)$
6) $\mathfrak{P}\mathfrak{T} =$	$-\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{1}{8}$
$-3)$ $PT =$		$+\frac{1}{4}$	$-\frac{1}{4}$
$-\frac{3}{2})$ $B =$	$+1$	$-\frac{3}{2}$	$+\frac{1}{2}$
$+6)$ $\mathfrak{P}A =$	$+\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$
iunctim $M =$	0	$+\frac{3}{4}$	$-\frac{3}{4}$

	$\sin q$	$\sin(q-2r)$	$\sin(q+2r)$
$-3)$ $\mathfrak{P}T =$		$+\frac{1}{8}$	$-\frac{1}{8}$
$-3)$ $P\mathfrak{T} =$	$+\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$
$-\frac{3}{2})$ $AP =$	-1	$+\frac{1}{2}$	$+\frac{1}{2}$
ergo $N =$	0	$-\frac{3}{8}$	$+\frac{3}{8}$
hinc $\mu =$	n	$-n$	$+3n$
$-\frac{2mN}{\mu} =$	0	$-\frac{3}{4}$	$-\frac{1}{4}$
$+M =$	0	$+\frac{3}{4}$	$-\frac{3}{4}$
Numer. $=$	0	0	-1
Demon. $=$	0	0	3
$L =$	0	0	$-\frac{1}{3}$
$\frac{N}{\mu\mu} =$	0	$-\frac{3}{8}$	$+\frac{1}{24}$
$-\frac{2mL}{\mu} =$	0	0	$+\frac{1}{12}$

$$U = 0.\sin q - \tfrac{3}{8}\sin(q-2r) + \tfrac{1}{8}\sin(q+2r)$$

$$\text{at } \mathfrak{U} = 0.\cos q + 0\cos(q-2r) - \tfrac{1}{8}\cos(q+2r).$$

29. His duobus ordinibus acquiefcamus, quoniam facile patet, quemadmodum etiam aequationes tertii ordinis refolui queant, quare pro quantitatibus x et y, praeter valores iam in praecedente cafu euolutos, habebimus nunc infuper

$$x = \ldots\ldots - \tfrac{1}{4}ii + \tfrac{1}{4}ii\,\text{cos.}\,2r + 0.iik\,\text{cos.}\,(q - 2r)$$
$$- \tfrac{1}{8}iik\,\text{cos.}\,(q + 2r)$$
$$y = \ldots - \tfrac{1}{4}ii\,\text{fin.}\,2r - \tfrac{3}{8}iik\,\text{fin.}\,(q - 2r) + \tfrac{3}{8}iik\,\text{fin.}\,(q + 2r)$$

quibus adiungi poteſt valor pro z iam ante adhibitus.

30. His correctionibus pro x et y inuentis nunc demum etiam valorem pro z accuratius definire poterimus, ſiquidem hactenus in tertia aequatione, terminum z^3 neglеximus, perſpicuum autem eſt inde ad valorem z, acceſſuros inſuper terminos indicibus i^3 et $i^3 k$ affectos, ad quos inueniendos ſtatuamus

$$z = i\,\text{fin.}\,r + ikX + ikkY + ik^2Z, + i^3 ♄ + i^3 k 𝔜$$

I. Ordo

$$i^3)\quad \frac{dd♄}{mm\,dt^2} + ♄ - 3\,\mathfrak{T}\,\text{fin.}\,r - \tfrac{3}{2}\,\text{fin.}\,r^3 = 0$$

II. Ordo

$$i^3 k)\quad \frac{dd𝔜}{mm\,dt^2} + 𝔜 - 3\mathfrak{T}X - 3\,\mathfrak{U}\,\text{fin.}\,r + 12\,\mathfrak{P}\mathfrak{T}\,\text{fin.}\,r - 3PT\,\text{fin.}\,r$$
$$- \tfrac{9}{2}X\,\text{fin.}\,r^2 = 0.$$

31. En igitur duas nouas aequationes, methodo ſupra tradita reſoluendas, ac pro priori quidem charactere i^3 inſignita minores termini praebent vt ſequitur

$-3)$	$\mathfrak{T}\,\text{fin.}\,r =$	$-\tfrac{3}{8}\,\text{fin.}\,r$	$+\tfrac{1}{8}\,\text{fin.}\,3r$
$-\tfrac{3}{2})$	$\text{fin.}\,r^3 =$	$+\tfrac{3}{4}\,\text{fin.}\,r$	$-\tfrac{1}{4}\,\text{fin.}\,3r$
	$K =$	0	0

vnde hinc nulla correctio ad valorem ipſius z accedit, ſeu fit ♄ $= 0$.

32.

32. Pro altera aequatione charactere $i^3 k$ signata euolutio terminorum minorum praebet

	sin.$(q-r)$	sin.$(q+r)$	sin.$(q-3r)$	sin.$(q+3r)$
$(-3)\ \mathfrak{T}\mathrm{X} =$	$+\frac{5}{16}$	$-\frac{1}{16}$	$-\frac{5}{16}$	$-\frac{1}{16}$
$(-3)\ \mathfrak{U}$ sin. $r =$		$+\frac{1}{16}$		$-\frac{1}{16}$
$(+12)\ \mathfrak{P}\mathfrak{T}$ sin. $r =$	$+\frac{3}{16}$	$-\frac{3}{16}$	$-\frac{1}{16}$	$+\frac{1}{16}$
(-3) P T sin. $r =$	$+\frac{1}{8}$	$+\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$
$(-\frac{9}{2})$ X sin. $r^2 =$	$-\frac{5}{8}$	$+\frac{1}{8}$	$+\frac{5}{8}$	$+\frac{1}{8}$

ergo $\mathrm{K} = +\frac{15}{4}\,\text{sin.}(q-r) - \frac{51}{16}\,\text{sin.}(q+r) - \frac{3}{2}\,\text{sin.}(q-3r) + \frac{15}{16}\,\text{sin.}(q+3r)$

$\mu =$	0	$+2m$	$-2m$	$+4m$
hinc Denom.	-1	$+3$	$+3$	$+15$

consequenter

$$\zeta = -\tfrac{15}{4}\,\text{sin.}(q-r) - \tfrac{17}{16}\,\text{sin.}(q+r) - \tfrac{1}{2}\,\text{sin.}(q-3r) + \tfrac{1}{16}\,\text{sin.}(q+3r).$$

33. Colligamus nunc omnes partes, quibus coordinatae nostrae x, y et z pro casu posteriore definiuntur, ac prodibunt sequentes expressiones:

$$\begin{aligned} x = &-\tfrac{1}{2}kk + k\,\text{cos.}\,q + \tfrac{1}{2}kk\,\text{cos.}\,2q - \tfrac{3}{8}k^3\,\text{cos.}\,3q + \tfrac{67}{192}k^4\,\text{cos.}\,4q \\ &\;\tfrac{23}{64}k^4 \qquad\quad -\tfrac{17}{24}k^4 \\ &-\tfrac{1}{4}ii \qquad\quad +\tfrac{1}{4}ii\,\text{cos.}\,2r - \tfrac{1}{8}iik\,\text{cos.}(q+2r) \end{aligned}$$

$$\begin{aligned} y = &-2k\,\text{sin.}\,q + \tfrac{1}{4}kk\,\text{sin.}\,2q - \tfrac{7}{24}k^3\,\text{sin.}\,3q + \tfrac{29}{96}k^4\,\text{sin.}\,4q \\ &+\tfrac{9}{8}k^3 \qquad -\tfrac{29}{48}k^4 \\ &-\tfrac{1}{4}ii\,\text{sin.}\,2r - \tfrac{3}{8}iik\,\text{sin.}(q-2r) + \tfrac{1}{8}iik\,\text{sin.}(q+2r) \end{aligned}$$

$$\begin{aligned} z = &\,i\,\text{sin.}\,r - \tfrac{3}{2}ik\,\text{sin.}(q-r) - \tfrac{1}{2}ik\,\text{sin.}(r+q) + \tfrac{3}{8}ikk\,\text{sin.}(2q+r) \\ &\qquad\qquad\qquad +\tfrac{5}{16}ik^3 \\ &-\tfrac{15}{4}i^3k \qquad -\tfrac{17}{16}i^3k \\ &-\tfrac{1}{48}ik^3\,\text{sin.}(3q-r) - \tfrac{1}{8}ik^3\,\text{sin.}(3q+r) - \tfrac{1}{2}ik^3\,\text{sin.}(q-3r) \\ &\qquad\qquad +\tfrac{1}{16}ik^3\,\text{sin.}(q+3r) \end{aligned}$$

Verum

Verum vti iam notauimus neutiquam consultum erit motus planetarum secundum hunc posteriorem casum exigere; praecipue si eorum orbitae ad eclipticam notabiliter fuerint inclinatae, tum enim nequidem multitudo terminorum ab inclinatione *i* pendentium forte sufficeret, has autem ambages felicissime euitabimus, si motum cuiusque planetae statim ad ipsum planum in quo mouetur referamus, hocque pacto totam determinationem ad casum priorem perducamus.

DIS-

LENT

QVAE VE
RIANT, V
A R

Cum hoc
praecip
rumque ter
aptare affun
hilo haberi
exsequi lice
habentes ne
nere debean
tanto interu
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